A FORMULA FOR APPROXIMATION OF THE SATURATION VAPOR PRESSURE OVER WATER

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The use of electronic digital computers in scientific fields has made common the application of mathematical formulations of a much higher order of complexity and sophistication than was thought possible a generation ago. Yet, paradoxically, computer economics have made it urgent to seek mathematical simplifications, especially for commonly computed functions, to minimize computer time and the need for storage capacity. The kind of simplification to be sought in a formula is the use of only the elementary arithmetic operations in as few computer steps as possible.

Many approximation formulae have been developed for functions common to general mathematics. A widely used and well-worn collection of such approximations is that of Hastings [1]. Some areas of specific interest to meteorology, however, are as yet relatively untouched.

The hygrometric functions are basic and are used extensively in meteorology. Therefore, approximation formulae for these functions can be a valuable tool for meteorological computer applications. The author [2] presented such an equation for approximating relative humidity from dry bulb and dew point temperatures.

The equation presented here approximates the saturation vapor pressure over water with reasonable accuracy for temperatures in the range $-60°\text{F.} < t < 130°\text{F.}$:

$$e_s \cong (0.0041t+0.676)^8-0.000019|t+16|+0.001316 \quad (1)$$

where e_s is the saturation vapor pressure over water in inches of mercury, and t is the temperature in °F.

The equivalent equation in metric units is:

TABLE 1.—*Comparison of values of e_s (in. Hg) computed from the Goff-Gratch formula and from approximation formula (1).*

t (°F.)	Goff-Gratch	Approximation	Error
−60	0.001651	0.001649	−0.000002
−40	0.005584	0.005582	−0.000002
−20	0.01668	0.01674	+0.00006
0	0.04477	0.04462	−0.00015
20	0.10960	0.10962	+0.00002
40	0.24767	0.24813	+0.00046
60	0.52160	0.52209	+0.00049
80	1.0323	1.0319	−0.0004
100	1.9334	1.9339	+0.0005
120	3.4477	3.4625	+0.0148

$$e_s \cong 33.8639\ [\ (0.00738\ t+0.8072)^8 -0.000019|1.8t+48|+0.001316] \quad (2)$$

where e_s is in mb., and t is in °C.

If values of e_s computed by the accurate (but complicated) Goff-Gratch formula [3] are taken as a standard, the error of approximation using formula (1) is indicated in table 1. The relative error of approximation, in terms of percentage divergence from Goff-Gratch computed values, is shown in figure 1.

It may be of interest to note that the generally accepted equations, such as the Goff-Gratch formula for saturation vapor pressure over water, are themselves approximation equations in that they are the "best fit" to the known body of experimental measurements. If physical theory is ignored, and one seeks only to construct an empirical equation to fit the data reasonably well for a useful range, a simpler mathematical formula is frequently possible.

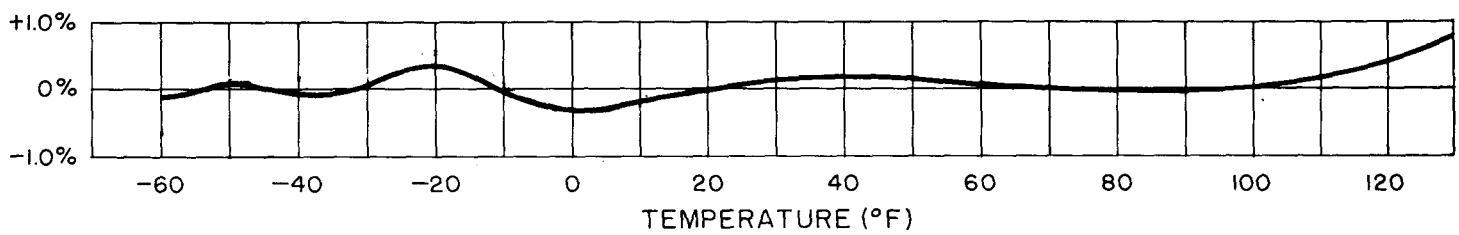

FIGURE 1—Percent error in the approximation $e_s \cong (0.0041\ t+0.676)^8-0.000019\ |t+16|+0.001316$.

REFERENCES

1. Cecil Hastings, Jr., *Approximations for Digital Computers*, Princeton University Press, Princeton, N.J. 1955, 201 pp.
2. Julius F. Bosen, "An Approximation Formula to Compute Relative Humidity from Dry Bulb and Dew Point Temperatures," *Monthly Weather Review*, vol. 86, No. 12, Dec. 1958, p. 486.
3. R. J. List (Ed.), "Smithsonian Meteorological Tables," 6th Rev. Ed., *Smithsonian Miscellaneous Collections*, vol. 114, The Smithsonian Institution, Washington, D.C., 1951, 527 pp. (p. 350).

An Extension of a Table of Absorption for Elsasser Bands

The table referred to above was published by D. Q. Wark and M. Wolk in the July 1960 issue of the *Monthly Weather Review*. Regrettably, a number of entries in the table are illegible because of faulty printing. Therefore, Dr. Wark is offering a properly printed copy of the table to any reader who needs to make use of the information therein. His address is:

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