A NOMOGRAM TO DETERMINE MONTHLY POTENTIAL EVAPOTRANSPIRATION

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ABSTRACT

A nomogram for the solution of Thornthwaite's empirical relationships among mean monthly temperature, latitude, and potential evapotranspiration is constructed. In the form of alignment charts, the nomogram is simple to use and has the added advantages of the temperature scale being labeled in both degrees Celsius and degrees Fahrenheit and of the potential evapotranspiration scale being labeled in both centimeters and inches.

1. INTRODUCTION

Potential evapotranspiration is defined as the amount of water that would be evaporated from the ground or be transpired by the vegetation if there were no shortage of water at any time during the period of observation. The concept has proven to be a valuable tool in the classification of climates as well as in the computation of water balances [1], [2], [3].

Thornthwaite [4] found an empirical relationship among the mean monthly temperatures, the latitude of the location, and the potential evapotranspiration. The determination of the latter involves the use of two tables and a nomogram and is rather time consuming, especially if series of years for different locations have to be computed.

The nomogram presented here was the result of the desire to have a fast, accurate, and convenient method to determine the potential evapotranspiration according to Thornthwaite's method. The manuscript was ready for publication when the author's attention was drawn to a recent article by Palmer and Havens [5], dealing with the same subject. The alignment charts described in the present paper are easier to use and either alone or in combination with the graphical technique of Palmer and Havens will expedite the computations of potential evapotranspiration.

2. THE THORNTHWAITE FORMULA

The computation of potential evapotranspiration according to Thornthwaite's [4] formula consists of three steps. First, the heat index (J) has to be found for the year and consists of the summation of 12 monthly indices (i), where $i=(t/5)^{1.514}$ and t=the mean temperature of the month in °C. Second, the unadjusted potential evapotranspiration (E) must be computed according to the formula:

$$E=1.6\ (10t/J)^a \qquad (1)$$

in which

$$a=0.000000675\,J^3-0.0000771\,J^2+0.01792\,J+0.49239. \qquad (2)$$

Third, E must be multiplied by a factor for daylight (f), adjusting E to the mean possible duration of sunlight in each month of the year. This factor depends on the latitude of the location.

The heat index (J) can be computed from Thornthwaite's [4] table IV, but it is simpler to construct a conversion scale where both ° F. and ° C. are inserted. Such a scale is presented at the top of figure 1. By adding the i's for each of the twelve months of the year J is determined. This completes the first step.

The second step involves the use of equation (1) which, if we take logarithms and put $a=F(J)$, becomes:

$$\log E-\log 1.6=F(J)\ \log 10t-F(J)\ \log J. \qquad (3)$$

This equation can be simplified to:

$$\log E=F(J)\ \log t'-F'(J) \qquad (4)$$

by ignoring the constant log 1.6, putting $10t=t'$ and $F(J)$ $\log J=F'(J)$. Equation (4) in determinant form is:

$$\begin{vmatrix} -\log E & 1 & 0 \\ \log t' & 0 & 1 \\ F'(J) & 1 & F(J) \end{vmatrix} =0. \qquad (5)$$

Finally the potential evapotranspiration (PE) is calculated according to:

$$Ef=(\text{PE}) \qquad (6)$$

which can be written:

$$\begin{vmatrix} -m & km\log E & 1 \\ n & kn\log f & 1 \\ 0 & kmn\log(\text{PE}) & m+n \end{vmatrix} =0 \qquad (7)$$

with k, m, and n constants.

3. CONSTRUCTING THE NOMOGRAM

For the purpose of constructing the nomogram the determinant (5) must be multiplied by the non-zero determinant (see [6])

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$$\begin{vmatrix} p_1 & q_1 & r_1 \\ p_2 & q_2 & r_2 \\ p_3 & q_3 & r_3 \end{vmatrix} \neq 0 \qquad (8)$$

which gives:

$$\begin{vmatrix} -p_1 \log E+p_2 & -q_1 \log E+q_2 & -r_1 \log E+r_2 \\ p_1 \log t'+p_3 & q_1 \log t'+q_3 & r_1 \log t'+r_3 \\ p_1F'(J)+p_2+p_3F(J) & q_1F'(J)+q_2+q_3F(J) & r_1F'(J)+r_2+r_3F(J) \end{vmatrix} =0. \qquad (9)$$

Inspection shows that it is permissible to put $p_2=p_3=q_1=q_3=r_1=0$, but p_1, q_2, r_2, and r_3 must be non-zero. Therefore:

$$\begin{vmatrix} -p_1 \log E & q_2 & r_2 \\ p_1 \log t' & 0 & r_3 \\ p_1 F'(J) & q_2 & r_2+r_3F(J) \end{vmatrix} =0 \qquad (10)$$

in which r_2 may be equal to r_3. By multiplying each row by the reciprocal of its element in the third column, ignoring subscripts, rearranging, and inserting the constant log 1.6, we find the parametric equations:

$$\begin{aligned} y_1 &= (p/r) \log 10t & x_1 &= 0 \\ y_2 &= -(p/r)\ (\log E - \log 1.6) & x_2 &= q/r \\ y_3 &= (p/r)\frac{F'(J)}{1+F(J)} & x_3 &= (q/r)\frac{1}{1+F(J)}. \end{aligned} \qquad (11)$$

The nomogram can now be constructed and consists of two parallel logarithmic scales of equal modulus running in opposite direction and a third line not parallel to these. The slanted line is not straight because $F(J)$ is a polynomial. However, within the range of J's for which Thornthwaite's formula has proven to give adequate results, the line is straight, except for slight curves below $J=10$ and above $J=160$. Below $J=5$ the line approaches zero asymptotically toward $x_3=(1/1.49239)(q/r)$ since by equation (2), $F(J)=0.49239$ when $J=0$; and above $J=160$ the line curves similarly toward $x_1=0$. Between $J=10$ and $J=160$ the slope of the J line is determined by $t=26.5°$ C. on the t scale and $E=13.5$ cm. on the E scale. At this temperature E is 13.5 cm. regardless of index, For $E=1.6$ cm., equation (3) becomes:

$$F(J)\ \log 10t = F(J)\ \log J \qquad (12)$$

which, for $F(J)\neq 0$, gives $\log J = \log 10t$. The t scale can therefore be used to find the J's on the sloping line by connecting 1.6 cm. on the E scale with the different values (in cm.) on the t scale.

For temperatures above 26.5° C. (79.7° F.) values of E are given by Thornthwaite [4]. These are added to the E scale above 13.5 as indicated in figure 1.

A convenient size of nomogram is obtained by making the distance between two vertical scales $q/r=17$ cm. and the length of one logarithmic cycle on the vertical lines $p/r=12.5$ cm.

In order to find the actual potential evapotranspiration a second nomogram is added to the first. Determinant (7) becomes with moduli 12.5 for the E line and 10 for the PE line:

$$\begin{vmatrix} -2.5 & 12.5 \log E & 1 \\ 10 & 50 \log f & 1 \\ 0 & 10 \log \text{PE} & 1 \end{vmatrix} =0 \qquad (13)$$

so the f line has a 50-cm. cycle, while the distance between the E and PE lines is 2.5 cm. and between the PE and f lines 10 cm. Since the E line now functions only as a pivot line it does not need to be marked. Markings for °F. and inches can be added as desired to the t and PE lines.

4. ACCURACY AND PRACTICAL USE OF THE NOMOGRAM

The nomogram gives results at least as accurate as those obtained by the old method and when used in routine procedures has proven to be much faster, especially if one mean J value is established for any one station and a series of years of record is being processed. The deviations occurring in consequence of the use of a mean J value are negligible.

Speed and accuracy both are increased even more if the nomogram is drawn on "hard board" or similar material. It can then be equipped with a system of pins and slots (for an illustration, see [7]). The J and the pivot line should be grooved. A pin is fastened at the appropriate place on the J line. A slotted transparent arm is made to slide over this pin and is connected via a sliding pin on the pivot line with another transparent arm. When the left arm is placed on the appropriate place on the t line, the pin on the pivot line slides automatically into the right position and the right arm can be placed immediately on the proper place along the f line to obtain the PE.

The f values depend on the month of the year and the latitude of the location. They must be obtained from tables as published by Thornthwaite ([4], p. 93) or Thornthwaite and Mather ([1], p. 98). Although it would be possible to arrange these tables in the form of a graph, it would make a nomogram unnecessarily cluttered and awkward to handle.

This nomogram, as any nomogram, is of principal advantage if series of computations have to be made, such as series of years of monthly PE's at one station or mean monthly PE's at different stations. (In the latter case the stations should be grouped by latitude.)

It is, therefore, convenient to mark the f values belonging to the latitude of the station, or stations, before taking a series of readings. As an example the mean temperature for May in Bridgeton, N.J., is 17.5° C.; $J=58.3$; f, taken from Thornthwaite's table, is 1.23 at latitude 39° N.; and the PE is read to be 9.2 cm., as illustrated in figure 1.

APPENDIX

Table 1, as taken from Thornthwaite [4] presents the f values for latitudes 50° N. to 50° S. for the 12 months of

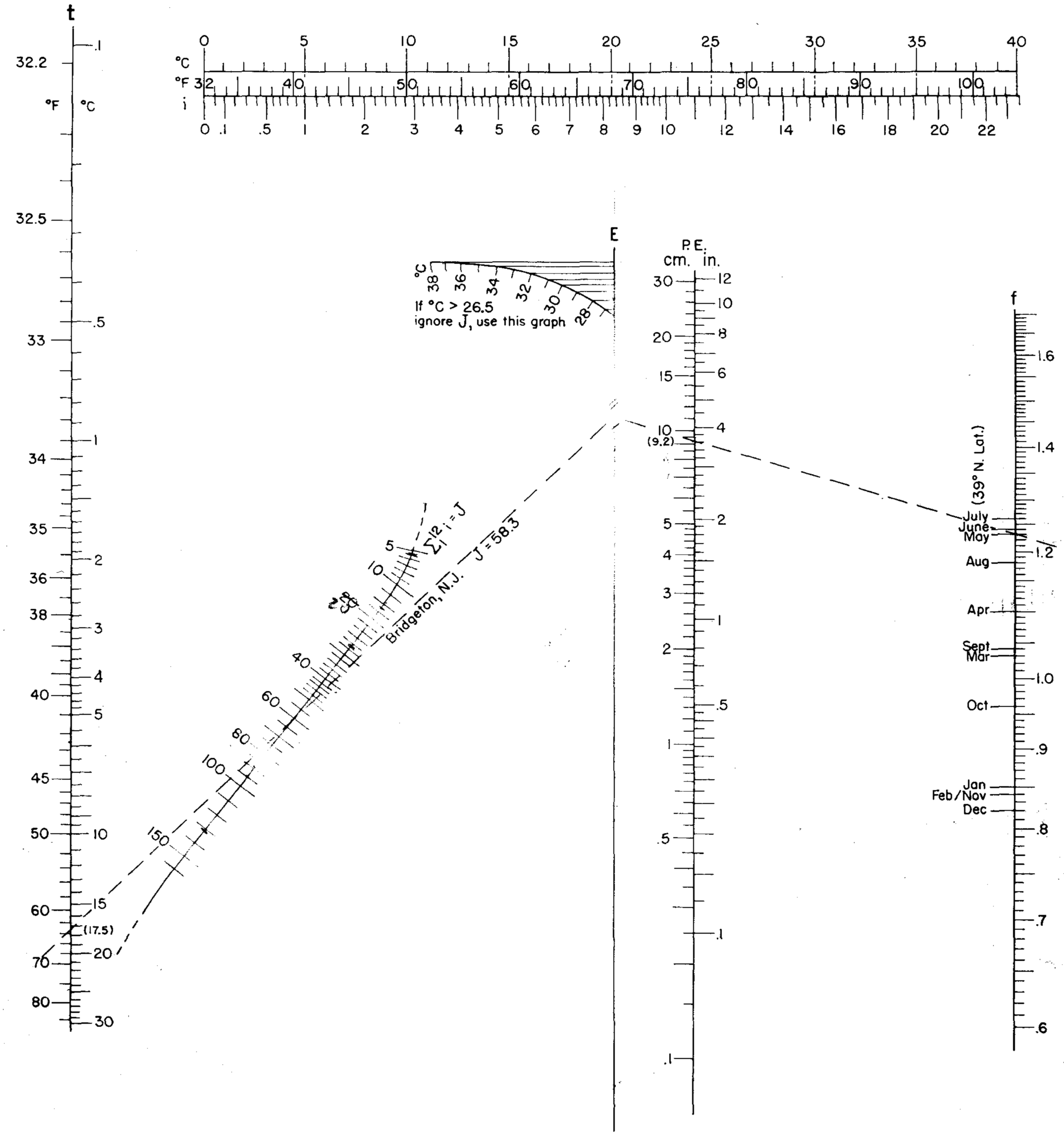

FIGURE 1.—A nomogram to determine the potential evapotranspiration according to Thornthwaite's formula.

TABLE 1.—*Mean possible duration of sunlight in the Northern and Southern Hemispheres expressed in units of 30 days of 12 hours each. (After Thornthwaite [4].)*

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
°N. Lat.												
0	1.04	.94	1.04	1.01	1.04	1.01	1.04	1.04	1.01	1.04	1.01	1.04
5	1.02	.93	1.03	1.02	1.06	1.03	1.06	1.05	1.01	1.03	.99	1.02
10	1.00	.91	1.03	1.03	1.08	1.06	1.08	1.07	1.02	1.02	.98	.99
15	.97	.91	1.03	1.04	1.11	1.08	1.12	1.08	1.02	1.01	.95	.97
20	.95	.90	1.03	1.05	1.13	1.11	1.14	1.11	1.02	1.00	.93	.94
25	.93	.89	1.03	1.06	1.15	1.14	1.17	1.12	1.02	.99	.91	.91
26	.92	.88	1.03	1.06	1.15	1.15	1.17	1.12	1.02	.99	.91	.91
27	.92	.88	1.03	1.07	1.16	1.15	1.18	1.13	1.02	.99	.90	.90
28	.91	.88	1.03	1.07	1.16	1.16	1.18	1.13	1.02	.98	.90	.90
29	.91	.87	1.03	1.07	1.17	1.16	1.19	1.13	1.03	.98	.90	.89
30	.90	.87	1.03	1.08	1.18	1.17	1.20	1.14	1.03	.98	.89	.88
31	.90	.87	1.03	1.08	1.18	1.18	1.20	1.14	1.03	.98	.89	.88
32	.89	.86	1.03	1.08	1.19	1.19	1.21	1.15	1.03	.98	.88	.87
33	.88	.86	1.03	1.09	1.19	1.20	1.22	1.15	1.03	.97	.88	.86
34	.88	.85	1.03	1.09	1.20	1.20	1.22	1.16	1.03	.97	.87	.86
35	.87	.85	1.03	1.09	1.21	1.21	1.23	1.16	1.03	.97	.86	.85
36	.87	.85	1.03	1.10	1.21	1.22	1.24	1.16	1.03	.97	.86	.84
37	.86	.84	1.03	1.10	1.22	1.23	1.25	1.17	1.03	.97	.85	.83
38	.85	.84	1.03	1.10	1.23	1.24	1.25	1.17	1.04	.96	.84	.83
39	.85	.84	1.03	1.11	1.23	1.24	1.26	1.18	1.04	.96	.84	.82
40	.84	.83	1.03	1.11	1.24	1.25	1.27	1.18	1.04	.96	.83	.81
41	.83	.83	1.03	1.11	1.25	1.26	1.27	1.19	1.04	.96	.82	.80
42	.82	.83	1.03	1.12	1.26	1.27	1.28	1.19	1.04	.95	.82	.79
43	.81	.82	1.02	1.12	1.26	1.28	1.29	1.20	1.04	.95	.81	.77
44	.81	.82	1.02	1.13	1.27	1.29	1.30	1.20	1.04	.95	.80	.76
45	.80	.81	1.02	1.13	1.28	1.29	1.31	1.21	1.04	.94	.79	.75
46	.79	.81	1.02	1.13	1.29	1.31	1.32	1.22	1.04	.94	.79	.74
47	.77	.80	1.02	1.14	1.30	1.32	1.33	1.22	1.04	.93	.78	.73
48	.76	.80	1.02	1.14	1.31	1.33	1.34	1.23	1.05	.93	.77	.72
49	.75	.79	1.02	1.14	1.32	1.34	1.35	1.24	1.05	.93	.76	.71
50	.74	.78	1.02	1.15	1.33	1.36	1.37	1.25	1.06	.92	.76	.70
° S. Lat.												
5	1.06	.95	1.04	1.00	1.02	.99	1.02	1.03	1.00	1.05	1.03	1.06
10	1.08	.97	1.05	.99	1.01	.96	1.00	1.01	1.00	1.06	1.05	1.10
15	1.12	.98	1.05	.98	.98	.94	.97	1.00	1.00	1.07	1.07	1.12
20	1.14	1.00	1.05	.97	.96	.91	.95	.99	1.00	1.08	1.09	1.15
25	1.17	1.01	1.05	.96	.94	.88	.93	.98	1.00	1.10	1.11	1.18
30	1.20	1.03	1.06	.95	.92	.85	.90	.96	1.00	1.12	1.14	1.21
35	1.23	1.04	1.06	.94	.89	.82	.87	.94	1.00	1.13	1.17	1.25
40	1.27	1.06	1.07	.93	.86	.78	.84	.92	1.00	1.15	1.20	1.29
42	1.28	1.07	1.07	.92	.85	.76	.82	.92	1.00	1.16	1.22	1.31
44	1.30	1.08	1.07	.92	.83	.74	.81	.91	.99	1.17	1.23	1.33
46	1.32	1.10	1.07	.91	.82	.72	.79	.90	.99	1.17	1.25	1.35
48	1.34	1.11	1.08	.90	.80	.70	.76	.89	.99	1.18	1.27	1.37
50	1.37	1.12	1.08	.89	.77	.67	.74	.88	.99	1.19	1.29	1.41

the year. For latitudes farther north or south the *f*'s of the 50th parallel should be used, presumably because the effect of longer daylight is offset by the diminishing effectiveness of radiation.

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