UDC 551.509.33:338.978:63(79)

The Economics of Extended-Term Weather Forecasting

LEE G. ANDERSON—*Department of Economics, University of Miami, Coral Gables, Fla.*

ABSTRACT—The first part of this paper deals with the theoretical aspects of the economies of extended-term weather forecasting. The discussion centers around the optimal way of reacting to extended-term weather information, formulations of value of such information, and methods of comparing different forecasting systems. The second part deals with case studies where the value of extended-period forecasting to two firms is determined. The first study is of pea farming, which is sensitive to temperature, and the second is of logging, which is sensitive to rain. In these cases, it is shown that the firms can benefit from extended-period forecasts.

1. INTRODUCTION

The economics of weather forecasting has advanced greatly in the past 20 yr. Maunder (1970) provides a good introduction to this material as well as an extensive set of references. Thompson and Brier (1955) and Nelson and Winter (1964) have developed models that determine the optimal way of reacting to a weather forecast and how to measure the value of weather information. The first half of this paper will extend their analyses in two ways. First, extended-period forecasts will be included in the model; and second, the graphical analysis of weather forecasting will be formalized and will then be used to define what is meant by an improvement in a forecasting system. In view of the government's attempts to improve the forecasting ability of its meteorologists and especially to improve extended-term forecasting techniques, these extensions are of more than theoretical importance. The second half of this paper consists of two case studies to determine the value of extended-period forecasts to particular industries.

Because much of what follows relies heavily on the models of Nelson and Winter (1964), the remainder of this introduction consists of a brief introduction to the definitions and formulations of their work. The model is set up in game form with a firm playing against "Mother Nature." The firm tries to pick the activity that will yield the highest expected returns or lowest expected losses no matter what the variations of weather. As aids in the decision making process, the firm has climatological records and the forecasting ability of meteorologists.

For the simplest case, assume a firm that faces two weather states, W_1 (unfavorable weather) and W_2 (favorable weather), and that has two possible activities, A_1 (take protective activity against the weather) and A_2 (no action). The game box is depicted in figure 1.

L is the loss per day the firm will bear if bad weather comes and it is not prepared, and C is the daily cost of preparing for bad weather. Naturally, C is assumed to be less than L. If, for a given time period, P_1 is the climatological probability of W_1 occurring on any day in that period and P_2 (which equals $1-P_1$) is the climatological probability of W_2, then, with no weather forecasting, the firm will minimize losses by taking A_2 if P_1L is less than C (i.e., $P_1<C/L$) and by taking A_1 if the opposite is true.

Let us now assume a weather forecasting system that has the following parameters:

Π_1 = the relative frequency of forecast F_1, a forecast of W_1.

$\Pi_2=(1-\Pi_1)$ = the relative frequency of F_2, a forecast of W_2.

Π_{11} = the conditional probability of W_1 given that a forecast of W_1 is received.

$\Pi_{21}=(1-\Pi_{11})$ = the conditional probability of W_2 given that a forecast of W_1 is received.

Π_{12} = the conditional probability of W_1 given that a forecast of W_2 is received.

$\Pi_{22}=(1-\Pi_{12})$ = the conditional probability of W_2 given that a forecast of W_2 is received.

It can be shown that if the forecasting system is to have any net benefits (i.e., that it will cause the manager to make different decisions than climatology does), the following must hold:

$$\Pi_{12}<\frac{C}{L}<\Pi_{11}.$$

If this inequality does hold, then the least expensive activity, given a forecast of W_1 (i.e., an F_1 forecast), is A_1; and the least expensive activity, given an F_2 forecast, is A_2. Therefore, the expected cost for one day to the firm in the long run is

$$\Pi_1 C+\Pi_2\ \Pi_{12}L.$$

The value of the forecasting system is the money saved over what would be spent if climatology provided the only source of information on expected weather states. If $P_1>C/L$, the value of the system will be C minus the expected daily cost, which can be simplified to

$$V=\Pi_2(C-\Pi_{12}L). \tag{1a}$$

When $P_1<C/L$, the value of the forecasting system can be expressed as

$$V=\Pi_1(\Pi_{11}L-C). \tag{1b}$$

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	W_1	W_2
take action A_1	C	C
A_2	L	0

FIGURE 1.—Game box for hypothetical firm.

A perfect forecast system would have the following parameters:

$$\Pi_1 = P_1,\ \Pi_2 = P_2,\ \Pi_{11} = 1,\ \Pi_{21} = 0,\ \Pi_{22} = 1,\ \Pi_{12} = 0.$$

Substitution of these numbers into whichever of eq (1a) or (1b) is relevant results in an expression of the value of a perfect forecasting system for the firm involved. By subtracting the value of the existing system from that expression, one obtains a measure of the maximum gains possible from improvements in forecasting abilities.

2. THE ECONOMIC THEORY OF EXTENDED-TERM FORECASTING

A Simple Model with Extended-Term Forecasting

Since accurate forecasting does not change the weather but only leads to a choice of actions that maximizes gains or minimizes losses from a predicted event, it will only have value if there are alternatives available when one is facing a weather event. Likewise, extended-term weather forecasting will have value over short term forecasting of comparable accuracy only if the number of choices of actions is increased. That is, the difference between a short-term system and an extended-term forecast system is that the latter will allow sufficient extra warning time such that new choices of action become possible. If one forecasting system provides more warning time than another, and yet does not increase the choice of actions, this extra warning time is not economically significant. Accordingly, for purposes of this paper, "extended-term forecast" will be used from an economic rather than a meteorological point of view.

Figure 2 contains a game box for a firm that can take advantage of extended-term forecasting. There are two possible protection activities: one that can be completed in a short period of time, A_1, and a second that takes longer but is cheaper, A_2 (i.e., $C_2 < C_1 < L$). It is also possible that the two are the same, but when more time is available the activity can be performed more cheaply. Obviously when the firm has only climatology as an aid in decision making, it will use A_2 as its protection activity. From this particular firm's point of view, a short-term forecasting system would be one that would allow only enough time to use A_1 as a protective activity, while an extended-period system would be one that allows for the use of A_2.

	W_1	W_2
A_1	C_1	C_1
A_2	C_2	C_2
A_3	L	0

FIGURE 2.—Game box for firm that can use extended-period forecast.

The analysis of short-term forecasts in this more general model is different from the Nelson and Winter model when the climatological conditions make protection all the time the wisest policy (i.e., when $P_1 > C_2/L$). Here, the value of the system is expressed as

$$V = (C_2 - C_1) + \Pi_2 (C_1 - \Pi_{12} L). \tag{2}$$

The first term, which is always negative, distinguishes eq (2) from eq (1a). For the value of the forecasting system to be positive, the following must hold:

$$\Pi_{11} > \frac{C_1}{L} + \frac{P_1 - \dfrac{C_2}{L}}{\Pi_1}.$$

P_1 is greater than C_2/L by assumption; therefore, the second term on the right side of the inequality must be positive.

In the Nelson and Winter model, the condition $\Pi_{11} > C_1/L > \Pi_{12}$ will guarantee that the optimal response to the forecast system will differ from that when climatology is used. It will also guarantee that the system will always have positive value. From the above, however, it can be seen that, when $P_1 > C_2/L$, more stringent specification must be placed upon the accuracy of the forecast for unfavorable weather (i.e., Π_{11}) for the system to have positive value. How much more accurate it must be will depend upon the relative sizes of its relative frequency, Π_1, the climatological probability of unfavorable weather, P_1, and the cost loss ratio when climatology is used, C_2/L. If everything else is equal, the greater the difference between C_1 and C_2, the higher Π_{11} must be.

Useful information can be obtained by comparing two different types of forecasting systems: a short-term one that gives only enough lead time for A_1 to be finished, and an extended-term one that gives enough lead time for either A_1 or A_2 to be completed. These forecast systems can be represented by the parameters listed in the previous section. To distinguish between the two, the Πs of the extended-term system will be primed (i.e., Π_1' is the relative frequency of forecast F_1', a forecast of W_1' from the extended-term forecast.)

The results can be expressed briefly in the following way. If $P_1 > C_2/L$, then the value of the long-term forecast over the short-term one is

$$[\Pi_2'(C_2-\Pi_{12}'L)]-[(C_2-C_1)+\Pi_2(C_1-\Pi_{12}L)]. \tag{3}$$

The first term in brackets is the value of the long-term system while the second term is the value of the short-term system. If $C_2=C_1$, then the only difference in the long-term forecast would be a difference in accuracy; therefore, the change in value due to the change in accuracy alone can be expressed by substituting C_1 for C_2 and expression (3) becomes

$$\Pi_2'(C_1-\Pi_{12}'L)-\Pi_2(C_1-\Pi_{12}L). \tag{4}$$

In almost all conceivable cases (except when it is the result of a theoretical or a technological breakthrough), the extended-term forecast system will be less accurate than the short-term one. This means that expression (4) will be negative in most cases. By subtracting expression (4) from (3), we can obtain the following expression for the difference in value due to the increased lead time:

$$\Pi_1'(C_1-C_2). \tag{5}$$

On the other hand, if $P_1 < C_2/L$, then the value of the long-term system over the short-term one is

$$\Pi_1'(\Pi_{11}'L-C_2)-\Pi_1(\Pi_{11}L-C_1). \tag{6}$$

The difference in value due to the change in accuracy is

$$\Pi_1'(\Pi_{11}'L-C_1)-\Pi_1(\Pi_{11}L-C_1), \tag{7}$$

and the difference due to the increase in lead time is again given by expression (5).

The above breakdown of benefits into changes due to increased warning time and to increased forecast accuracy makes explicit the ways in which improvements in forecasting can come. Obviously, both of these types of changes do not have to be positive for the total gain to be positive; the only requirement is that the positive one be larger than the absolute value of the negative one. If, for example, the value of expression (4) for a certain extended-forecast system is negative, as would normally be expected, but the value of expression (5) is positive and larger in absolute value, then it makes economic sense to accept the losses due to decreased accuracy to achieve the gains from increased lead time.

Before spending large amounts of time and money to develop or improve extended-period forecasting systems, we should estimate the size of the benefits from using them. The maximum improvement the long-term forecast could make would be perfect information. If this were the case, and if $P_1 > C_2/L$, the value of the long-term system over the current short-term system for our hypothetical firm would be

$$P_2C_2-(C_2-C_1)-\Pi_2(C_1-\Pi_{12}L),$$

which can be simplified to

$$\Pi_1C_1-P_1C_2+\Pi_2\Pi_{12}L. \tag{8}$$

If $P_1 < C_2/L$, the value is

$$P_1(L-C_2)-\Pi_1(\Pi_{11}L-C_1),$$

which can also be simplified to expression (8).

The maximum gain from perfect information, therefore, is the same regardless of the relationship of P_1 and C_2/L. Expression (8) can be broken into two parts, as were expressions (3) and (6), to show the gain from increased accuracy and the gain from increased warning time. By substituting C_1 for C_2 in expression (8), we can derive an expression that shows the gain from increased accuracy that could be delivered by a perfect long-term forecasting system. It is

$$(P_2-\Pi_2)C_1+\Pi_2\Pi_{12}L. \tag{9}$$

This is the same as the expression for the maximum possible gain from perfect information in the simple model of Nelson and Winter. In other words, the maximum to be gained from increased accuracy is the change in protection costs (which can be positive or negative) plus the amount of money currently being lost by the incorrect prediction of good weather.

By subtracting expression (9) from (8), we obtain an expression for the gain from increased warning time of a perfect long-term system; that is,

$$P_1(C_1-C_2). \tag{10}$$

It is apparent that expression (10) shows the extra value of having perfect information at an earlier time, whereas expression (9) gives the value of perfect information in the short-term period.

A Model That Allows for Decisions to be Made Over the Period of Analysis

A more complicated and possibly more useful extension of the model allows for new information to be assimilated during the decision making process. For the simplest case, assume a forecast that covers 2 days in advance; that is, a forecast that is issued day 1 will predict the weather on day 2 and day 3. The firm with game box 2 (fig. 2) that is planning to undertake some weather sensitive activity on day 3 can actually do four things if it receives this type of forecast on day 1. (Assume that A_1 needs 1 day lead time and A_2 needs 2 days.) First, it can decide to use A_2; therefore, to complete the action by day 3, the firm must initiate action immediately. Second, it can definitely decide to use A_1 on day 2. Third, it can decide to do nothing. Fourth, it can decide to put off the decision until the day 2 forecast arrives with its prediction of the weather for days 3 and 4. The last three decisions are similar, but it is best to make distinctions among them. It is also best to modify the game box in such a way that the forecast received is the prime concern instead of the actual

	F_1'	F_2'
A_1	C_1	C_1
A_2	C_2	C_2
A_3	$\Pi_{11}'L$	$\Pi_{12}'L$
A_4	$EA_4\|F_1'$	$EA_4\|F_2'$

FIGURE 3.—Modified game box.

weather state. This modified game box is depicted in figure 3.

If, in figure 3, A_4 is a relevant activity (i.e., the 1-day forecast is better than climatology), then the expected costs of A_4, given F_1' and F_2', respectively, are

$$EA_4|F_1'=\Pi_{01|F_1'}C_1+\Pi_{02|F_1'}\Pi_{012|F_1'}L$$

and

$$EA_4|F_2'=\Pi_{01|F_2'}C_1+\Pi_{02|F_2'}\Pi_{012|F_2'}L.$$

As before, the primed Fs represent the extended-term forecasts. $\Pi_{01|F_1'}$ refers to the frequency of the short-term forecast for day 3 received on day 2, given that W_1 was predicted on day 3 in the forecast received on day 1, and so forth.

There are six possible ways that such a system will have value over climatology (or over another type of forecast for that matter). Whether a given system falls into one of these six categories depends upon values of C_1, C_2, L, and of the various Πs. The six possible categories, as described by their decision rules, are:

1. If the forecast is F_1', take A_2; if the forecast is F_2', take A_3.
2. If the forecast is F_1', take A_2; if the forecast is F_2', take A_4.
3. If the forecast is F_1', take A_3; if the forecast is F_2', take A_4.
4. If the forecast is F_1', take A_4; if the forecast is F_2', take A_2.
5. If the forecast is F_1', take A_4; if the forecast is F_2', take A_3.
6. If the forecast is F_1', take A_4; if the forecast is F_2', take A_4.

There are other possible rules, but close observation shows that they would either be less valuable than climatology or less valuable than some other decision rule. For example, any rule that calls for A_1 on either an F_1' or an F_2' forecast has to be less valuable than the same rule with A_2 substituted for A_1 since C_2 is less than C_1.

It will be helpful to explain in greater detail one of the above categories. Number 3 will be discussed because, on the surface, it may look suspect. This rule leads the firm to do nothing if a forecast of bad weather is received for 2 days from now, but leads it to put off making a decision until tomorrow if it gets a forecast of fair weather for the day after tomorrow. For this to be logical, the following conditions must hold (see fig. 3):

1. $\Pi_{11}'L<C_1$,	4. $EA_4|F_2'<C_1$,	
2. $\Pi_{11}'L<C_2$,	5. $EA_4|F_2'<C_2$,	and
3. $\Pi_{11}'L<EA_4|F_1'$,	6. $EA_4|F_2'<\Pi_{12}'L$.	

From condition 2, it can be shown that $\Pi_{11}'<C_2/L$, and since $\Pi_{11}'>P_1$ (Nelson and Winter 1964, p. 429), then $C_2/L>P_1$. Climatology would, therefore, also have indicated that the firm should do nothing. Hence, in the forecasting system under consideration, the 2-day forecast will be of no more use than climatology. The 1-day forecast on days following a 2-day forecast of fair weather is of value, however.

The expected daily cost to the firm for any of the above decision rules follows directly from game box 3 (fig. 3). Simply multiply the relative frequency of each of the types of long-term forecasts times the expected cost of the activity used, given that forecast. For rule number 3, the expected daily cost is $\Pi_1'\Pi_{11}'L+\Pi_2'EA_4|F_2'$.

The value of the forecasting system can be found by subtracting the above from C_2 or P_1L, depending upon whether P_1 is greater than or less than C_2/L. The value of this long-term forecasting system over a specified short-term system is obtained in the manner described previously, as are the expressions for the changes in value due to changes in accuracy and due to the increase in warning time. Similarly, the conditions necessary for each of the other decision rules to be optimal and the relevant value expressions can be derived from game box 3.

Graphical Analyses: Isovalue and Isoparameter Curves

The value expressions derived previously can be plotted in either (Π_1, Π_{11}) or (Π_2, Π_{22}) space depending upon which expression is used (i.e., depending upon whether P_1 is less than or greater than either C/L or C_2/L). For the remainder of the analysis (Π_1, Π_{11}) space will be used but the results are common to (Π_2, Π_{22}) space.

Each point in (Π_1, Π_{11}) space represents a forecasting system. This holds true because of the following relations between the parameters of any system:

$$\Pi_1+\Pi_2=1, \tag{11}$$

$$\Pi_{11}+\Pi_{21}=1, \tag{12}$$

$$\Pi_{22}+\Pi_{12}=1, \tag{13}$$

and

$$\Pi_1\Pi_{11}+\Pi_2\Pi_{12}=P_1. \tag{14}$$

Therefore, if P_2, which is equal to $(1-P_1)$, is known, and if Π_1 and Π_{11} are specified, then all of the parameters are specified.

It is obvious, however, that any portion of (Π_1, Π_{11}) space other than that where $0<\Pi_1<1$ and $P_1<\Pi_{11}\leq 1$ is of no practical interest, because these are the ranges over which these parameters can vary (fig. 4). In fact, even all of this area is not relevant since stipulations have been placed on the value of Π_{22}.

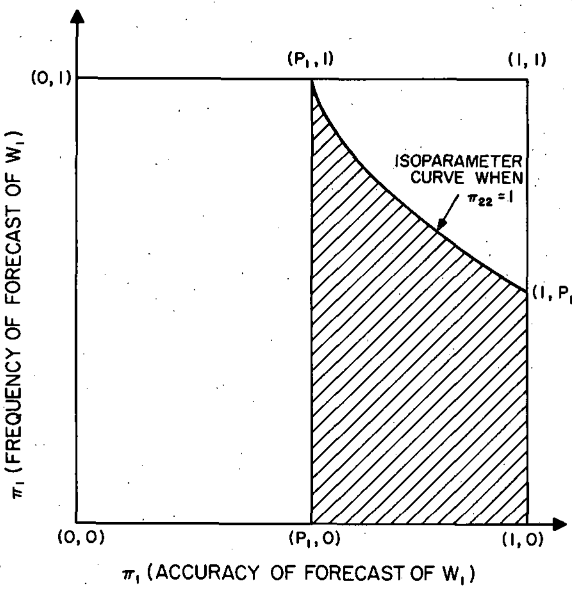

Figure 4.—Area of physically possible forecast systems.

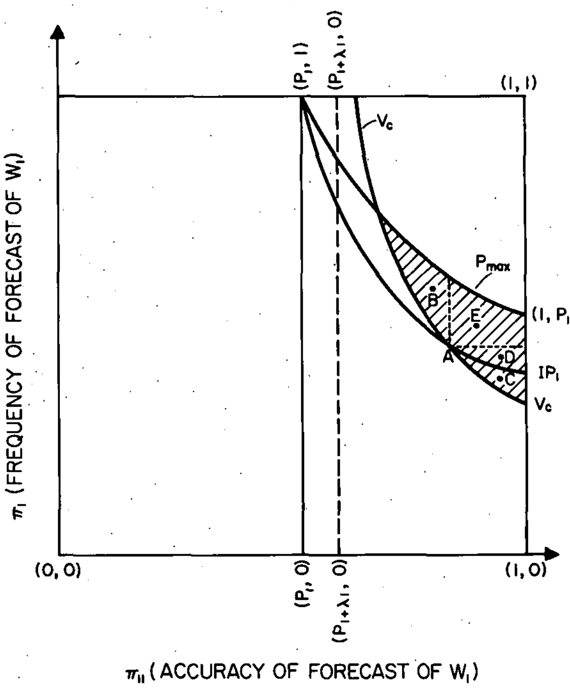

Figure 5.—Area of superior forecasting systems.

With some simple manipulations, however, eq (14) can be modified to

$$\Pi_1=\frac{P_1-1+\Pi_{22}}{\Pi_{11}-1+\Pi_{22}}. \tag{15}$$

For a specified value of Π_{22}, eq (15), when plotted in (Π_1, Π_{11}) space, is a curve that connects all those forecasting systems that have the same value for Π_{22}. The set of curves represented by eq (15) will be called isoparameter curves. In this study, we are interested in only that part of the set where $P_2<\Pi_{22}\leq 1$. By taking the first and second derivative of eq (15), one can show that these curves are downward sloping and convex to the origin for values of Π_{22} over this range. All of these curves will go through the point $(P_1, 1)$ because, whenever $\Pi_{11}=P_1$, $\Pi_1=1$. The higher the value of Π_{22}, the farther away the isoparameter curve is from the origin. The isoparameter curve for $\Pi_{22}=1$ (call this curve $P_{\max}$) is pictured in figure 4. Therefore, out of the area specified above, only those points on or to the left of and below $P_{\max}$ represent physically possible forecast systems. This area is the hatched area in figure 4.

Plotting the value expression $V_0=\Pi_1(\Pi_{11}L-C)$, hereafter called an isovalue curve, in this part of (Π_1, Π_{11}) is equivalent to connecting all the points that represent forecasting systems that yield a value of V_0 to a specified firm. By expressing the above value expression in terms of Π_1 and taking the first and second derivatives with respect to Π_{11}, one can show that an isovalue curve is downward sloping and convex to the origin over the range where $\Pi_{11}>P_1$. One can also show that the curve will asymptotically approach the vertical line $\Pi_{11}=P_1+\lambda$ where $\lambda=(C-P_1L)/L$. As V increases, the isovalue curve shifts to the right.

Using the graphical analysis just introduced, one can delineate sections within the area of physically possible forecast systems wherein all points will represent forecasting systems superior to a given system. Let point A on figure 5 represent the current forecasting system, which is assumed to have a positive value. The isovalue curve through point A is V_c. The isoparameter curve for Π_{22} through A is IP_1. All points between V_c and $P_{\max}$ or on $P_{\max}$ (the hatched area in fig. 5) represent physically possible forecasting systems that will yield higher economic values than odes the system at point A.

Close observation of figure 5 will reveal four different ways in which a forecasting system can be improved. At point B, Π_1 and Π_{22} have increased while Π_{11} has decreased. At point C, Π_1 and Π_{22} have decreased while Π_{11} has increased. At point D, Π_{11} and Π_{22} have increased while Π_1 has decreased. Finally, at point E, Π_1, Π_{11}, and Π_{22} have all increased.

Let us now compare an extended-term forecasting system with a short-term one. The isovalue curve for a short-term system can be expressed in terms of Π_1 as

$$\Pi_1=\frac{V_s}{(\Pi_{11}L-C_1)}. \tag{16}$$

These curves will asymptotically approach the vertica line at which $\Pi_{11}=P_1+\lambda_2$, where $\lambda_2=(C_1-P_1L)/L$. The isovalue curve for a long-term forecasting system expressed in terms of Π_1' is

$$\Pi_1'=\frac{V_L}{(\Pi_{11}'L-C_2)}. \tag{17}$$

These curves will asymptotically approach the vertical line at which $\Pi_{11}'=P_1+\lambda_1$, where $\lambda_1=(C_2-P_1L)/L$. Since $P_1<C_2/L$, it is obvious that $\lambda_1>0$ and $\lambda_2>0$ and that $\lambda_2>\lambda_1$.

If A in figure 6 is the current forecasting system and V_{sc} is the short-term isovalue curve through A, and if V_{1c} is the long-term isovalue curve equal in value to the current short-term forecasting system [one can see that this curve will be to the left of V_{sc} by studying eq (16) and (17)], then those points between V_{sc} and $P_{\max}$ or on

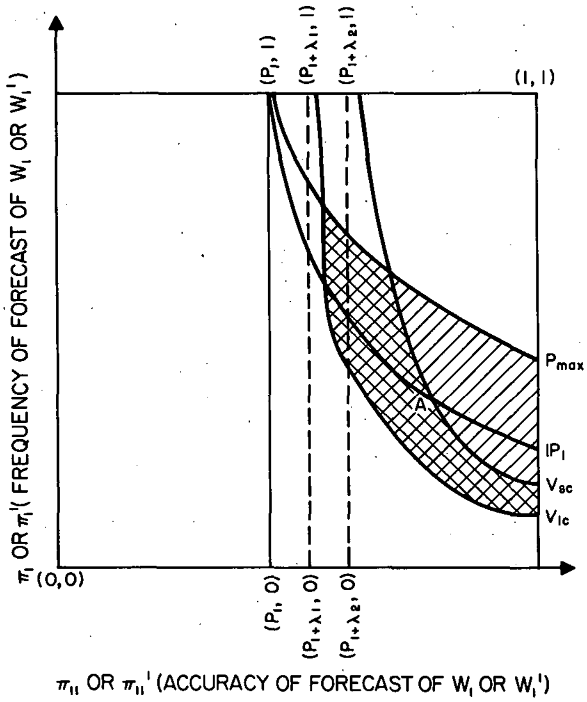

FIGURE 6.—Area of superior extended-term forecasting systems.

P_{max} (the hatched area in fig. 6) represent short-term forecast systems economically superior to the one at A. and those points between V_{1c} and V_{sc} (the cross-hatched area) represent extended-term forecast systems that have negative gains in accuracy but larger absolute gains in increased time warnings. Points between V_{sc} and P_{max} (the hatched area) represent extended-term forecast systems that, if possible, have positive gains in both accuracy and increased time warnings. Here, any possible combination of changes in Π_1, Π_{11}, and Π_{22} are possible in an improved system. These graphical analyses make it possible to see at a glance the potential for improvements (the size of the hatched area) and also the changes in accuracies and frequencies that are necessary to make a new forecast an improvement from an economic point of view. Along this same line, they also clearly demonstrate the relationship between the relative sizes of the parameters of forecast systems and their economic value.

It must be remembered that all of the analyses so far in this paper were in terms of one firm with a specified level of costs and losses due to weather events. To make the mathematical formulation general, one has to perform the operations on each firm in an economy (a formidable task indeed) and then take the sum, making sure that no double counting has taken place.

To make the graphical depiction of improved forecasting systems general, one must find the set of forecasting systems that will lead to an improvement for each firm. Any forecasting system that is in the intersection of all those sets will be a forecasting system that will definitely be an improvement for the whole society. Those forecasting systems that are in some of the sets but not others, will help some sections of the economy but hurt others. In this case, one runs into the familiar problem of interpersonal comparisons of utility.

3. CASE STUDIES

Introductory Comments

The remainder of the paper deals with two case studies that make use of some of the techniques described above in evaluating the benefits of 6- to 10-day weather forecasts to specific industries. The first study involves pea farming, an activity very sensitive to temperature, while the second example focuses upon logging, which is sensitive to rain.

One difficulty with these studies is that only current management techniques could be used, and these are, in the main, only short-range activities. To assess the true value of future forecasting systems, one should use in the study the activities that will be used by firms having access to the systems. It is probably not too rash a prediction, in light of past improvements, to say that the management techniques (or activities) open to a firm in the future will probably differ from those presently available. In fact, it is highly probable that the existence of long-term forecasts per se may encourage the development of new activities to take advantage of increased lead time.

The Pea Study

The growing of peas is greatly affected by temperature. To reach maturity, the peas must receive a specified number of degree-days. Such other elements as soil moisture content affect the maturation rate of peas, but, for purposes of this study, degree-days will be the only factor considered.

Peas are especially sensitive to temperature because they become valueless if they receive an excessive number of degree-days. Because of this, they have to be harvested at their prime or the grower has lost his crop; accordingly, the planting is done in such a way that the farmer will be able to harvest them as they ripen, given his existing equipment.

However, if during the harvesting season the temperature remains above normal for 2 or more days, more acres of peas will accumulate the desired number of degree-days in one 24-hr period than can be handled. Extended-term weather forecasting will help the pea grower if it can give him enough warning to make the harvesting of those extra acres possible.

Operational records from the National Fruit Canning Company of Burlington, Wash.[1], a food processor that handles peas, were studied to estimate how much this advance warning would be worth. The particular variety of peas that they grow requires 1,600 degree-days with a base of 40°F.

National Fruit Canning Company does not own the land on which its crops are grown but has contracts with farmers in the area for the use of their land. The main provisions of these contracts are that the farmer will buy the seed from National Fruit, will sow it themselves, and will cultivate and apply weed killers to the crop during its growth. National Fruit will harvest the crop at its own expense and will pay the grower a price per ton determined

[1] Mention of a commercial company does not constitute an endorsement.

by the tenderometer reading of the peas. After a certain point, the higher the tenderometer reading, the less the farmer will receive. Tenderometer readings vary directly with the number of degree-days accumulated. National Fruit also has the option to bypass some acreage if the weather conditions are such that too many acres are ripening at the same time. If they choose to do this, the terms of the contract stipulate that they must pay the farmer $105.00 an acre, and they must clear the land of the pea vines at a later date, which costs about $4.00 an acre, according to representatives of the firm. If this decision is made, the individual farmer makes practically nothing for the year, because the $105.00 barely covers the cost of seed and pesticide. The Company loses also: it has paid $109.00 an acre, it has no peas to sell, and it also has an angry farmer to deal with in future years.

To put this problem into the model described earlier, we had to determine the relationship between extra degree-days and extra acres to be harvested. This was found in the following way. Using climatological information on degree-days by week during the planting and harvesting periods, we derived an optimum long-run planting schedule. Then, assuming this schedule has been used and assuming that normal temperatures have prevailed up to the time of study, we determined the effect of the accumulation of 5 extra degree-days per day over a period of 8 days on the maturation rate for each week of the harvest period. With this information, it was possible to determine how many acres would ripen early during each 8-day period. The dates of these hypothetical 8-day hot spells and the number of days (N) when extra batches of peas needed immediate harvest because of them are July 9 (2), July 16 (2), July 23 (1), July 30 (1), August 6 (2), August 13 (3), and August 20 (2).

If the weather returns to normal, the anticipated number of degree-days will accumulate each day and the problem will be over; therefore, no long lasting effects follow a given spell. The effects are fully accounted for by the number of extra batches listed above.

For weather forecasting to be of value, a firm must know how the weather will affect it and what alternatives are available to avoid or minimize the adverse effects of the activities; these will be discussed in the context of the week of July 23, when a hot spell ripens 100 extra acres of peas.

As a first alternative, the firm can simply continue its normal routine and do the best it can if hot weather comes. (Call this A_1.) Since it is capable of handling 120 acres a day and a normal batch is 100, "the best it can" means that 20 of the extra 100 acres can be harvested on the day hot weather arrives and another 20 on the following day. The remaining 60 acres will have to be left to rot.

The second alternative is to plan on early harvesting; that is, on the 2 days before the predicted extra batch arrives, the firm harvests 20 extra acres a day. If the bad weather does come, the worst it can do is to bring forth 60 extra acres, 40 of which can be handled in the manner described in the previous paragraph. Hence, only 20 acres are left to rot.

		W_1 Normal Weather	W_2 Hot Spell
A_1	By-pass	0	20 acres (-$90) + 60 acres (-$199) or -$13,740
A_2	Harvest early	40 acres (-$10) or -$400	40 acres (-$10) + 20 acres (-$90) + 20 acres (-$199) or -$6,180
A_3	Harvest early and contract out extra	40 acres (-$10) + X [40 acres (-$90)]	40 acres (-$10) + X [40 acres (-$90)] + (1-X)[20 acres (-$90) + 20 acres (-$199)]

FIGURE 7.—Game box for week of July 23.

Finally, the firm can contract for other firms to harvest and process the extra acres. This is not always possible, however. Whether another firm can take on extra work depends upon its own harvesting and processing capacity, its planting schedule, and the weather in its area.

From these descriptions one can see that extended forecasts would be of help to this firm. It could use the extra warning time either to begin harvesting early or to make arrangements for help from other firms, including the shifting of packages and shipping labels to the other firm's plant.

Keeping this in mind and using the following cost data, one can construct a game box for this week. The relevant cost data are as follows:

1. Normal profit per acre is $90.00.
2. Harvesting the acreage 1 or 2 days early reduces the profit by $10.00.
3. Harvesting the acreage 1 day late eliminates the $90.00 profit.
4. Contracting the peas out eliminates the $90.00 profit.
5. Letting the peas rot cost the firm $109.00 plus the loss of normal profit.
6. Harvesting the acreage more than 1 day late cost more than letting them rot.

The above were derived with the help of National Fruit Canning Company management personnel. Some of the figures were based on actual operational records (i.e., profit per acre) but others are necessarily estimates. Similar figures were given by another processing firm in the area. It is felt, therefore, that they are reliable.

Figure 7 contains the game box that the firm would use during the week of July 23. The X in A_3 represents the probability of having another firm take care of the extra acres. Because less is lost on those acres harvested early than on those contracted out, it makes sense to harvest early and to ship out only what cannot be handled, There is no $(1-X)$ term in the (A_3-W_1) box because. in the event of W_1, no extra acres develop and nothing is lost in the absence of arrangements to handle the extra peas.[2]

If the exact make-up of the future 6- to 10-day forecast system were known, it would be an easy matter to estimate the dollar value of the system using the game box. Unfortunately, that information will not be available until several years after the system has begun operation. The next best thing is to find the value of a series of forecasts. Fleagle (1969) estimated that the accuracy of predicting hot spells will be 0.8; that is, he says that Π_{22} will be 0.8. Therefore, by specifying either Π_1, which automatically determines Π_2, or Π_{11}, which determines Π_{21}, a specific forecast system will be defined. For purposes of this study, Π_{11} was specified because we felt that it gives more descriptive information. If Π_{22} is specified to be 0.8 (i.e., Π_{12} is 0.2), then it is possible to solve for Π_1 and Π_2—which is $(1-\Pi_1)$—by using the identity $\Pi_1\Pi_{11}+\Pi_2\Pi_{12}=P_1$.

A few more points must be investigated before going on with the actual study, however. First, it is clear that A_3 should be taken when a forecast of hot weather is received; but should A_1 or A_2 be used when the forecast is for normal weather? That is, which will have the lower expected cost, given a forecast of F_1? This can be answered by solving the following equation, which tells the value of Π_{11} for which the two activities have the same expected cost:

$$\text{Expected cost } A_1|F_1=\text{Expected cost } A_2|F_1$$

or

$$\Pi_{11}(0)+\Pi_{21}(13{,}740)=\Pi_{11}(400)+\Pi_{21}(6{,}180).$$

Substituting $(1-\Pi_{11})$ for Π_{21} and solving for Π_{11}, we obtain

$$\Pi_{11}=0.9498.$$

Using numbers above and below this value, one can show that A_2 has the lower expected cost for values of Π_{11} less than 0.9498 and that A_1 has the lower expected cost for values greater than 0.9498. When the value of different systems is being considered for different levels of Π_{11}, this fact must be taken into consideration.

To find the maximum gain to the firm for the week of July 23 for specified levels of Π_{11}, one must consider the game box when $X=1$ (fig. 8). For the sake of convenience, all costs are represented as positive numbers in figure 8. The probability of normal weather (P_1) during this week is 0.83. [This figure as well as the other climatological data used in these studies was supplied by Phillips (1970).] A_2 would, therefore, be the climatologically superior activity since 0.83 (400) +0.17 (6,180) or $1,382.60 is less than 0.83 (0) +0.17 (13,740) or $2,235.80.

Remember that the value of a forecast system is the expected cost of using climatology minus the expected cost of using the forecast system. When Π_{11} is less than 0.9498 the value of a system is

$$V=1{,}382.60-\Pi_1\ (\Pi_{11}\ 400+\Pi_{21}\ 6{,}180)-\Pi_2\ (4{,}000).$$

[2] In the previous theoretical discussion, W_1 represented unfavorable weather. In the following case studies, however, W_1 and associated parameters, Π_1, Π_{11}, and P_1 represent normal (or favorable) weather conditions.

	W_1	W_2
A_1	0	13,740
A_2	400	6,180
A_3	4,000	4,000

FIGURE 8.—Simplified game box for week of July 23.

TABLE 1.—*Value of different accuracies during week of July 23, given that* $\Pi_{22}=0.8$

Π_{11}	Π_2	Value	Marginal value
		($)	($)
0.84	0.016	16.38	-----
.86	.046	47.10	30.72
.88	.074	75.77	28.67
.90	.010	102.40	26.63
.92	.1249	127.89	25.49
.94	.1486	152.16	24.27
.96	.1711	243.13	90.97
.98	.1924	392.00	148.93

When Π_{11} is greater than 0.9498, the value of a forecasting system is

$$V=1{,}382.60-\Pi_1\ (\Pi_{21}\ 13{,}740)-\Pi_2\ (4{,}000).$$

Table 1 shows the value of the appropriate value expression for different values of Π_{11} assuming that $\Pi_{22}=0.8$. The value of Π_2 corresponding to the value of Π_{11} is also given. The last column shows the marginal value or the increment in value as Π_{11} increases by 0.02. This is a useful figure because it shows what will be gained by small improvements in a system. The marginal values do not decrease continuously because, when Π_{11} changes from 0.94 to 0.96, the activity used when good weather is forecast changes. To complete the study, we made a similar game box and similar calculations for each of the remaining weeks in the harvest season, assuming that $X=1$.

Table 2 summarizes the results of these studies and shows total maximum values to the firm for the whole season for different levels of Π_{11} (i.e., for different accuracies) given $\Pi_{22}=0.8$.

The marginal values are not continuously decreasing as one would expect because the total value is the summation over the weeks of the summer. Each week has its own climatological probabilities of fair weather; therefore, they each have their own minimum level of accuracy (Π_{11}) for the forecast to have value. Thus, as Π_{11} increases, certain points are reached where the value of more weeks are added to arrive at total yearly savings. The marginal increase in value then is made up of the increase in weekly

TABLE 2.—*Economic benefits for the season of different accuracies if* $\Pi_{22}=0.8$

Π_{11}	Total value	Marginal value	Marginal value as Π_{22} goes to 0.9	Savings as a percentage of profit	Savings to Skagit County	Savings to Skagit and Snohomish Counties	Savings to Skagit, Snohomish, & Whatcom Counties
	($)	($)	($)	(%)	($)	($)	($)
0.80	126.65	126.65	21.09	0.029	493.29	610.74	681.21
.82	244.96	118.31	42.30	0.056	952.56	1,179.36	1,315.44
.84	490.37	245.41	89.69	0.111	1,888.11	2,337.66	2,607.39
.86	736.48	246.11	142.49	0.167	2,840.67	3,517.02	3,922.83
.88	1,160.17	423.69	178.58	0.263	4,473.63	5,538.78	6,177.87
.90	1,800.76	640.59	328.20	0.408	6,940.08	8,592.48	9,583.92
.92	2,471.39	670.63	481.74	0.560	9,525.60	11,793.60	13,154.40
.94	3,121.58	650.19	609.88	0.708	12,043.08	14,910.48	16,630.92
.96	4,946.44	1,824.86	764.48	1.122	19,085.22	23,629.32	26,355.78
.98	6,913.95	1,967.51	919.06	1.568	26,671.68	33,022.08	36,832.32

TABLE 3.—*Economic benefits for the season of different accuracies if* $\Pi_{22}=0.9$

Π_{11}	Total value	Marginal value	Savings as a percentage of profit	Savings to Skagit County	Savings to Skagit and Snohomish Counties	Savings to Skagit, Snohomish, and Whatcom Counties
	($)	($)	(%)	($)	($)	($)
0.80	147.74	147.74	0.034	578.34	716.04	798.66
.82	287.26	139.52	0.065	1,105.65	1,368.90	1,526.84
.84	580.06	292.80	0.132	2,245.32	2,779.92	3,100.68
.86	878.97	298.91	0.199	3,384.99	4,190.94	4,674.51
.88	1,386.75	507.78	0.314	5,341.40	6,612.84	7,375.86
.90	2,128.96	742.21	0.483	8,215.83	10,171.98	11,345.67
.92	2,961.22	832.26	0.671	11,413.71	14,131.26	15,761.79
.94	3,755.16	793.94	0.852	14,492.52	17,943.12	20,013.48
.96	5,801.60	2,046.44	1.316	22,385.16	27,714.96	30,912.84
.98	7,956.67	2,155.07	1.804	30,686.04	37,992.24	42,375.96

savings due to increase in accuracy plus the additional savings of the added weeks.

Column 5 of table 2 is savings shown as a percentage of total profits. (Total profits equal 4,900 acres times $90.00, or $441,000.) These figures were used in the following manner to extrapolate the gains from a specific firm to the industry in a region where weather conditions are similar to those affecting the firm. The number of acres of peas grown in Washington has increased steadily over the past 20 yr; it recently has been averaging 90,000 acres/yr (Anon. 1969). Skagit County, Washington, in which National Fruit is located, normally grows 21 percent of the state's crop, or 18,900 acres (Anon. 1957). Since the normal profit per acre is $90.00, the normal total profit for the county is $1,701,000/yr. This figure was multiplied by the percentage of profit figure for each level of Π_{11} to get a total saving possible to the county.

Columns 7 and 8 show this possible saving, respectively, to Skagit and Snohomish counties (which together account for 26 percent of the state's crop) and to Skagit, Snohomish and Whatcom counties (which account for 29 percent). Snohomish and Whatcom counties border Skagit county.

Table 3 is identical to table 2 except that here Π_{22} was assumed to be 0.9. Column 4 of table 2 is the marginal gain from going from $\Pi_{22}=0.8$ to $\Pi_{22}=0.9$ at all levels of Π_{11}. It is interesting to note that the gain from increasing Π_{22} by 0.1 is less than the gain from increasing Π_{11} by 0.02.

The values in tables 2 and 3 are actually underestimates of gains since they were arrived at by assuming some acres were harvested early or late; but, if extra equipment were available, they could be harvested on time and, hence, more profitably.

Lave (1963) pointed out that in some cases improved weather forecasting that is beneficial to individual firms may be harmful to the industry as a whole because the selling price may fall due to increased output. If the fall in price is large enough relative to the increase in output, total revenues for the industry may fall. It is unlikely that this would be the case for the pea industry for two reasons. First, there is a nationwide market for peas, and unfavorable weather in one growing area will have a very small effect on the price. Second, the expected amount of peas saved in the local area studied was small in relation to total production.

There is one problem with the above extrapolation, however. If all firms are warned that bad weather is coming, and if bad weather in one part of the area is generally accompanied by bad weather in all parts, then it is unlikely that any one firm will be able to contract out any of its extra batches of peas.

If and when new forecasting skills are developed, however, it may prove profitable for companies to combine in buying standby equipment because of the potential savings of tens of thousands of dollars to an area. For one firm alone, this expedient would probably not be profitable because its use of the additional equipment

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	W_1 (less than 0.06 in./week)	W_2 (more than 0.06 in./week)
A_1 (Dirt road)	0.4	1.4
A_2 (Gravel road)	1.0	1.0

FIGURE 9.—Game box for logging study.

would not be frequent enough. More work will be needed to determine the areawide need for such equipment and the terms of payment and use of it.

The Logging Study

Logging is also very sensitive to weather. Too much rain, for instance, can make logging roads impassably muddy unless gravel is applied. Too little rain accompanied by hot spells increases fire danger, which can make logging and slash removal impractical and sometimes unlawful.

The effects of improved forecasting systems on logging road costs were chosen for study because this is the only aspect of logging for which a game box can be adequately defined under present research conditions. For example, it would be difficult to build a game box for slash removal, because of the wide range of costs that bad weather can impose. Unfavorable weather brings only the possibility of fire, and, even if a fire is started, its costs will depend on the location and geography of the area and on the nearness of firefighting equipment. An expected cost of bad weather could be developed, but it would have a large variance and would require a great amount of microanalysis.

When a decision is made to log a certain area, a main road is laid through and spur roads are built from it. In the Northwest, main roads are always surfaced with loose gravel to prevent them from becoming too muddy. Spur roads are customarily graveled also; but, on occasion when dry weather (less than 0.06 in./week) is expected for a week or more, no gravel is applied since it normally takes only about 1 week to work the area serviced by a spur. If loggers had 7- to 10-day forecasts of rainfall, they would have a better idea of when they could safely use spurs without gravel.

Two different branches of the Weyerhaeuser Company, a large wood-products firm, were studied to see by how much a 7-day forecast system could reduce road costs.

The Springfield, Oreg., Area Study. The Springfield area of Weyerhaeuser Company is located in the west-central part of the State of Oregon. Because of company policy, only the following percentage data were available. When dirt roads are used, road costs per million board feet (Mbf) are 40 percent of those when graveled roads are used. On the average, 5 percent of the annual harvest is over dirt roads. This means that the average cost per Mbf is 97 percent of the cost of harvesting one Mbf over graveled roads. [0.4 (0.5)+1.0 (0.95)=0.97.] That is, the current forecasting system, which is a combination of existing short-term forecasts, climatology, and "seat of the pants" forecasting by the foreman in charge, reduces costs per unit by 3 percent over climatology. If a long-term forecasting system is to be worthwhile, it must reduce costs even more.

Using the percentage data given above and the fact that no dirt logging is done unless P_1 is at least 0.40, we developed a game box for the area (fig. 9). The value 1.4 in the (A_1-W_2) box was estimated by solving the following equation for X:

$$P_1(0.4)+P_2X=1.0.$$

Since no dirt roads are used unless P_1 (the probability of there being less than 0.06 in. of rain in a week) is at least 0.4, and since they are not used all of the time even then, solving for X will yield a downward-biased estimate of road cost using dirt roads in the event of "bad weather." The equation becomes

$$0.40(0.4)+0.60X=1.0$$

and

$$X=1.4.$$

This means that, if the cost of dirt roads in bad weather is 1.4 times normal, the firm will be indifferent between using graveled roads or dirt roads if $P_1=0.4$. The firm is not actually indifferent when $P_1=0.4$, but, in practice, dirt logging is often used even when $P_1>0.4$. Since the cost of dirt roads has been underestimated for bad weather, the estimate of savings from forecasting systems will be underestimated.

By using value equations derived from this game box (fig. 9) and the known climatological probabilities of dry weather by weeks, one can find the value of the specified forecast system over climatology. In this case the value will be the percentile reduction in cost from using graveled roads all of the time for the period involved, which in this case is 1 week. The initial specification for the systems was set in the same way as in the pea study except that here Fleagle (1969) predicted that Π_{22} would be equal to 0.7.

Column 2 of table 4 is the percentile increase or decrease in yearly costs over the current system that the various systems will yield. (The minus signs indicate negative savings.) These figures were derived from the weekly cost reductions using the following equation:

$$\text{Savings over current method}=1-\left(\frac{52-\sum V}{52}\,\frac{1}{0.97}\right)$$

where ΣV equals the sum of the weekly cost reductions. Since harvesting goes on 52 weeks a year in this area, $52-\Sigma V/52$ will be the yearly average cost of harvesting a thousand board feet expressed as a percentage of the cost of harvesting logs on rock roads. Dividing this by 0.97 (the average cost of using the current system expressed in the same terms) results in an expression of the average cost of harvesting a thousand board feet expressed as a percentage of the average cost of using the current system. One minus this figure is the percentile saving derived by using the forecasting system.

TABLE 4.—*Savings to Springfield, Oreg., area in percentage of road costs*

	$\Pi_{22}=0.7$			$\Pi_{22}=0.8$	
Π_{11}	Total for summer	Margin	Margin as Π_{22} goes to 0.8	Total for summer	Margin
	(%)	(%)	(%)	(%)	(%)
0. 45	−2. 814			−2. 550	
. 50	−2. 150	0. 664		−1. 605	0. 945
. 55	−1. 725	0. 425		−0. 924	0. 681
. 60	−0. 897	0. 828	0. 140	0. 140	1. 064
. 65	−0. 741	0. 156	0. 499	0. 499	0. 359
. 70	−0. 057	0. 684	1. 357	1. 357	0. 858
. 75	0. 717	0. 774	1. 582	2. 299	0. 942
. 80	2. 867	2. 150	0. 927	3. 794	1. 495
. 85	5. 072	2. 205	1. 896	6. 968	3. 494
. 90	5. 218	0. 146	2. 070	7. 288	0. 320
. 95	5. 355	0. 137	2. 240	7. 595	0. 307

TABLE 5.—*Savings to Chehalis, Wash., area in percentage of road costs*

	$\Pi_{22}=0.7$			$\Pi_{22}=0.8$	
Π_{11}	Total for summer	Margin	Margin as Π_{22} goes to 0.8	Total for summer	Margin
	(%)	(%)	(%)	(%)	(%)
0. 45	−2. 058		0. 282	−1. 776	
. 50	−1. 676	0. 382	0. 562	−1. 114	0. 662
. 55	−1. 236	0. 440	0. 767	−0. 469	0. 645
. 60	−1. 010	0. 226	0. 872	−0. 138	0. 331
. 65	0. 335	1. 345	1. 159	1. 494	1. 632
. 70	1. 583	1. 248	1. 143	2. 926	1. 432
. 75	1. 726	0. 143	1. 530	3. 256	0. 270
. 80	1. 853	0. 127	1. 672	3. 525	0. 269
. 85	1. 950	0. 097	1. 794	3. 744	0. 219
. 90	2. 032	0. 082	1. 920	3. 952	0. 208
. 95	2. 100	0. 068	2. 170	4. 117	0. 165

It can be seen that Π_{11} must be at least 0.75 when $\Pi_{22}=0.7$ for the forecasting system to be an economic improvement over the current system. The savings for the area can get as high as 5.3 percent of road costs if $\Pi_{11}=0.95$.

Column 3 shows the marginal percentile increase in savings as Π_{11} increases by increments of 0.05. Column 4 shows the marginal increase in savings if Π_{22} increases from 0.7 to 0.8.

In columns 5 and 6, it is assumed that $\Pi_{22}=0.8$. For the forecast system to be an economic improvement over the current system, Π_{11} has only to equal 0.60, and savings can be as large as 7.6 percent of the annual road budget if $\Pi_{11}=0.95$.

The Chehalis, Wash., Area Study. The Chehalis area is located in the southwest part of Washington. Besides the climatological data, the only difference between this case and the previous one is that here only 4 percent of the total harvest is taken out over dirt roads. This means that the average cost of roads per thousand board feet is 0.976 of the cost of harvesting a thousand board feet over graveled roads. Table 5 is similar to table 4. In Chehalis, when $\Pi_{22}=0.7$, Π_{11} has to be at least 0.65 before a forecasting system can be better than the current manner of making decisions and the savings can go as high as 2.1 percent of current road costs if $\Pi_{11}=0.95$. When $\Pi_{22}=0.8$, Π_{11} again has to be at least 0.65 and the savings can get as high as 4.1 percent when $\Pi_{11}=0.95$.

4. CONCLUSIONS

Two models of the economics of long-term forecasting are presented. Both models show the optimal way to react to a forecast, and mathematical expressions of their value are derived. The value of extended-period forecast systems to two industrial firms was determined, using these economic models.

For the two industries studied, 10-day forecasts of fairly high accuracy will be of great value. Given the assumptions made in these studies, the values presented are probably underestimates. To obtain the total value of such extended-period forecasts, one must undertake further studies of all weather-sensitive operations, be they firms, households, or government agencies. The cost of such a task would very likely be higher than the value of the information provided. However, research is required on a sufficiently large sample of the most important weather information users to make sure that current forecasting abilities are being used to their greatest advantage and to suggest new areas where meteorological research may be beneficial.

ACKNOWLEDGMENTS

This study is part of the author's doctoral dissertation submitted to the University of Washington. The author would like to thank his chairman, James Crutchfield, and Robert Fleagle, Yoram Barzel, and Gardner Brown. Thanks also to the representatives of the firms studied. The author assumes all responsibility for any errors.

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[*Received July 30, 1971; revised November 29, 1972*]