UDC 551.508.313:551.511.2

CORRESPONDENCE

Comments on "A Note on the Computational Stability of the Two-Step Lax-Wendroff Form of the Advection Equation"

G. FISCHER and V. RENNER—*Meteorologisches Institut, Universität Hamburg, Hamburg, Germany*

In a recently published article, Gerrity (1972) showed that the Lax-Wendroff method for integrating the advection equation

$$\frac{\partial V}{\partial t}=-A(x_i t)\frac{\partial V}{\partial x} \tag{1}$$

is subject to nonlinear computational instability. We want to point out that this deficiency can easily be avoided by adopting a staggered grid in time, which, in any case, is better fitted for this kind of numerical procedure.

Applying the same notations as in Gerrity's study and using a time-staggered grid, we obtain the following as an approximation to eq (1).

Step 1 for odd time steps:

$$V_{j+1/2}^{n+1}=\frac{1}{2}(V_j^n+V_{j+1}^n)-\frac{1}{2}\frac{\Delta t}{\Delta x}(A_{j+1}^n+A_j^n)(V_{j+1}^n-V_j^n). \tag{2}$$

Step 2 for even time steps:

$$V_j^{n+2}=V_j^n-\frac{\Delta t}{\Delta x}(A_{j+1/2}^{n+1}+A_{j-1/2}^{n+1})(V_{j+1/2}^{n+1}-V_{j-1/2}^{n+1}). \tag{3}$$

Equations (2) and (3) combined yield the single equation

$$V_j^{n+2}=V_j^n-\frac{1}{2}\frac{\Delta t}{\Delta x}(A_{j+1/2}^{n+1}+A_{j-1/2}^{n+1})(V_{j+1}^n-V_{j-1}^n)$$
$$+\frac{1}{2}\left(\frac{\Delta t}{\Delta x}\right)^2(A_{j+1/2}^{n+1}+A_{j-1/2}^{n+1})[(A_j^n+A_{j+1}^n)(V_{j+1}^n-V_j^n)$$
$$-(A_j^n+A_{j-1}^n)(V_j^n-V_{j-1}^n)]. \tag{4}$$

Writing

$$A_j^n\approx A_0^n+A_1^n e^{i\pi j} \tag{5}$$

and inserting this expression into eq (4), one immediately realizes that $A_1 e^{i\pi j}$ is "smoothed out" completely.

Thus, one obtains the "linear" stability condition

$$|\xi^2|^2=1-(4\epsilon)^2\left(\sin\frac{k\Delta x}{2}\right)^4[1-(2\epsilon)^2]\leq 1 \tag{6}$$

which is satisfied if $2\epsilon\equiv 2A_0^n\Delta t/\Delta x\leq 1$ for any value of $0\leq k\Delta x\leq\pi$. This favorable result is obtained because, with a time-staggered grid using the Lax-Wendroff method, (1) no lattice separation can occur and (2) high wave numbers of A are "automatically" suppressed by the inherent space averaging.

The latter point exactly meets the property that Robert et al. (1970) have proposed for circumventing nonlinear instability. Moreover, the time staggered grid needs only about half the number of space gridpoints as the non-staggered grid to achieve the same accuracy. With respect to the complete set of meteorological equations, we have had good experiences with this version of the Lax-Wendroff method. (See also Fischer and Ehlers 1968.)

REFERENCES

Fischer, G., and Ehlers, R., "Vergleich dreier Differenzen-Verfahren zur numerischen Integration der 'primitiven Gleichungen' für ein barotropes divergentes Modell der Atmosphäre" (Comparison of Three Different Schemes for Numerical Integration of the 'Primitive Equations' for a Divergent Barotropic Model of the Atmosphere), *Tellus*, Vol. 20, No. 2, Stockholm, Sweden, **1968**, pp. 318–329.

Gerrity, Joseph P., Jr., "A Note on the Computational Stability of the Two-Step Lax-Wendroff Form of the Advection Equation," *Monthly Weather Review*, Vol. 100, No. 1, Jan. **1972**, pp. 72–73.

Robert, André J., Shuman, Frederick G., and Gerrity, Joseph P., Jr., "On Partial Difference Equations in Mathematical Physics," *Monthly Weather Review*, Vol. 98, No. 1, Jan. **1970**, pp. 1–6.

[*Received April 17, 1972; revised April 21, 1972*]