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A Note on Mixed Resolution for Numerical Models

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ABSTRACT—The hypothesis that a numerical model with variables of mixed spatial resolution is as accurate as one having variables of equally high resolution is tested using a global, barotropic, free-surface primitive-equations model and the spectral formulation of Bourke. It is found that models with some variables of high resolution and others of low resolution are superior to one having uniformly medium resolution.

Numerical models for simulation of the atmosphere have generally adopted equal spatial resolution for all dependent variables. It is conceivable, however, that some of the dependent variables are carried with unnecessarily high spatial resolution while others have insufficient resolution. If this hypothesis were true, it would be possible to construct models with higher resolution for some variables and lower resolution for others, which would have the same accuracy as models with high resolution for all variables. With some programming ingenuity, this could result in much-increased computational efficiency.

This hypothesis was tested with the aid of a global free-surface primitive-equations model using the spectral formulation of Bourke (1972).

Rossby's (1938) adjustment theory predicts that in the small scales the mass or geopotential adjusts to the wind field while in the large scales the opposite is true. This theory suggests that for small enough spatial scales it might not be necessary to explicitly predict the mass field since it is controlled by the small-scale wind field. Thus, it would be sufficient to calculate the small-scale wind field by the filtered barotropic equation (conservation of absolute vorticity). The small-scale mass field could be obtained from the small-scale wind field at any time by the reverse balance equation. For the larger scales, the full primitive equations would be used, even though in principle it should be possible to diagnostically calculate the large-scale wind field from a predicted large-scale mass field.

Bourke's spectral formulation of the free-surface equations allows the various spatial scales of the vorticity, divergence, and geopotential fields to be separated easily. We can rewrite Bourke's spectral equations (21), (23), and (24) as

$$\frac{\partial \xi_l^m}{\partial t} = R_1(m, l, t), \tag{1}$$

$$\frac{\partial D_l^m}{\partial t} - l(l+1)\Phi_l^m = R_2(m, l, t), \tag{2}$$

and

$$\frac{\partial \Phi_l^m}{\partial t} + \bar{\Phi} D_l^m = R_3(m, l, t) \tag{3}$$

for $-J \leq m \leq J$ and $|m| \leq l \leq |m| + J$, where

$$\xi_l^m = -l(l+1)\psi_l^m \qquad \text{vorticity,}$$

$$D_l^m = -l(l+1)\chi_l^m \qquad \text{divergence,}$$

and

$$\bar{\Phi} = \text{time-independent global mean geopotential.}$$

R_1, R_2, and R_3 are the residual terms remaining on the right sides of Bourke's equations (21), (23), and (24), respectively. J defines the resolution of the model (considered equal in the latitudinal and longitudinal directions).

We will define three resolutions—a high resolution ($J = J_H$), a medium resolution ($J = J_M$), and a low resolution ($J = J_L$). We chose $J_H = 31$ to correspond roughly with the resolution of the standard 381-km grid, though it should be noted that the truncation error of a spectral model is smaller than for a gridpoint model with equivalent resolution. The low resolution was chosen to be $J_L = 15$, approximately half the resolution of the high-resolution model. This value of J_L was picked because it was found experimentally that the adjustment of the height field to the wind field was fairly complete for scales smaller than this. For the medium-resolution model, we selected $J_M = 25$ because a primitive-equations model with this resolution is approximately as efficient as the mixed resolution model we shall describe below.

In the high-resolution experiment, all variables, ξ, D, and Φ, are truncated at $J = J_H$. Similarly, for the medium- and low-resolution experiments, all variables would be truncated at $J = J_M$ and $J = J_L$, respectively. In a mixed-resolution experiment, ξ would be truncated at $J = J_H$, while D and Φ would be truncated at $J = J_L$. Thus, for the large-scale spectral modes where $-J_L \leq m \leq J_L$ and $|m| \leq l \leq |m| + J_L$, eq (1)-(3) would apply. For the smaller scale spectral modes where $-J_H \leq m < -J_L$ or $J_L < m \leq J_H$ or $|m| + J_L < l \leq |m| + J_H$, only the first equation (the conservation of vorticity) would apply. At any time during the integration, the geopotential field could be recovered via the reverse balance equation, but this is not necessary to carry out the integration. We are attempting to show

TABLE 1.—*The resolutions of the ξ, D, and Φ fields for the five different cases run*

Run	Resolution, J				Method of obtaining geopotential output
		Φ	D	ξ	
0	high	31	31	31	All scales predicted.
1	low	15	15	15	All scales predicted.
2	medium	25	25	25	All scales predicted.
3A	mixed	15	15	31	Large scale only.
3B	mixed	15	15	31	All scales from balance equation.
3C	mixed	15	15	31	Large-scale predicted. Small-scale from balance equation.
4	high	0	0	31	All scales from balance equation.

that the mixed-resolution model is much more accurate than a medium-resolution model.

The experimental procedure was as follows. First, a high-resolution control run was performed and the geopotential field stored every 8 hr. Then low-, medium-, and mixed-resolution runs were performed and the root-mean-square (rms) difference from the control was calculated. A final comparison was made by integrating the nondivergent vorticity equation at high resolution and recovering the height field by the reverse balance equation for output purposes.

All runs were performed with an initial 500-mb hemispheric stream function field obtained from the Canadian operational gridpoint nonlinear forward-balance routine. This stream function was then harmonically analyzed and truncated for all cases at $J=J_L$, to ensure that all models were run with identical initial wind fields. The initial wind field was nondivergent and the initial height field was obtained by the (spectral) reverse nonlinear balance equation. Because of the nonlinearity of the balance equation, the initial height fields differ slightly for the different cases even though the initial stream functions are identical. The mean-free-surface height was 5603 m. The runs are summarized in table 1.

The rms differences over the hemisphere between cases 1–4 and the control (case 0) for a 4-day forecast are plotted in figure 1.

If we assume that the high-resolution experiment (run 0) is accurate, then the best results are obtained from the mixed-resolution model in which the large-scale mass field is predicted and the small-scale mass field is obtained from the reverse balance equation when needed for output purposes (run 3C). To make sure that the results were independent of the mean height of the free surface or the initial wind field, we repeated the experiment with a different initial stream function and a free-surface mean height of 1.5 km, yielding substantially the same results as given in figure 1.

The results clearly demonstrate that, at least for the barotropic primitive-equations system, very little nonredundant information is carried in the smaller scales of the mass field.

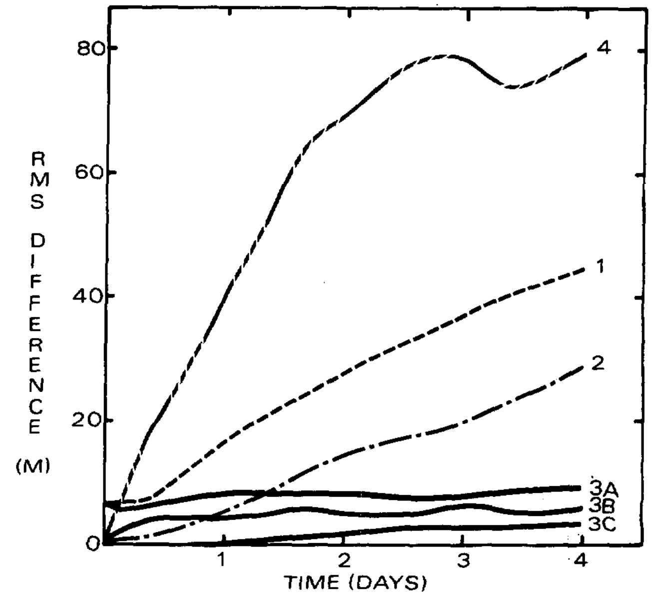

FIGURE 1.—Global root-mean-square differences of the geopotential as a function of time between cases 1, 2, 3, and 4 and the control (case 0). Particulars of the various cases are described in table 1.

The application of the mixed-resolution method to baroclinic models is not straightforward. Okland's (1972) investigations of the adjustment process in a linearized *f* plane baroclinic model imply that the adjustment of the horizontally small-scale geopotential field to the horizontally small-scale wind field becomes less rapid and less complete as the vertical wave number increases. This suggests that the conservation of potential vorticity rather than the conservation of absolute vorticity should be the governing equation for the smaller horizontal scales in a mixed-resolution, multilevel model.

The use of mixed resolution could also be considered to increase the efficiency of more complex models that include moisture or other physical variables with dominant scales of variation very different from that of the wind or the mass field.

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