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COMPUTATION OF CLIMATOLOGICAL VERTICAL VELOCITIES BY THE THERMODYNAMIC METHOD

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ABSTRACT

Vertical velocities compatible with climatological heating functions are computed. The physical assumptions in the thermodynamic equation incorporate radiational heating, heat flux from the underlying boundary, and heat of condensation. The computation is performed for the four seasons, and a comparison with other studies indicates that the method gives fairly good results that may be related to climatological cloudiness and rainfall.

1. INTRODUCTION

A variety of methods to compute vertical velocities are available. For the purposes of this study, we shall consider a classification of two types, the adiabatic approach and methods where the diabatic heating is incorporated.

In calculating time-averaged vertical motion fields, two basic methods can be applied. In the first method, a time series of vertical motion fields is computed; and the average of the series is then obtained. In such an approach, the adiabatic method seems satisfactory if the time interval in the time sequence is not too large. Use of the diabatic method under these conditions leads to the additional difficulty of parameterizing the heating that must be done with a high degree of accuracy.

The second possibility makes use of preliminary averaging of the governing equations in such a way that the data entering the computation are already averaged with respect to time. In this method, the adiabatic method is clearly unrealistic, while the diabatic method enjoys the advantage of considering only long time-averaged heating fields.

It is the purpose of this study to apply the time-averaged diabatic method to the computation of climatological, hereafter referred to as "normal," vertical velocities associated with particular monthly normal heating fields. The results of the computation may then be used to examine the quality of the heating through a comparison with those aspects of seasonal vertical motion that are well established.

The particular procedure used was adopted to take advantage of the availability of a machine program for a time-averaged thermodynamic model that contains most of the needed fields of climatological quantities.

2. FORMULATION

We consider the thermodynamic equation

$$\frac{d}{dt}\ln\theta=\frac{H}{c_pT} \tag{1}$$

where $\theta=T(p/p_0)^{-R/c_p}$ and H is the rate of heating per unit mass.

With pressure as the vertical coordinate, we have (using standard notation):

$$\frac{\partial}{\partial t}\ln T+\mathbf{V}\cdot\nabla\ln T+\omega\left(\frac{\partial}{\partial p}\ln T-\frac{R}{c_pp}\right)=\frac{H}{c_pT} \tag{2}$$

which may be written as

$$\frac{\partial T}{\partial t}+\nabla\cdot T\mathbf{V}+\frac{\partial}{\partial p}(\omega T)-T\left(\nabla\cdot\mathbf{V}+\frac{\partial\omega}{\partial p}\right)-\frac{\omega RT}{c_pp}=\frac{H}{c_p}$$

or, since the total divergence vanishes, as

$$\frac{\partial T}{\partial t}+\nabla\cdot T\mathbf{V}+\frac{\partial}{\partial p}(\omega T)-\frac{\omega RT}{c_pp}=\frac{H}{c_p}. \tag{3}$$

We now write the variables in the form

$$(\ \ )=(\overline{\ \ })+(\ \ )' \tag{4}$$

where the bar represents a time average over a given interval (normal). Substituting eq (4) into eq (3), using the gas law, and applying the bar operator, we have

$$\frac{\partial\overline{T}}{\partial t}+\nabla\cdot(\overline{T}\overline{\mathbf{V}}+\overline{T'\mathbf{V}'})+\frac{\partial}{\partial p}(\overline{\omega}\overline{T}+\overline{\omega'T'})-\frac{1}{c_p}(\overline{\omega}\overline{\alpha}+\overline{\omega'\alpha'})=\frac{\overline{H}}{c_p}. \tag{5}$$

Since one of the purposes of this study is to examine the vertical velocities resulting from the thermodynamic approach, the heating function $\overline{H}$ will be incorporated directly from the thermodynamic model of Adem (1962, 1964) and is summarized in section 3 of this paper.

For compatibility purposes, eq (5) will be integrated in the vertical:

$$\int_{p_S}^{p_T}\left\{\frac{\partial\overline{T}}{\partial t}+\nabla\cdot(\overline{T}\overline{\mathbf{V}}+\overline{T'\mathbf{V}'})-\frac{1}{c_p}(\overline{\omega}\overline{\alpha}+\overline{\omega'\alpha'})\right\}dp+[\overline{\omega}\overline{T}+\overline{\omega'T'}]_{p_S}^{p_T}=\frac{1}{c_p}\int_{p_S}^{p_T}\overline{H}dp \tag{6}$$

where p_S is the surface pressure and p_T is the pressure at the top of the layer, taken as the 300-mb level.

We now make the following assumptions:

1. $\overline{T'\mathbf{V}'}=-K\nabla\overline{T}$ where K is a mixing coefficient, obtained from observed normal data.
2. $\frac{1}{c_p}\int_{p_S}^{p_T}\overline{\omega'\alpha'}\,dp$ will be neglected.
3. $\overline{\omega}=0$ and $\overline{\omega'T'}=0$ at $p=p_T$.
4. $\int_{p_S}^{p_T}\frac{\partial\overline{T}}{\partial t}dp$ will be neglected.

Assumption (1), or something much like it, is needed since the heat transported by the transient eddies must be parameterized in some way. Assumption (2) involves the transformation between potential and kinetic energy and may be important. Assumption (3) is justified in middle and high latitudes, provided the vertical variation of ω is not to be computed. Finally, it is reasonable to assume, as in (4), that time-averaging over several seasons results in a steady-state situation.

We therefore have

$$\int_{p_s}^{p_T}\left\{\nabla\cdot(\overline{T}\,\overline{\mathbf{V}})-\nabla\cdot K\nabla\overline{T}-\frac{1}{c_p}\,\overline{\omega}\,\overline{\alpha}\right\}dp-[\overline{\omega}\,\overline{T}]_{p_s}=\int_{p_s}^{p_T}\frac{\overline{H}}{c_p}\,dp+[\overline{\omega' T'}]_{p_s}\cdot \quad (7)$$

Consider now the term $\int_{p_s}^{p_T}\overline{\omega}\,\overline{\alpha}dp$. Since $\overline{\alpha}$ is monotonic in p, we may write

$$\int_{p_s}^{p_T}\overline{\omega}\overline{\alpha}dp=\overline{\omega}(p^*)\int_{p_s}^{p_T}\overline{\alpha}dp \quad (8)$$

where p^* is some intermediate pressure level that cannot be further specified.

Using eq (8) and (7), we may solve for $\overline{\omega}(p^*)$:

$$\overline{\omega}(p^*)=\frac{c_p}{\int_{p_s}^{p_T}\overline{\alpha}dp}\left\{\int_{p_s}^{p_T}(\nabla\cdot\overline{T}\,\overline{\mathbf{V}}-\nabla\cdot K\nabla\overline{T})\,dp-[\overline{\omega}\,\overline{T}]_{p_s}-\left(\int_{p_s}^{p_T}\frac{\overline{H}}{c_p}dp+[\overline{\omega' T'}]_{p_s}\right)\right\}\cdot \quad (9)$$

Evaluation will next be made of the terms in eq (9):

$$\frac{1}{c_p}\int_{p_s}^{p_T}\overline{\alpha}dp=\frac{1}{c_p}\left(-g\int_{z_s}^{z_T}dz\right)=\frac{g}{c_p}\,(z_s-z_T) \quad (10)$$

where z_s, z_T are the heights of the lower and upper surfaces, respectively.

Since the winds are obtained from a balance equation, $\nabla\cdot\mathbf{V}=0$. When using this condition,

$$\int_{p_s}^{p_T}\nabla\cdot\overline{T}\,\overline{\mathbf{V}}dp\simeq(p_T-p_s)\overline{\mathbf{V}}(p^0)\cdot\nabla\tilde{\overline{T}} \quad (11)$$

where p^0 is some pressure level (say 700 mb) and $\tilde{\overline{T}}$ is the vertically averaged normal temperature. For the approximation (11), also see Adem (1967). Some support for this approximation is obtained from the empirical observation that the *time-averaged* temperature gradient does not change rapidly with elevation within the troposphere. Because of the thermal wind relationship, the wind direction will also change slowly; and the wind speed will vary linearly with elevation. Therefore, the use of mid-tropospheric (700-mb) winds and temperature fields is likely to yield a reasonable approximation to (11). Now,

$$\int_{p_s}^{p_T}\nabla\cdot K\nabla T dp\simeq(p_T-p_s)(K\nabla^2\tilde{\overline{T}}+\nabla K\cdot\nabla\tilde{\overline{T}}) \quad (12)$$

where K is assumed to be a function of the horizontal coordinates only.

The contribution from the mean vertical velocity at the surface is assumed to be induced by bottom topography. We write

$$\omega_s=-\rho_s g\overline{\mathbf{V}}_s\cdot\nabla z_s \quad (13)$$

where ρ_s and $\mathbf{V}_s$ are the surface density and velocity, respectively. To a good degree of approximation, ρ_s can be taken as its value at 1000 mb.

3. COMPUTATION PROCEDURE

The long-term average (normal) vertical-motion fields covering most of the Northern Hemisphere were computed from eq (9), using eq (10) through (13), for each quarter of the year and are shown in figures 1A through 1D.

The last two terms on the right of eq (9) were evaluated using the normal fields of atmospheric heating. The vertical transport of sensible heat at the surface (last term) is that estimated by Budyko (1963); the condensation heating was obtained from the normal precipitation fields of Möller (1951); and the heating by shortwave and longwave radiation was generated following the procedure described by Adem (1962, 1964), using observed normal fields of cloudiness and snow cover together with the observed normal temperatures at the surface of the oceans and at 700 mb.

The topographically forced upward motion [eq (13)] was obtained from fields of smoothed topography and normal sea-level air density and geostrophic winds. The austausch term [eq (12)] was evaluated using fields of mixing coefficients (Clapp 1970), obtained using 5 yr of eddy heat-transport data from the Massachusetts Institute of Technology General Circulation Library (Oort and Rasmusson 1970*a*). Finally, the mean heat advection [eq (11)] was obtained using observed normal 700-mb geostrophic winds and temperatures.

4. RESULTS

In testing the reality of these vertical motion fields, it is first essential to know what they represent; but it is not possible to state categorically which level or levels are associated with them. However, they probably come closest to representing the mean pressure-weighted vertical motion throughout the troposphere—or perhaps, preferably, that in the mid-troposphere between 600 and 700 mb.

If this is correct, we should compare the fields in figures 1A through 1D with corresponding ones computed independently from the vertically integrated divergence of the observed normal winds. Unfortunately, this is not possible for each season, on a hemisphere-wide basis, because the divergence fields are too sensitive to errors and gaps in the available winds. However, an attempt to compute zonally averaged vertical motions from mean south-to-north wind components for the entire year 1950 was made by Murakami (1963) and shown as a function of latitude in figure 2—thin curve labeled "M." This may be compared with

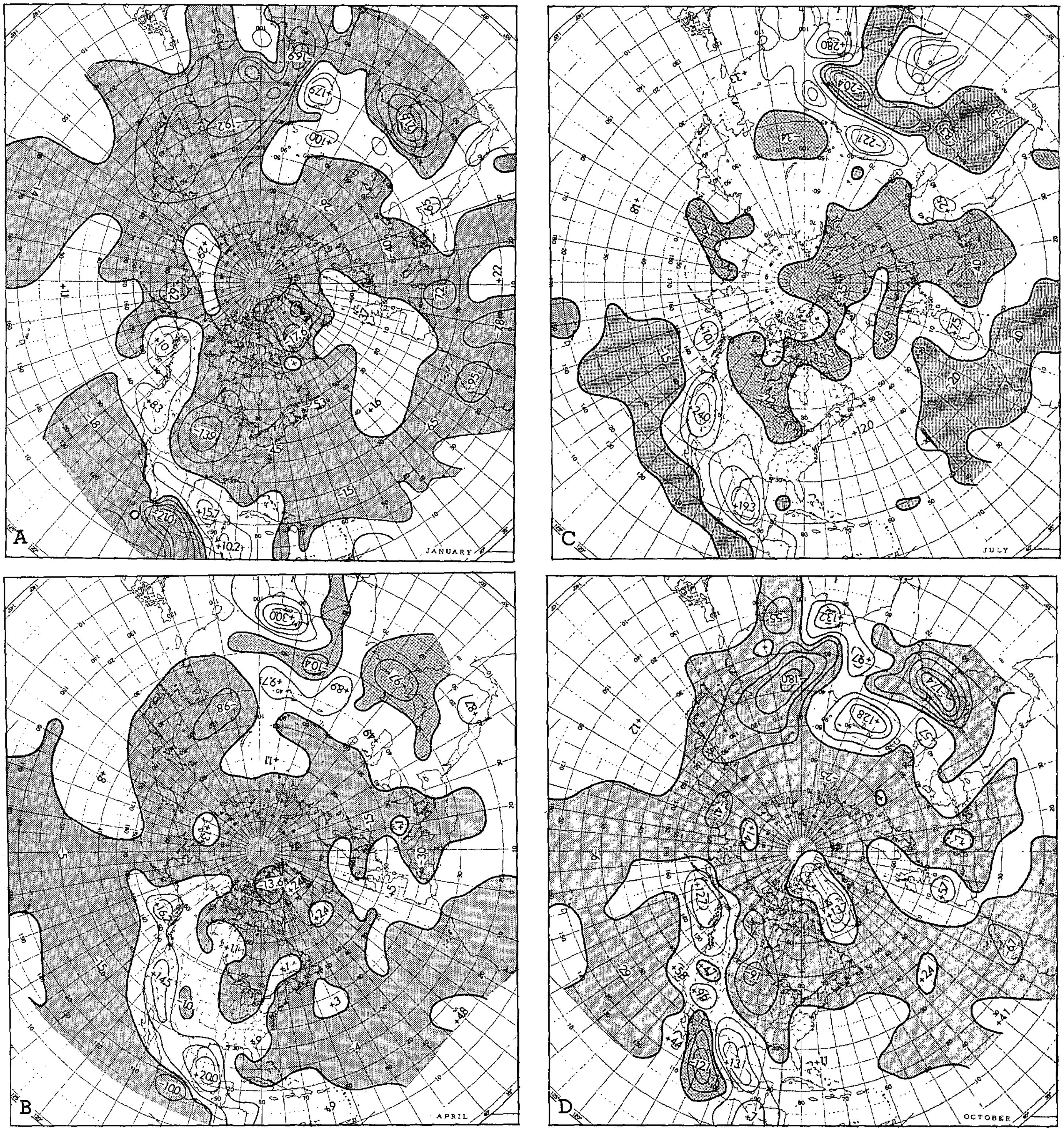


FIGURE 1.—Normal fields of vertical motion (mm/s) from the thermodynamic equations; (A) January, (B) April, (C) July, and (D) October.

zonally averaged values of vertical motion for each quarter from figure 1—heavy curves, indicated by the appropriate month. The agreement is not good; but Murakami states that his values need correcting to allow for more realistic (near-zero) vertical motions near 100 mb. A study (Oort and Rasmusson 1970*b*) in which real winds are used has recently become available. This indicates mean vertical motion fields (not shown here) which, for January at 700 mb, are in agreement with Murakami's annual mean, but with maximum upward motion near 55°N. Other months show a double maximum, as do our values.

The two curves labeled "JJ" and "JA" in figure 2 are zonally averaged vertical motions at about 700 mb for the months of January and April 1958 (Jensen 1960) computed each day by the so-called "adiabatic" method, which is essentially an abbreviated version of eq (9).

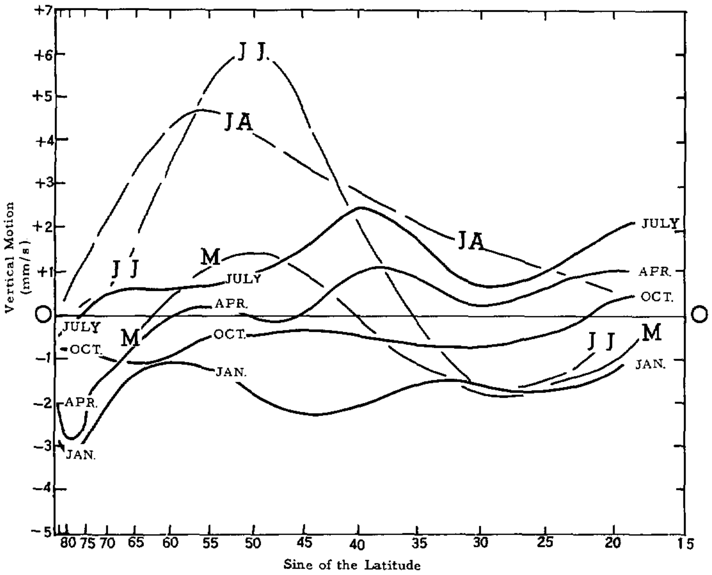
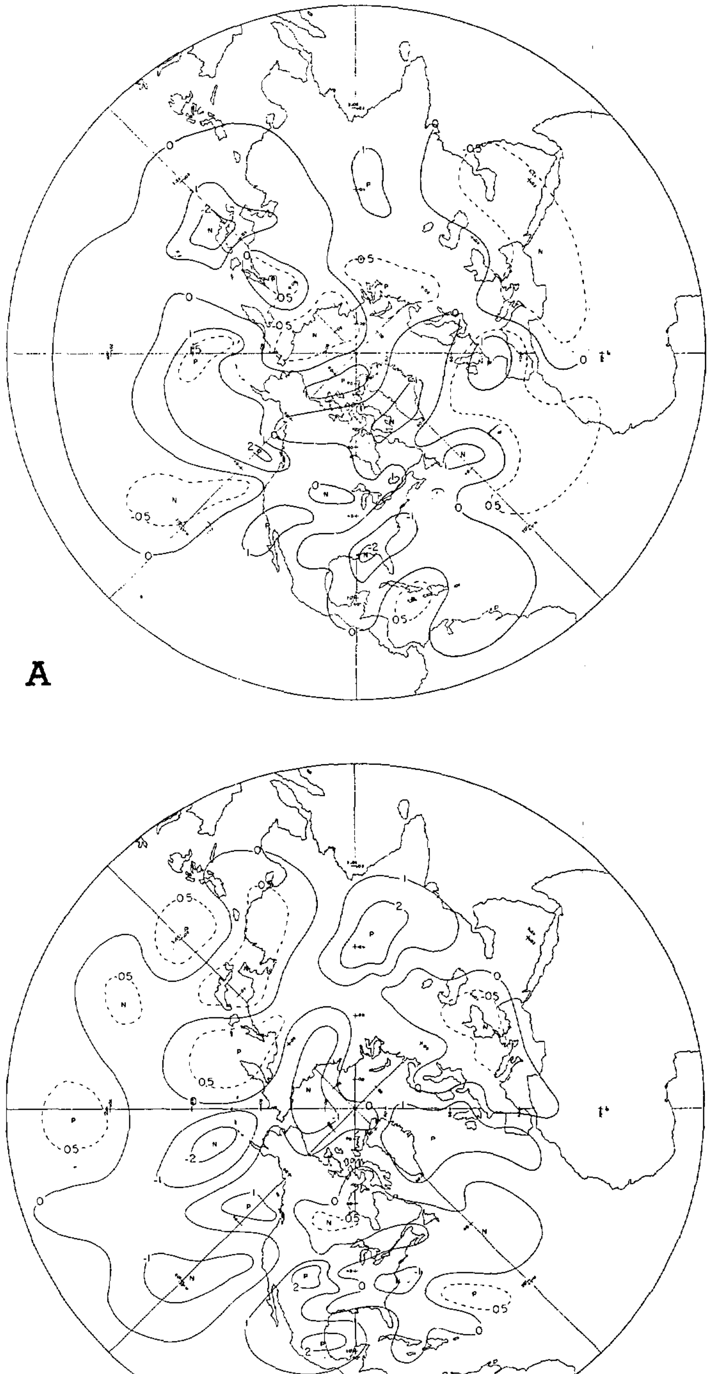


FIGURE 2.—Zonally averaged vertical motion (mm/s) versus sine of latitude. The four heavy curves labeled with the appropriate month are from figure 1.

However, since the computations were made each day and since only the values in the mid-troposphere (where diabatic heating is small) are shown here, the computed values probably are reasonable approximations to the true vertical motions for these 2 individual months.

It can be seen that the general form of the latitudinal distribution and the positions of maxima and minima from Jensen's data agree much better with the values of Murakami than with the zonal averages from figure 1. Therefore, it may be safely concluded that our mean zonal values are in error, especially with regard to the magnitude and position of the maximum upward motion between latitudes 40° and 60°N. These errors may be due to the use of nondivergent winds, which automatically exclude the heat flux convergence due to the mean meridional transport. More will be said of these errors later.

The Northern Hemisphere fields of vertical motion for January and April 1958, between 700 and 500 mb, from Jensen (1960) are reproduced in figures 3A and 3B and may be compared with figures 1A and 1B. Many areas of major disagreement can be seen, probably resulting from errors in both methods. However, there is enough overall pattern resemblance to suggest that some of the differences may be due to real anomalies in the vertical motion fields during 1958. Particularly noteworthy areas of agreement are the generally sinking motions in the southeastern North Pacific, eastern continents, and western oceans—the rising motions in the eastern oceans and western continents and the sinking motions centered over North Africa in January and over southern Europe in April. The rising motions north of the Himalayas in both seasons are also in fair agreement. There is little evidence for the strongly contrasting centers of rising and sinking motions shown in figure 1 near Mexico and the northern Bay of Bengal, and these can probably be attributed to boundary errors.



A

B

FIGURE 3.—Mean vertical motion (cm/s) for the layer 700 to 500 mb for (A) January and (B) April 1958, after Jensen (1960). The "P" represents centers of upward motion and "N" represents those of downward motion.

Although no hemisphere-wide vertical motions are available (computed by the wind-divergence method), it is possible to approximate these by combining available rough estimates of the normal surface divergence for January and July over the oceans (Mintz and Dean 1952, not shown here) with estimates of normal divergence at 10,000 ft for February, May, August, and November (Namias and Clapp 1946, reproduced in figs. 4A to 4D).

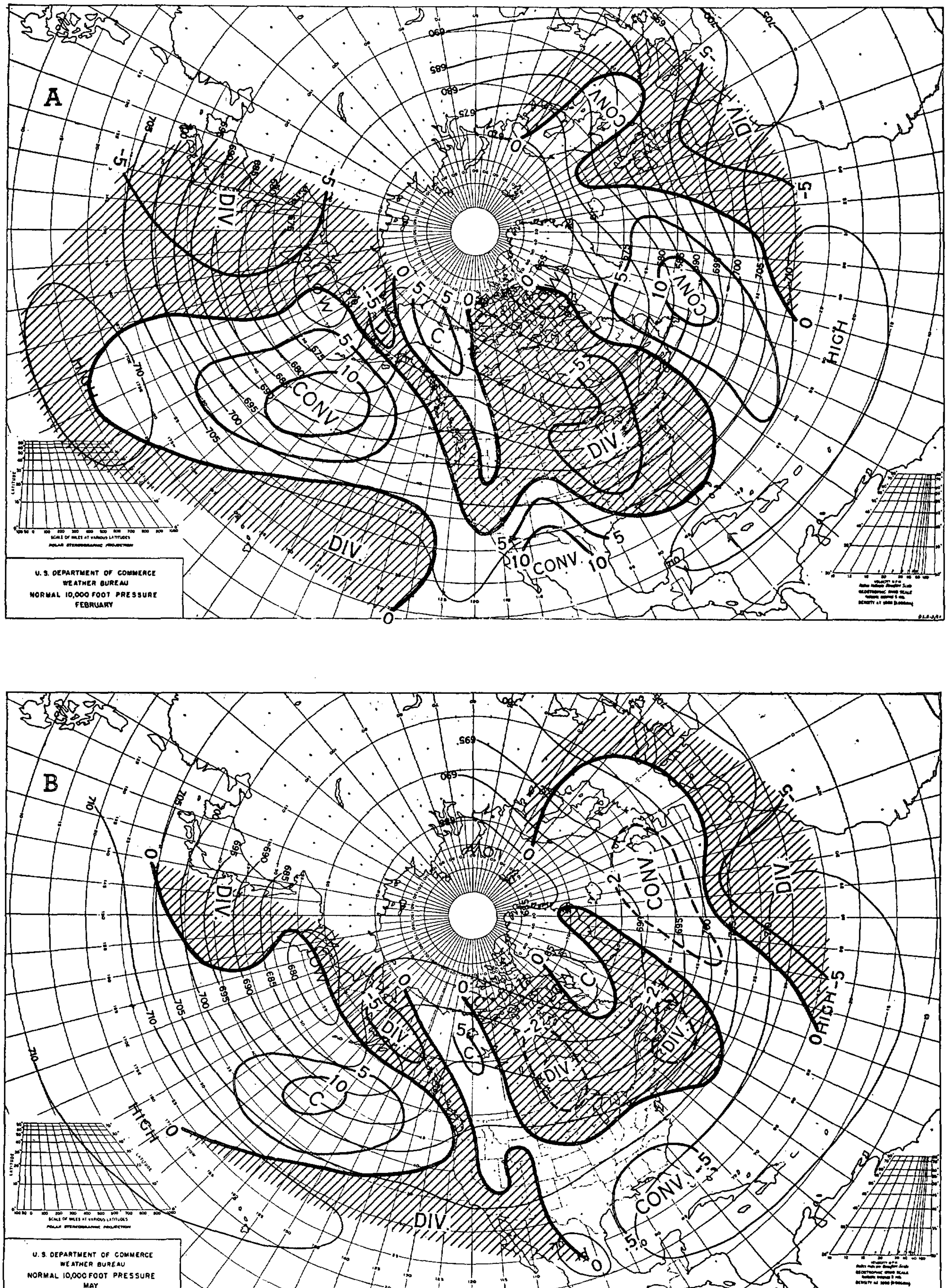
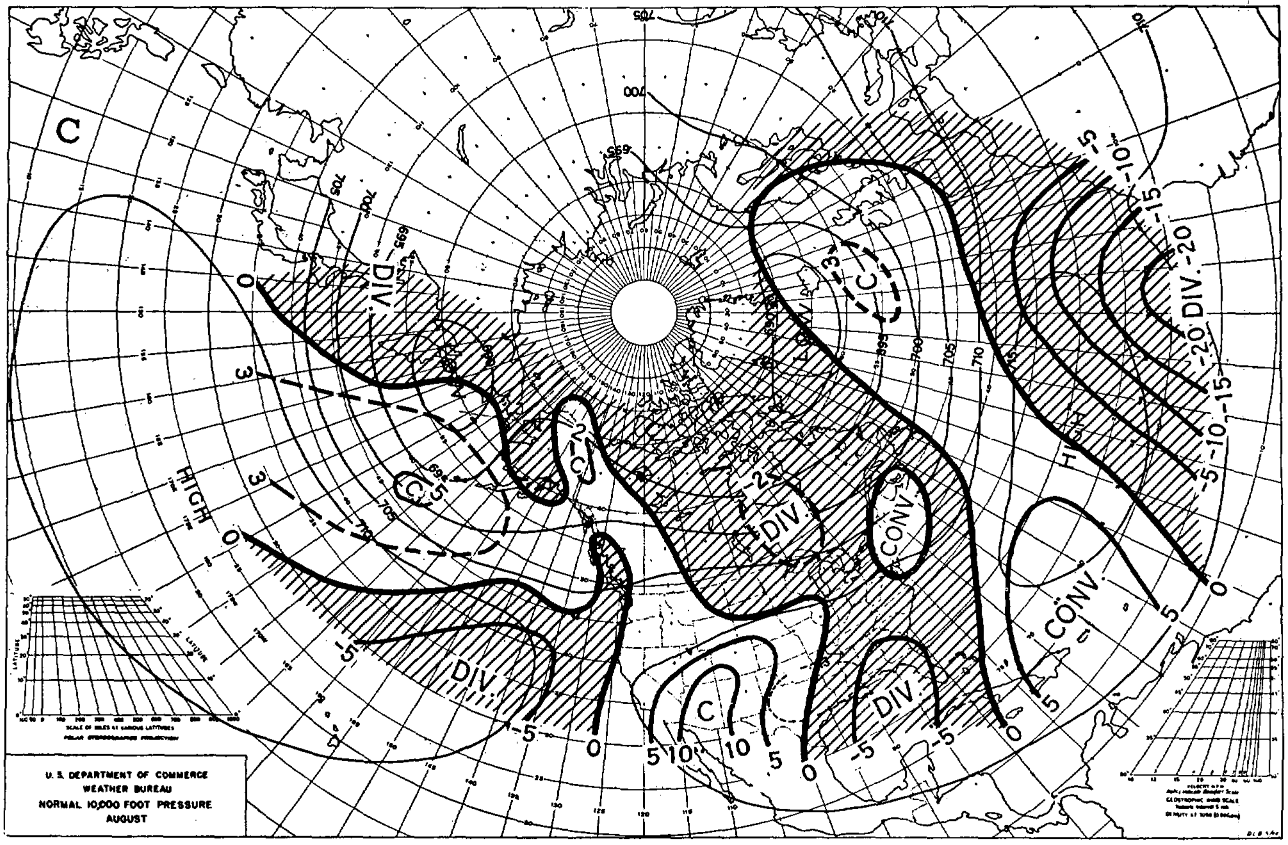


FIGURE 4.—Normal fields of divergence at 10,000 ft, after Namias and Clapp (1946); units, 10^{-7} s^{-1}; convergence, positive; (A) February, (B) May, (C) August, and (D) November.



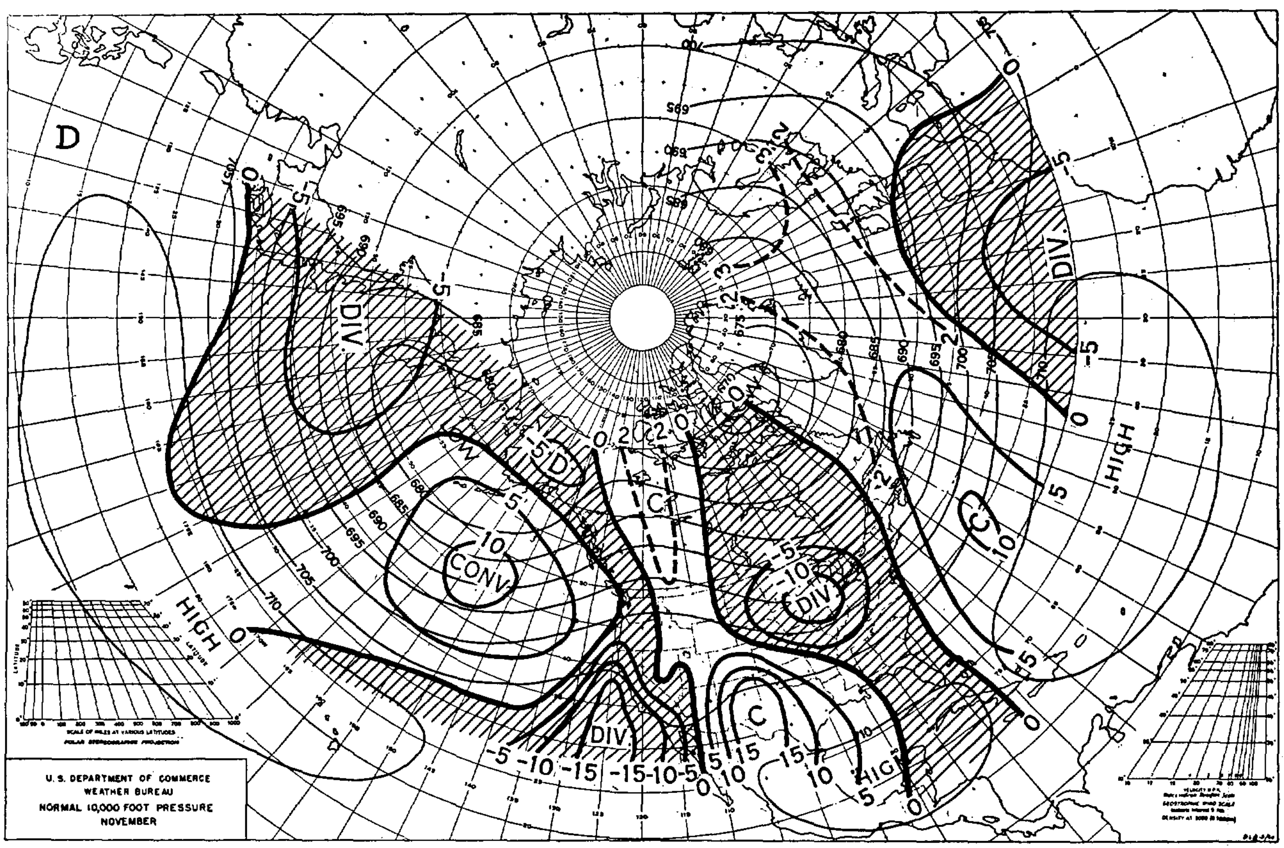


FIGURE 4.—Concluded.

The divergence fields in figure 4 are based on the steady-state vorticity equation; and although several important approximations were made, these do not include the adiabatic assumption.

In general, the sea-level divergence fields of Mintz and Dean for January and July agree in sign with the corresponding fields of figures 4A and 4C, except in the western oceans. We will return to these areas of disagreement in the next section; but if for the moment it may be assumed that the signs of the mean divergence fields do not change from the surface to 700 mb, then the patterns of figure 4 should resemble the patterns of vertical motion—with upward motion in areas of convergence and downward motion in areas of divergence. Indeed, when the fields of figure 1 are compared to those of figure 4, the patterns clearly resemble each other as well as those of figure 3, despite the fact that the corresponding charts are displaced in time by 1 mo.

The major exceptions to this good pattern resemblance are in the central oceans and along the west coasts of the continents. The upward motion in the latter areas shown in figure 1 is due largely to topographical lifting and therefore need not correspond in sign to the mid-tropospheric divergence fields of figure 4, even though the latter are also influenced by topography.

The strong convergence fields in the central oceans in figure 4 imply centers of upward motion that are only weakly indicated or even reversed in figure 1. This suggests that it is the failure to capture these centers of upward motion which is responsible for the absence of a peak of upward motion near 50°N in the heavy curves of figure 2. Possibly, the atmospheric heat sources over the oceans in the cold season, which are due largely to condensation and conduction from the ocean surface, have been underestimated in making these computations.

5. RELATION TO NORMAL WEATHER

It is tempting to relate the vertical motion fields in figure 1 to observed normal fields of cloudiness and precipitation. However, this should be done with caution, realizing that the bulk of the moisture (and, therefore, condensation) in the atmosphere is found in the lower layers (Benton et al. 1953). Furthermore, condensation is most often associated with transient wave cyclones or mesoscale convection, which often have vertical motions several orders of magnitude greater than those associated with the broad scale centers of action.

Nevertheless, it can be anticipated that regions of gradual rising motion, persistent over long time periods, ought to favor clouds and precipitation by transporting moisture aloft and by generating static instability—with the opposite effect true in areas of gradual sinking motion.

Indeed, on comparing the vertical motion fields of figure 1 with average cloud cover, obtained from weather satellite photographs for each season of the year March 1962 to February 1963 (Clapp 1964, not shown), a reasonably good relationship is found of the type suggested in the previous paragraph. Major exceptions are the bands of cloudiness found during the cold seasons along the storm tracks in the western oceans, which are associated with low-level convergence (Mintz and Dean 1952) but not with the mid-tropospheric sinking motions shown in figures 1, 3, and 4.

Other areas of discrepancy are the cloudy regions in the southeastern North Pacific and North Atlantic Oceans, which coincide with the sinking motion in the eastern lobes of the subtropical anticyclones shown in figures 1, 3, and 4, as well as in the low-level divergence fields of Mintz and Dean. Clearly, these are stratus clouds that form under a subsidence inversion in regions of relatively cool ocean water.

The pronounced centers of upward motion in the western conterminous United States (fig. 1) are well related to the mean cloudiness only during the cold season.

6. CONCLUSIONS

An attempt is made to compute normal vertical motion fields over the Northern Hemisphere for each season of the year, using a time-averaged thermodynamic energy equation, in a manner similar to the model of Adem (1962, 1964); but independent estimates are themselves so subject to error that it has not been possible to effectively assess the reality of the computed fields. Climatological fields of cloudiness and precipitation can be described somewhat more accurately than the mean tropospheric vertical motions, but these fields cannot be directly related.

The assumptions and climatological heating fields used here can undoubtedly be greatly improved. As more reliable estimates of the fields of vertical motion become available for comparison, it will be possible to test the validity of new formulations and heating fields.

When this is accomplished, the next logical step is to compute anomalies of vertical motion for individual months by replacing the normal heating and temperature fields with computed and observed values, respectively, for the selected month. The anomalies in the heating fields can be approximated using the same parameterization as in the thermodynamic model (Adem 1964). It would be interesting to compare computed anomalies of vertical motion with observed cloud and precipitation anomalies because these are the most valuable and at the same time the most difficult by-products of a monthly weather forecast.

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