



FIGURE 1.—Mean daily minimum temperature. City Office, Phoenix, Arizona (December 1895–February 1956).

2. STATISTICAL INVESTIGATION

Before we can investigate this possible singularity statistically, we must set up a working definition of a "singularity." For the purposes of this discussion we define a singularity as a period of higher or lower values of an element, that is superimposed on the seasonal trend curve of the element, and that exhibits a strong tendency to recur at about the same time each year. If we accept the official U. S. Weather Bureau normal daily minimum temperatures [2] as a "best estimate" of the smoothed seasonal trend of this element during the period of interest, we can investigate statistically the question of whether or not we have a singularity in terms of our definition.

The following procedure was used. The 9-day period in question (January 9–17) was bracketed by two other 9-day periods (December 31–January 8, and January 18–26) and average values in each year during each of these three 9-day periods were computed using daily minimum temperature data for the City Office for December 1895 through February 1956. These average values were used instead of individual daily values to minimize the effects of serial correlation in the data. One approach might be to compute the mean minimum temperatures during the three periods for the entire length of record, then compare them statistically by using Student's "t" test on the final averages. Throughout the period of record of the City Office, however, the instruments have been exposed at a number of different locations and elevations, making the entire series of observations non-homogeneous. The instrument exposures during the three periods in *each individual year* are comparable, however, so the method of "pairing" was used. Hypotheses are harder to prove (or disprove) by this method than by averaging the whole series, because the number of degrees of freedom in the estimate of the variance is reduced by about one-half; however, the nature of the data in this problem forces us to pair the observations.

Let us call the period December 31–January 8 "period A," the period January 9–17 "period B," and the period January 18–26 "period C;" and call the *observed* average minimum temperature in the *i*th year in periods A, B, and C, respectively, T_{ai} , T_{bi} , and T_{ci} . As a first step, averages during each year for periods A and B were paired and the algebraic difference $T_{bi} - T_{ai}$ computed, and a list of differences was made for all years. Similarly for periods B and C, differences $T_{bi} - T_{ci}$ were computed and listed for all years.

Table 2 shows the official U. S. Weather Bureau normal daily minimum temperatures at the Phoenix City Office throughout the three 9-day periods in question. Let us call \bar{T}_a , \bar{T}_b , and \bar{T}_c the average of the *normal* mean minimum temperatures in periods A, B, and C, respectively. Then,

$$\bar{T}_a = 38.00, \bar{T}_b = 37.67, \bar{T}_c = 38.33$$

and,

$$\bar{T}_b - \bar{T}_a = -0.33, \bar{T}_b - \bar{T}_c = -0.66$$

On the basis of these differences in the average of *normal* values, we would expect the *observed* average minimum temperatures T_{ai} , T_{bi} , and T_{ci} , in the *i*th year in periods A, B, and C, respectively, to satisfy the following hypotheses:

$$\lim_{n \rightarrow \infty} \left[\frac{\sum_{i=1}^n (T_{bi} - T_{ai})}{n} \right] \rightarrow -0.33$$

$$\lim_{n \rightarrow \infty} \left[\frac{\sum_{i=1}^n (T_{bi} - T_{ci})}{n} \right] \rightarrow -0.66$$

These hypotheses can be tested using Student's "t" test

by testing the equivalent hypotheses:

$$\frac{\sum_{i=1}^n T_{bt}}{n} - \frac{\sum_{i=1}^n T_{at}}{n} + 0.33 = 0$$

and

$$\frac{\sum_{i=1}^n T_{bt}}{n} - \frac{\sum_{i=1}^n T_{ct}}{n} + 0.66 = 0$$

for $n=61$, allowing $n-1$ degrees of freedom. In this case, if both hypotheses are rejected, we will have good reason to believe that the average minimum temperature is higher or lower in period B than one would expect it to be due to chance variations from the smoothed seasonal trend. Choosing a 5 percent level of significance, we reject the hypotheses if $t < -2.00$ or if $t > +2.00$.

The value of t for the set of differences between B and A values is $+3.45$, while that for the differences between B and C values is $+2.30$, indicating a strong probability that the minimum temperature values in period B are higher than one would expect them to be due to chance variations alone.

The following points should be emphasized about the above statistical treatment:

(1) The significance test was applied after inspecting the data and this selection increases the probability of getting a high value of t .

(2) The number of degrees of freedom was determined on the assumption that the years are independent. This is not strictly true because of year-to-year persistence,

TABLE 2.—Normal daily minimum temperature. City Office, Phoenix, Ariz. [2]

Date	Normal minimum (° F.)	Date	Normal minimum (° F.)
Dec. 31.....	39	Jan. 14.....	38
Jan. 1.....	38	15.....	38
2.....	38	16.....	38
3.....	38	17.....	38
4.....	38	18.....	38
5.....	38	19.....	38
6.....	38	20.....	38
7.....	38	21.....	38
8.....	37	22.....	38
9.....	37	23.....	38
10.....	37	24.....	39
11.....	37	25.....	39
12.....	38	26.....	39
13.....	38		

which has the effect of making the number of degrees of freedom somewhat less.

(3) The test of significance used above applies only to random samples from a single, normally distributed population. Because of the nature of the meteorological problem, we cannot assume that weather phenomena behave in random fashion and that the next 61 years will belong to the same population as the past 61 years. Hence, the above test indicates only that the singularity has occurred during the past 61 years, and the question of whether or not it will occur in the future can only be answered by testing independent data; i. e., future records.

3. CONCLUDING REMARKS

The possible singularity discussed above is much more evident in the minimum temperature curves than in the mean or maximum temperature curves for Phoenix. The maximum temperature curves show little evidence of it (mean temperatures are computed from the formula: $\text{mean temperature} = \left(\frac{\text{max. temp.} + \text{min. temp.}}{2} \right)$). This is probably due to the fact that solar heating during the middle of the day plays a major role in determining the maximum temperature at this station and tends to make day-to-day maximum temperature changes less responsive to the subtle changes in the general circulation which Wahl showed accompanied the singularities at east coast stations. The Phoenix singularity has occurred roughly 4 days earlier than the one at Columbia, Mo., and about 7 days earlier than the east coast singularity [1]. One must be careful in concluding that the eastward propagation of some large-scale feature of the atmosphere is involved, but the possibility seems interesting enough to warrant further study.

REFERENCES

1. E. W. Wahl, "The January Thaw in New England (An Example of a Weather Singularity)", *Bulletin of the American Meteorological Society*, vol. 33, No. 9, Nov. 1952, pp. 380-386.
2. U. S. Weather Bureau, *Daily Temperature, Degree Day, and Precipitation Normals, Phoenix, Arizona CO*, (Based on, or adjusted to, the 1921-1950 period), Washington, D. C.