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EFFECTS OF HIGHER ORDER ADVECTION TECHNIQUES ON A NUMERICAL CLOUD MODEL

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ABSTRACT

Crowley's second- and fourth-order nonconservative techniques for treating the advection term in numerical solutions of the hydrothermodynamic equations are tested on a model of cumulus cloud growth over mountains. The results are compared with previous integrations using upstream differencing, a first-order method.

All three methods give comparable results on a symmetric case. However, numerical damping which is a characteristic of the upstream-differencing method is considerably reduced in the Crowley techniques, as evidenced by curves of kinetic energy changes and sources and sinks for this energy.

In an ambient wind case the Crowley second-order method and the upstream-differencing method give comparable results if the eddy diffusion coefficients used in the Crowley method are twice as large as those used in the upstream-differencing method.

The results illustrate that the value of the eddy coefficients is crucial for the formation of the numerical clouds in the ambient wind model. Coefficients that are too small or too large lead to weak circulation cells created by the heated slopes and insufficient penetration into the upper flow region to form a cloud.

The smaller numerical diffusion of the Crowley second-order method is illustrated in a test of the numerical ffusion of rainwater in the cloud model. Upstream differencing causes rainwater contents downwind an order of agnitude larger than those that occur using the Crowley technique for advection.

1. INTRODUCTION

Crowley (1968) presented results of numerical advection experiments using several higher order finite-difference techniques for the advection term of the hydrodynamic equations that commonly occur in fluid dynamics problems. He applied these techniques to simplified problems and stated that "The final evaluation of the usefulness of these higher order advection schemes can be made only after they are tested in more realistic hydrodynamic models."

We have applied his second- and fourth-order advection nonconservative techniques to a problem of upslope wind and cumulus initiation over an infinitely long mountain ridge (Orville 1968*a*). Previous work on this cumulus model had been done using upstream differencing, a first-order method, which Crowley showed to be quite dissipative (see also Roberts and Weiss 1966; and Molenkamp 1968). An advantage of the strong dissipation (which occurs principally in the shorter wave lengths) is the great stability of the numerical method. Disadvantages are the "indefiniteness" of the diffusion (since the mixing coefficient is dependent upon the grid spacing and fluid velocity), the phase error of the wave motion, and the fact that any field that is advected is also diffused—a situation that numerical cloud modelers may not want (particularly with regard to modeling the precipitation process). One of the advantages of using Crowley's second- or fourth-order schemes would be that most of the numerical diffusion would be eliminated.

Other schemes may work as efficiently. Arakawa's (1966) technique was used by Nickerson (1965) successfully but was not used as successfully by Molenkamp (1968) in small-scale numerical problems. Bryan's box technique (1966) may also be applicable. The important point to emphasize is that the techniques allow the diffusion to be represented explicitly in Fickian diffusion or other diffusion representations and not implicitly in the advection terms.

2. SUMMARY OF EQUATIONS

The derivations of the equations have been exhibited elsewhere (Orville 1965, Liu and Orville 1969). The equations will be briefly summarized below. They are applied to a two-dimensional region (x, z) in the atmosphere bounded below by an idealized mountain-valley surface and above by an inflexible boundary—no motion is allowed across the upper level. Air is allowed to flow across the side boundaries.

A vorticity equation is used. It is

$$\frac{\partial \eta}{\partial t}=-\mathbf{V}\cdot\nabla\eta+\frac{g}{\Theta}\frac{\partial\theta'}{\partial x}+.61\,g\frac{\partial r'}{\partial x}-g\frac{\partial l}{\partial x}+K\nabla^2\eta \qquad (1)$$

where η is vorticity, $\mathbf{V}$ is two-dimensional velocity with horizontal component u and vertical component w, Θ is the potential temperature of a reference state, θ' is the deviation of potential temperature from this state, r' is the deviation of water vapor mixing ratio, l is total cloud and rainwater content, K is the eddy coefficient, assumed constant and the same for momentum, heat, and water substance, g is the acceleration of gravity, x, z, and t are the horizontal, vertical, and time coordinates, respectively, and ∇^2 is a two-dimensional Laplacian.

Incompressible flow is assumed so that the continuity equation is

$$\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}=0 \tag{2}$$

which allows a streamfunction, ψ, to be defined,

$$u=-\frac{\partial \psi}{\partial z} \text{ and } w=\frac{\partial \psi}{\partial x},$$

and leads to

$$\nabla^2\psi=\eta. \tag{3}$$

The thermodynamic energy equation is

$$\frac{\partial \phi'}{\partial t}=-\mathbf{V}\cdot\nabla\Phi'+K\nabla^2\Phi' \tag{4}$$

with

$$\Phi'=\frac{\theta'}{\Theta}+\frac{Lr}{c_pT_{00}} \text{ unsaturated}=\frac{\theta'}{\Theta}+\frac{Lr_s}{c_pT_{00}} \text{ saturated}$$

where ϕ' is related to entropy, c_p is specific heat at constant pressure, r and r_s are water vapor mixing ratios, r_s the saturated value, and T_{00} is the Kelvin temperature at the base of the reference atmosphere.

The equations for conservation of water are

$$\frac{\partial q}{\partial t}=-\mathbf{V}\cdot\nabla q+K\nabla^2 q-P_r \tag{5}$$

and

$$\frac{\partial Q}{\partial t}=-\mathbf{V}\cdot\nabla Q+K\nabla^2 q+\frac{1}{\rho}\frac{\partial}{\partial z}(\rho V_t l_r) \tag{6}$$

with $q=r+l_c$ and $Q=r+l_c+l_r$ where q is the total cloud water substance, Q is the total water substance, l_c is the cloud water content, l_r is rainwater content, ρ is density of air, V_t is the terminal velocity of the mean size raindrops of the Marshall-Palmer (1948) distribution which the rainwater content is assumed to follow, and P_r is a production term that allows for evaporation of rainwater, conversion of cloud water to rainwater, and collection of cloud water by rainwater (Liu and Orville 1969). This production term is similar to Kessler's (1967). It should be noted in equation (6) that no explicit mixing of rainwater is modeled.

The initial stability in the atmosphere is represented by a potential temperature increase with height of 2.8°K km^{-1}. The water vapor decreases from a base value of 11 or 11.5 gm kg^{-1} at a rate of 2 gm kg^{-1} km^{-1}. Heating and evaporation at the slopes and valleys is treated in the same way as in Orville (1965).

3. SYMMETRIC CASE

The principal results of all of the tests are summarized in table 1. The higher order advection schemes first were applied to a symmetric model of cumulus growth over mountains (case 5 of Orville 1965, Orville 1968*b*). One method of analyzing the computational methods involves the comparison of the changes of kinetic energy and the sources and sinks of that energy. Equation (7) (Orville 1968*a*),

$$\frac{\partial \mathrm{KE}}{\partial t}=-\left[\frac{\psi g}{\Theta}\frac{\partial \theta'}{\partial x}+.61\psi g\frac{\partial r}{\partial x}-\psi g\frac{\partial l}{\partial x}+\psi K\nabla^2\eta\right], \tag{7}$$

shows that kinetic energy is changed by horizontal gradients of potential temperature, water vapor and liquid water, and by the diffusion of vorticity. The brackets $[\ ]=\int_0^H\int_0^L (\ )\,dx\,dz$ indicate an integration over the entire x, z plane. In the above, KE is kinetic energy and is given by the double summation of the squares of velocities over all grid points (i, j), that is,

$$\mathrm{KE}=\sum_j\sum_i[(u^2_{i,j}+w^2_{i,j})/2]\Delta x\Delta z$$

where u is the horizontal and w the vertical wind speed, Δx and Δz are the grid intervals. The right side of equation (7) will hereafter be referred to as GRADT. How well these two sides of the equation agree after calculation of the various quantities in the numerical integrations of the basic equations gives some idea of the effect of the truncation error on the solution.

TABLE 1.—*Various cases, initial parameters, and principal results of numerical integrations*

Case	Method	K(m^2 sec^{-1})	Δx, Δz (m)	Initial wind	Cloud formation (min)	Principal results
			SYMMETRIC MODEL			
1	UD	40	50	Zero	132	Cloud initiation and development is similar in all 3 cases
2	CM-2	"	"	"	122	
3	CM-4	"	"	"	123	
			AMBIENT WIND			
4	UD	40	100	Ambient wind, constant shear	100	Cloud initiation and development similar in only 2 cases
5	CM-2	"	"	" " "	None	
6	CM-2	40+K'	"	" " "	None	
7	CM-2	80	"	" " "	112.500	
8	CM-2	160	"	" " "	None	
			RAIN EVAPORATION SET TO ZERO			
9	CM-2	80	100	Constant shear	112.500	Less numerical diffusion of rain in CM-2, case 9
10	UD	10	"	" " "	112.625	

UD, upstream differencing
CM-2, Crowley second-order method
CM-4, Crowley fourth-order method

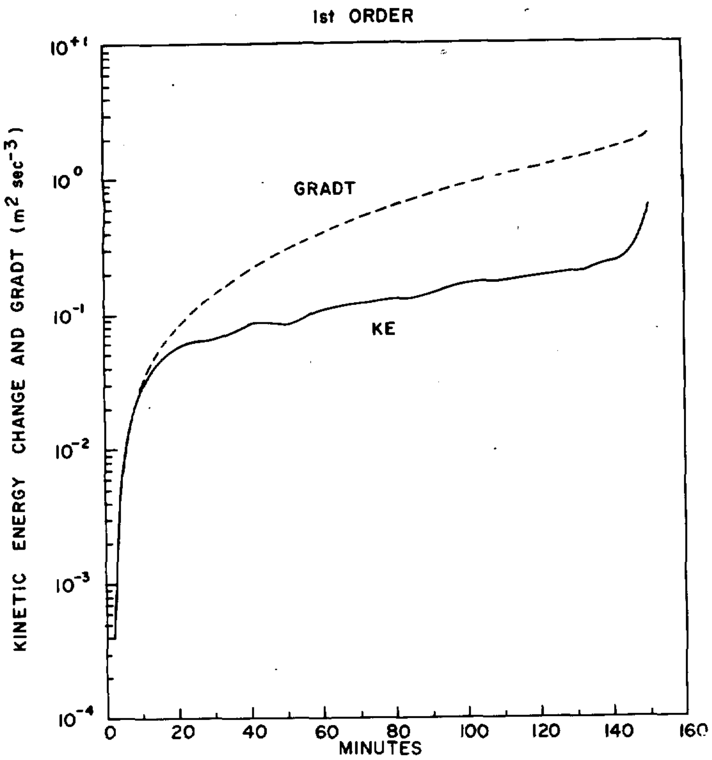

FIGURE 1.—Changes of kinetic energy and GRADT versus time *for the upstream-differencing method. The ordinate has units of* m^{-2} sec^{-3}. The abscissa is time in minutes. The solid curve refers to changes in kinetic energy, and the dashed curve is the change for GRADT, the sources and sinks of this energy.

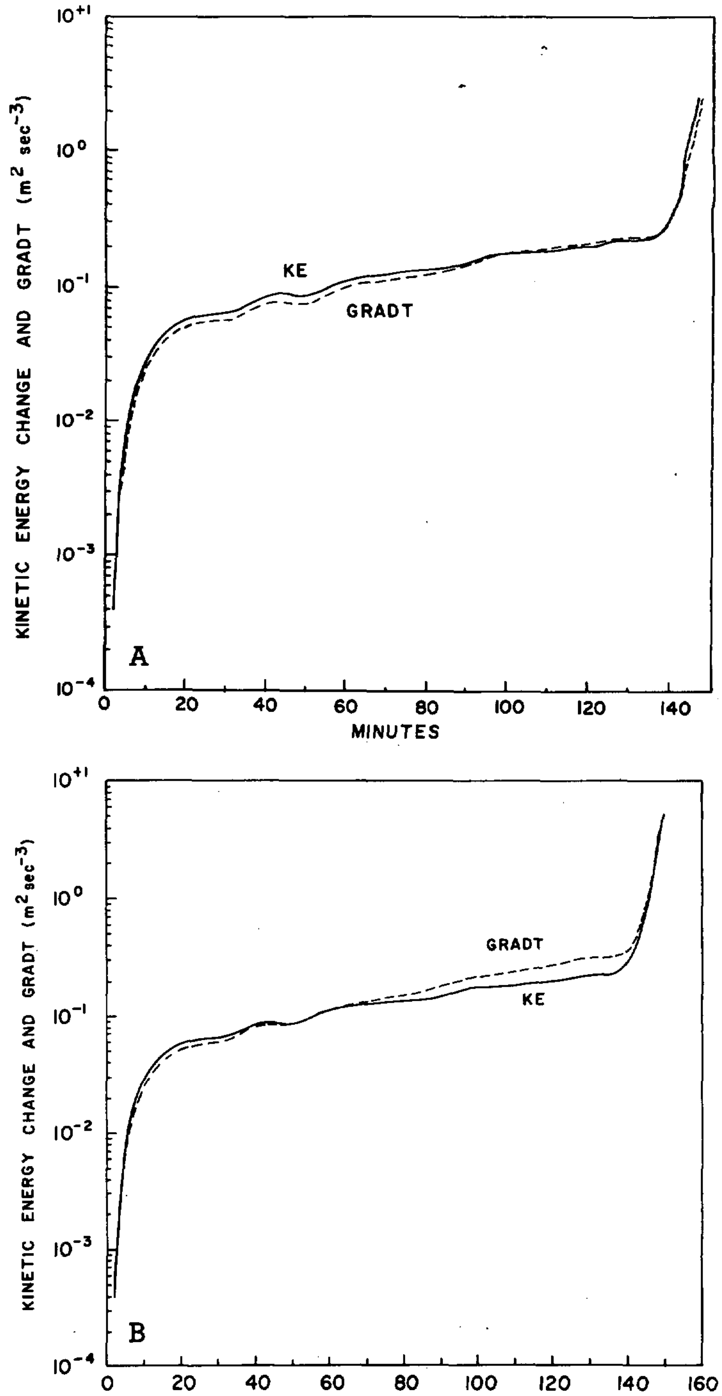

FIGURE 2.—Same as figure 1 except (A) is for Crowley's second-order method and (B) represents curves for kinetic energy changes and GRADT for Crowley's fourth-order method.

Figure 1 graphs this change in kinetic energy versus the source and sink terms for upstream differencing. The discrepancy in the curves is due to the fact that the numerical diffusion of the upstream-differencing term is not represented on the right side of equation (7). Implicit numerical diffusion equivalent to Fickian diffusion with a $K=35$ m^2 sec^{-1} is present (Molenkamp 1968). If we assume a K' as the eddy diffusion coefficient for the implicit mixing, then a $\psi K'\nabla^2\eta$ term would tend to decrease the absolute magnitude of GRADT. This occurs because the implicit diffusion term is of opposite sign to the $\frac{\psi g}{\Theta}\frac{\partial\theta'}{\partial x}$ term which is the largest term in equation (7) in general. Thus the discrepancy is in the right direction.

Figures 2A,B show the results of the second-order and fourth-order advection schemes of Crowley applied to the advective form of the equations. The kinetic energy is conserved at all times within a few percent in the second-order scheme, which appears to be best in this respect.

In addition, the results concerning the evolution of stream function, potential temperature, water vapor, cloud liquid water, and the cloud formation are at all times quite similar to those for upstream differencing except that less damping is evident. This is evidenced by the earlier appearance of cloud in the higher order schemes, at approximately 123 min versus 131 min in the first-order method, as seen in figure 3, a graph of maximum liquid water content versus time for the three integrations. Earlier cloud formation correlating with the smaller diffusion has been characteristic of the symmetric models using upstream differencing (Orville 1968*b*).

4. AMBIENT WIND CASE

These results of the higher order techniques applied to the symmetric model have been quite satisfying. They

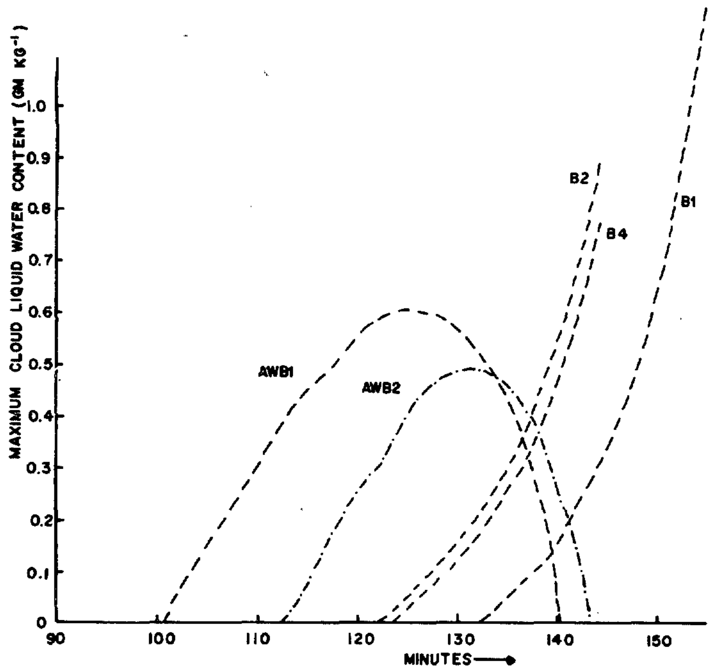

FIGURE 3.—Variations with time of maximum cloud liquid water content. B1, B2, and B4 refer to upstream differencing, Crowley's second-order and Crowley's fourth-order method, respectively. AWB1 and AWB2 refer to upstream differencing and Crowley's second-order method in the ambient-wind case.

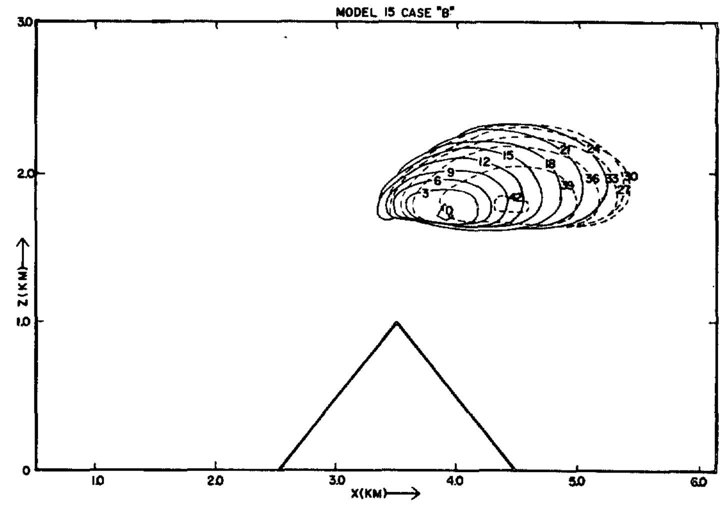

FIGURE 4.—Outline of the numerical cloud showing its life cycle. The curves are given for every 3 min during the lifetime of the cloud. The solid curves are for its growth period, and the dashed curves are for its dissipation. The cloud outlines are for all regions in the model where 100 percent relative humidity and liquid water are present.

may be used to establish the fact that the numerical diffusion is reduced considerably in the higher order schemes applied to this particular numerical model. This is important to establish since the conservation equation for kinetic energy is not so easily derived in the ambient wind models, principally because mass inflow is not necessarily matched by mass outflow through the sides and the influence of the mountain as an obstacle to the flow is difficult to account for energetically.

We have applied the second-order advection technique to an ambient wind model, one with constant wind shear initially from the mountain peak to the top of the grid (case B of Orville 1968*a*). A characteristic of this ambient wind model with upstream differencing is that a cloud forms at 100 min of integration time, grows, and dissipates in place in about 40 min as shown in figure 4. When the second-order advection scheme of Crowley was applied to the same model (in particular, $K=40$ m^2 sec^{-1}), no cloud formed. This was disconcerting, so attempts were made to attain correspondence between the models by varying the eddy coefficient in the second-order method.

Since the numerical damping of the upstream differencing has been eliminated by the second-order scheme, the total diffusion is less in the second-order scheme than in the first-order method. An attempt was made to model the numerical damping in an explicit fashion in the second-order scheme by defining a K' as

$$K'=(1/2)c[1-c(\Delta t/\Delta s)]\Delta s$$

where K' is the numerical eddy coefficient to be multiplied times the second derivative of the diffused field and is variable from grid point to grid point, c is the absolute magnitude of the u or w velocity, Δs represents Δx or Δz, the space increments in the model. With this simulation the numerical model acted more like the second-order scheme than the first-order scheme, and hence using this technique to obtain correspondence between the two models was not successful.

It is possible to increase the diffusion in the second-order model by increasing K in the Fickian diffusion terms (at the mountain slopes and valley surface as well as in the main flow region). With K increased by a factor of 4 over the entire region of integration, still no cloud forms in the model. The results are smoother than with the $K=40$ m^2 sec^{-1} case, as shown in figures 5 and 6 for $K=40$ and 160 m^2 sec^{-1}, respectively.

It is not until the eddy coefficient is increased by a factor of 2 in all areas of the model that a solution comparable to the first-order scheme emerges. A cloud forms in 112 min and grows and dissipates *in situ* downwind of the mountain. Its entire life history is quite similar to that for upstream differencing. The solution for $K=80$ m^2 sec^{-1} at 126 min of model time is shown in figure 7. Figure 8 shows the cloud growth curve for this case, the cloud lasting for about 32 min.

These solutions indicate that the upstream-differencing method and the second-order advection technique of Crowley give comparable solutions if the Crowley method uses a K-value increased by a factor of 2 over that of the upstream differencing that had K's of 40 m^2 sec^{-1}.

These results also indicate the crucial importance that mixing has on the initiation of cumulus clouds over

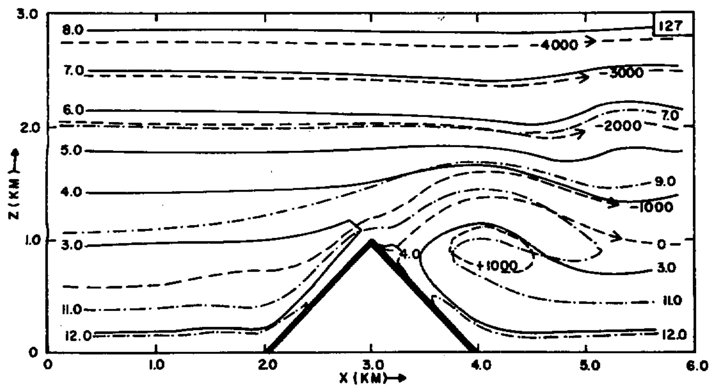

FIGURE 5.—Fields of motion, potential temperature, and water vapor at 127 min for an ambient-wind case using Crowley's second-order method. The eddy coefficient $K=40$ m² sec⁻¹. The dashed lines are streamlines in m² sec⁻¹, the solid lines are potential temperature deviations in degrees Kelvin or Celsius from the base state of 296°K, and the dot-dashed lines are isohumes in gm kg⁻¹. Real time in minutes is in the upper right-hand corner. No cloud is initiated in this case.

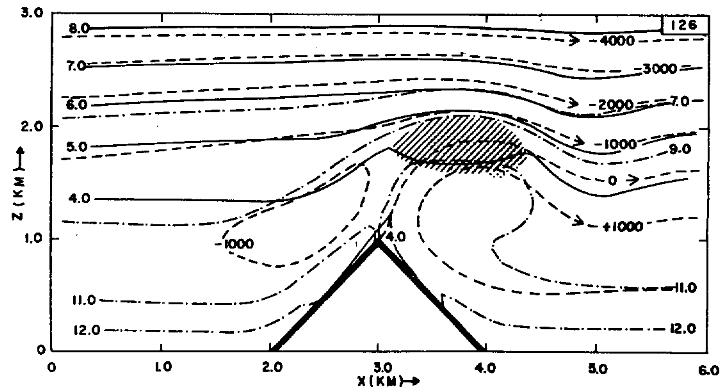

FIGURE 7.—Fields of motion, potential temperature, and water vapor at 126 min in an ambient-wind case using Crowley's second-order method with an eddy coefficient $K=80$ m² sec⁻¹. The lines have the same significance as in the previous two figures with the addition that the slanted line area depicts cloud water content. The dots within the slanted line area depict rainwater content.

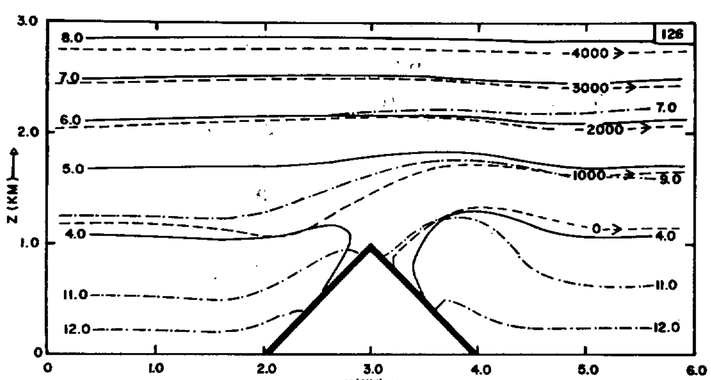

FIGURE 6.—Fields of motion, potential temperature, and water vapor at 126 min in an ambient-wind case using Crowley's second-order method with an eddy coefficient $K=160$ m² sec⁻¹. Significance of the isolines is the same as in figure 5. No cloud is initiated in this model.

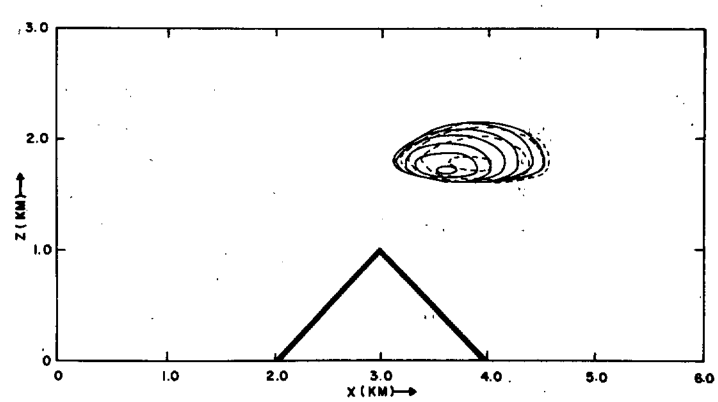

FIGURE 8.—Life cycle of the cloud shown in figure 7 is depicted here. The outlines are for every 3 min during the cloud life cycle with the solid lines indicating growth and the dashed lines indicating dissipation.

mountains. If the eddy diffusion coefficient is too small ($K=40$ m² sec⁻¹, CM–2) not enough heat and moisture are diffused from the slopes to create slope circulations of great enough strength to penetrate significantly the horizontal winds aloft. If the Fickian mixing is too large ($K=160$ m² sec⁻¹, CM–2), only small horizontal gradients in temperature and vapor are created which in turn lead to only weak slope circulations. An intermediate value of turbulent diffusion ($K=80$ m² sec⁻¹, CM–2 and $K=40$ m² sec⁻¹, UD) leads to the establishment of relatively large horizontal gradients and strong slope circulations that are able to penetrate into the environmental flow aloft and reach the condensation level. Upstream differencing aids this process by diffusing more strongly in the downwind direction (upward); hence an explicit diffusion coefficient of 40 m² sec⁻¹ with the UD method gives equivalent results to the CM–2 method with $K=80$ m² sec⁻¹.

5. NUMERICAL DIFFUSION OF RAINWATER

In section 1 it was mentioned that one advantage of higher order differencing schemes might be the elimination of diffusion of rainwater caused by the upstream numerical method. In order to test this hypothesis, cloud models using Crowley's second-order technique and the upstream-differencing method were brought into correspondence with respect to cloud initiation times (starting from the same record on a history tape holding the second-order method results at 105 min). The cloud formed at 112.500 min in the Crowley model and 112.625 min in the upstream-differencing model, a difference of one time step. This correspondence was obtained by adjusting the K-

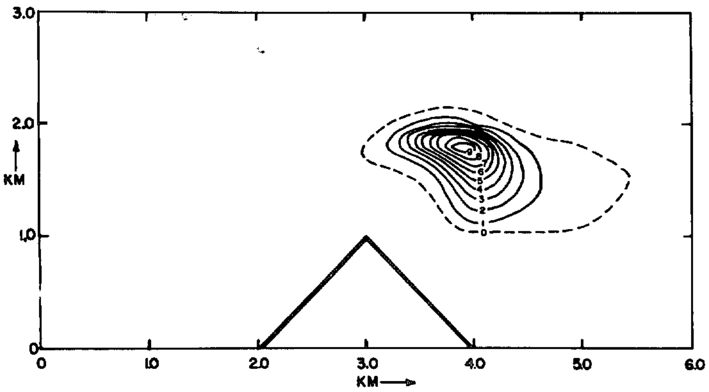

FIGURE 9.—Field of rainwater content at 126 min for the upstream-differencing method, ambient-wind case. The isolines are in units of 10^{-6} gm gm^{-1}. The dashed line encloses all values equal to or greater than 5×10^{-8} gm gm^{-1}. No explicit mixing has occurred in this model for rainwater, that is, the eddy coefficient for rainwater equals 0. No evaporation of rainwater occurs.

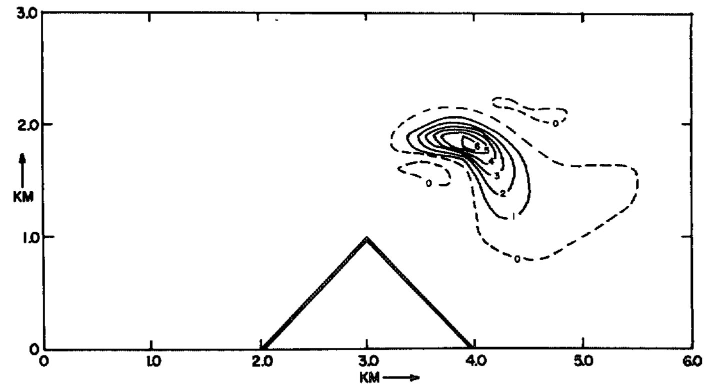

FIGURE 10.—Field of rainwater content at 126 min for the Crowley second-order method, ambient-wind case. The units for the isolines are the same as in figure 9. However, the dashed line encloses all rainwater content equal or greater than 5×10^{-9} gm gm^{-1}. No mixing or evaporation of rainwater occurs.

value in the upstream model to 10 m^2 sec^{-1}. In addition, the evaporation parameter was set to zero in both models so that all the rain that formed would remain in the model to be advected and numerically diffused. There is no *explicit* diffusion of rainwater in the numerical models.

The results may best be seen in figures 9 and 10 showing the rainwater content at 126.0 min for the upstream differencing and Crowley's method, respectively. The cloud water and other atmospheric variables are nearly identical to the fields in figure 7. The dashed line in figure 9 shows the extent of rainwater at least as large as 5×10^{-8} gm gm^{-1} and in figure 10 at least as large as 5×10^{-9} gm gm^{-1} so that the zero line for the Crowley method (fig. 10) encloses rainwater contents an order of magnitude smaller than the zero line in figure 9 for upstream differencing. The solid lines in both figures are for intervals of 10^{-6} gm gm^{-1} as analyzed by computer subroutines. The rainwater content is larger in figure 9, due indirectly to the size of the eddy coefficient (only 10 m^2 sec^{-1} compared to 80 m^2 sec^{-1} in fig. 10). The effect is indirect because rainwater has no explicit diffusion. However, the fields of entropy and total cloud water (vapor plus liquid) are diffused explicitly. The explicit mixing of these fields causes the cloud water to be reduced in the high diffusion case and hence the transformation from cloud water to rainwater is slower. This is particularly true in the interior of the cloud where the maximum rainwater content is .009 gm kg^{-1} compared to .006 gm kg^{-1} in the Crowley method (fig. 10). Downwind, this difference has widened so that the upstream method gives values an order of magnitude greater than the Crowley method due largely, we think, to the diffusion characteristics of the numerical method. The upstream method is considerably smoother than the Crowley method as indicated by the dashed lines. Nevertheless, the smaller numerical diffusion of the second-order method would appear to us to make it preferable to the first-order technique for precipitation models.

6. CONCLUSIONS

The Crowley second-order scheme gives results comparable to the upstream-differencing technique if the eddy coefficients in the Fickian diffusion terms are adjusted. Crowley's fourth-order advection method gives results not obviously better than the second-order advection method on the symmetric cases. Numerical damping is considerably reduced in the Crowley methods and the kinetic energy is approximately conserved by the nonlinear advective terms. The second-order method should prove quite useful in local scale numerical models of clouds and precipitation. It is more economical than the fourth-order method, and leads to fewer boundary problems. For practical use of the schemes, we would suggest that the second-order scheme be used due to the decreased amount of machine time required and simpler boundary computations compared to the fourth-order technique.

Prior results of Orville (1965, 1968*a*) using upstream differencing represent solutions to the basic equations of the model with a K-value of approximately twice that which is shown explicitly (this applies to the K=40 m^2 sec^{-1} cases only).

The fact that clouds form for only a restricted range of the eddy coefficient gives hope that when several of the methods for modeling turbulence are compared with observations the proper method may be selected for these local scale problems. We do not know the proper K-value or functional form. Several candidates exist (Lilly 1967, Smagorinsky 1963, Leith 1968, Fick, etc.). In addition, models in three dimensions will modify these results and pose other questions about the correct specification of turbulence parameters. Nevertheless, the connection between the formation of clouds and the various model parameters have interesting observational implications.

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CORRECTION NOTICE

Vol. 97, No. 9, Sept. 1969: p. 683, paragraph before ACKNOWLEDGMENT, McKinley is to be read instead of Whitney.