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The Effect of Thermal Stratification and Evaporation on Geostrophic Drag Coefficient in the Atmospheric Boundary Layer

GOUR-TSYH YEH [1]—*Visiting Research Associate, Earth Observation Division, NASA-Manned Spacecraft Center, Houston, Tex.*

ABSTRACT—The turbulent energy equation, together with a modified mixing length, is adopted to close the governing equations in the turbulent planetary boundary layer under stratification conditions. The system of governing equations is decoupled by introducing a stratification parameter, Q. Numerical integration of the system, in general, is outlined and should be easily implemented. Results for the momentum fluxes, eddy viscosity, geostrophic drag coefficient, and the angle between the geostrophic wind and the surface shear stress, which are obtained from the decoupled system with Q as a parameter, are presented for Q ranging from -100 to 100. They are compared, when possible, with the measurements. The effects of both the thermal and humidity stratification are indexed by Q. The model shows that, in the presence of evaporation or humidity stratification, the geostrophic drag coefficient may differ greatly from the case of thermal stratification only. The planetary boundary thickness and other main features of the turbulence depend on Q.

1. INTRODUCTION

The dependence of the geostrophic drag coefficient on the surface Rossby number and other turbulent characteristics in the planetary boundary layer under neutral conditions has been studied by several authors (e.g., Lettau 1959, Blackadar 1962, Blackadar and Tennekes 1968, Deardorff 1970). The effects of thermal stratification were subsequently introduced (e.g., Bobyleva et al. 1967, Yamamoto et al. 1968, Pandolfo 1969). In addition to the thermal stratification, it is intended in this paper to take the specific humidity into consideration. The present solution will furnish an auxiliary solution when the interaction between the atmosphere and water bodies is encountered.

A steady, horizontal, uniform case for an incompressible fluid is studied, in which the turbulent exchange is considered only in the vertical direction. This will provide the initial condition for a later investigation of the unsteady case. The governing equations of motion, energy, and water vapor with the semiempirical K-theory for the quantities of fluctuation correlations are (e.g., Laikhtman 1964)

$$\frac{d}{dz}\left(K\frac{du}{dz}\right)+f(v-v_g)=0, \tag{1}$$

$$\frac{d}{dz}\left(K\frac{dv}{dz}\right)-f(u-u_g)=0, \tag{2}$$

$$\frac{d}{dz}\left(a_hK\frac{d\theta}{dz}\right)+\frac{1}{c_p\rho}\frac{dR}{dz}=0, \tag{3a}$$

and

$$\frac{d}{dz}\left(a_vK\frac{dq}{dz}\right)+\frac{1}{\rho}\frac{dM}{dz}=0 \tag{4a}$$

where u and v are the x and y components of the mean velocity, K is the eddy viscosity, u_g and v_g are the x and y components of the geostrophic wind, ρ is the density of the air, c_p is the specific heat at constant pressure, θ is the mean potential temperature, q is the mean specific humidity, f is the Coriolis parameter, a_h and a_v are ratios of eddy conductivity and eddy diffusivity to the eddy viscosity, respectively, and R is the sum of net radiation flux over all wavelengths. M is the vapor flux due to mechanisms other than the turbulent fluctuation.

Integrating eq (3a) and (4a) yields

$$\theta=\theta_0-\int_{z_0}^{z}\frac{(H-R)}{c_p\rho a_h}\frac{dz}{K} \tag{3b}$$

and

$$q=q_0-\int_{z_0}^{z}\frac{E}{\rho a_v}\frac{dz}{K}+\int_{z_0}^{z}\frac{M}{\rho a_v}\frac{dz}{K} \tag{4b}$$

where H, E, θ_0, and q_0 are integration constants representing the heat flux, evaporation, potential temperature, and specific humidity at the surface, respectively. These four constants will be determined from the boundary conditions imposed on θ and q.

A fifth equation is needed to close the system [eq (1)–(4)] for five dependent variables u, v, θ, q, and K. Yamamoto et al. (1968) assumed that

$$K=\frac{kzu_*}{\phi} \tag{5}$$

where k is the von Karman constant, u_* is the frictional velocity, and ϕ is a dimensionless wind shear function obtained with the assumption of constant shear stress and heat flux (Yamamoto 1959). This is inconsistent with the boundary layer. Deardorff (1971) used a subgrid-scale eddy coefficient to obtain K in the process of numerical solution. Pandolfo (1969) used different semiempirical formulas for K under different stability conditions. This approach

[1] Now Senior Engineer, Ebasco Services Inc., 21 West Street, 11th Floor, New York, N.Y.

seems sound but fails to give a unified formulation for K. Furthermore, the stability condition cannot be chosen as a priori but should emerge as a result of the solution in the case of unsteady flow. Therefore, in the processes of finding a numerical solution with an iterative technique, one has to check the stability conditions and insert the correct K for each step; this greatly complicates the calculation. Hence, a more theoretically oriented turbulent energy equation, which will yield a unified formulation for K, is highly desirable.

Bobyleva et al. (1967) used the turbulent energy equation aided by von Karman's similarity hypothesis to determine the mixing length. Brutsaert and Yeh (1973) also applied that equation but with linearly varying mixing length to the surface layer. Thus, in the present modeling, this concept will be employed to close the system [eq (1)–(4)]; that is,

$$K^2=l^2\left[\left(\frac{du}{dz}\right)^2+\left(\frac{dv}{dz}\right)^2-\frac{\gamma g a_h}{T_0}\frac{d\theta}{dz}-0.61\ g a_v \gamma\frac{dq}{dz}\right] \quad (6)$$

where γ is an empirical constant, and T_0 is a reference temperature. The central problem lies in the determination of the mixing length, l. The von Karman theory is not valid throughout most of the planetary boundary (Blackadar 1962). Observations showed (Mildner 1932) that the mixing length could not continue to be proportional to the distance from the ground outside the surface layer. Therefore, l will be modified according to Blackadar (1962) such that

$$l=\frac{kz}{1+\frac{z}{\lambda}} \quad (7)$$

where λ is an empirical constant.

2. BOUNDARY CONDITIONS AND METHODS OF SOLUTION

The boundary conditions for the system [eq (1)–(4) and eq (6) and (7)] are obtained from the following physical considerations:

1. At infinity, the mean velocity approaches the geostrophic wind.
2. At the surface, the shear stress approaches a constant surface stress, τ_0.
3. The x coordinate will be set to coincide with the direction of shear stress at the surface.
4. At infinity, the mean potential temperature and mean specific humidity approach their free atmosphere values.
5. At the surface, the specific humidity is saturated and hence a unique function of the temperature.
6. At the surface, the net energy flux equals zero. In other words, the net solar plus the net atmospheric radiation fluxes are used up by sensible heat transfer from the surface into the atmosphere and into the water body, by evaporation, and by the long-wave back radiation from the surface into the atmosphere.

From items (1), (2), and (3), one has

$$u=u_g \text{ and } v=v_g \text{ at } z=\infty \quad (8)$$

and

$$K\frac{du}{dz}=u_*^2 \text{ and } K\frac{dv}{dz}=0 \qquad \text{at } z=z_0 \quad (9)$$

where z_0 is the roughness length of the surface and $u_*=\sqrt{\tau_0/\rho}$ is the frictional velocity at the surface. From items (4), (5), and (6), one has

$$\theta=\theta_g=\theta_0-\int_0^\infty\frac{(H-R)}{a_h c_p \rho}\frac{dz}{K} \qquad \text{at } z=\infty, \quad (10)$$

$$q=q_g=q_0-\int_0^\infty\frac{(E-M)}{\rho a_v}\frac{dz}{K} \qquad \text{at } z=\infty, \quad (11)$$

$$R_n^4-\epsilon\sigma\theta_0^4+c_p\rho a_h K\frac{d\theta}{dz}+L\rho a_v K\frac{dq}{dz}-S=0 \qquad \text{at } z=z_0 \quad (12a)$$

or

$$R_n^4-\epsilon\sigma\theta_0^4-H-LE-S=0 \qquad \text{at } z=z_0, \quad (12b)$$

and

$$q_0=q_s(\theta_0) \qquad \text{at } z=z_0. \quad (13)$$

Here, R_n is the net long-wave and short-wave radiation at the surface from the sun and the atmosphere, S is the energy lost to the water body, q_s is the saturated specific humidity (which is a unique function of θ_0,) q_g and θ_g are the values of q and θ in the free atmosphere, respectively, L is the latent heat for the water vapor, ϵ is the emissivity of the surface, and σ is the Stefan-Boltzmann constant.

The system [eq (1)–(4) and eq (6) and (7) subject to conditions (8)–(13)] will be solved numerically. It is usually more convenient to write the equations in dimensionless form, thus reducing the number of parameters involved. For this purpose, a velocity scale and a length scale are needed. It has been shown (Blackadar and Tennekes 1968, Bobyleva 1967) that u_* and u_*/f will be appropriate for such purpose. Hence, defining the characteristic quantities

$$T_*=\frac{T_0 u_* f}{k^2 g\gamma a_h} \quad \text{and} \quad q_*=\frac{u_* f}{0.61k^2 g\gamma a_v} \quad (14a)$$

and

$$H_*=\frac{u_*^2 f\rho c_p T_0}{k^2 g\gamma} \quad \text{and} \quad E_*=\frac{u_*^2 f\rho}{0.61k^2 g\gamma} \quad (14b)$$

and introducing the dimensionless variables

$$\zeta=\frac{(z-z_0)}{h},\quad u_+=\frac{ku}{u_*}, \quad \text{and} \quad v_+=\frac{kv}{u_*} \quad (14c)$$

and

$$\kappa=\frac{K}{ku_* h},$$

$$\theta_+=\frac{k\theta}{T_*},$$

and (14d)

$$q_+=\frac{kq}{q_*}$$

where $h=ku_*/f$ is about the order of the planetary boundary layer thickness, we can reduce eq (1)–(4) and eq (6)–(13) to the following:

$$\frac{d}{d\zeta}\left(\kappa\frac{du_+}{d\zeta}\right)+v_+-\frac{k}{u_*}v_g=0, \tag{15}$$

$$\frac{d}{d\zeta}\left(\kappa\frac{dv_+}{d\zeta}\right)-u_++u_g=0, \tag{16}$$

$$\kappa^4=\iota^4\left[\left(\kappa\frac{du_+}{d\zeta}\right)^2+\left(\kappa\frac{dv_+}{d\zeta}\right)^2-Q\right], \tag{17}$$

and

$$\iota=\frac{\zeta+\zeta_0}{1+\lambda_+(\zeta+\zeta_0)}. \tag{18}$$

Here

$$\left.\begin{aligned}\kappa\frac{du_+}{d\zeta}&=1\\ \kappa\frac{dv_+}{d\zeta}&=0\end{aligned}\right\}\text{ at }\zeta=0, \tag{19}$$

$$\left.\begin{aligned}u_+&=\frac{ku_g}{u_*}\\ v_+&=\frac{kv_g}{u_*}\end{aligned}\right\}\text{ at }\zeta=\infty, \tag{20}$$

$$\theta_+=\theta_{0+}-\int_0^\zeta H_+\frac{d\zeta}{\kappa}+\int_0^\zeta R_+\frac{d\zeta}{\kappa}, \tag{21}$$

and

$$q_+=q_{0+}-\int_0^\zeta E_+\frac{d\zeta}{\kappa}+\int_0^\zeta M_+\frac{d\zeta}{\kappa} \tag{22}$$

where

$$\frac{k\theta_g}{T_*}=\theta_{0+}-\int_0^\infty H_+\frac{d\zeta}{\kappa}+\int_0^\infty R_+\frac{d\zeta}{\kappa}\text{ at }\zeta=\infty, \tag{23}$$

$$\frac{kq_g}{q_*}=q_{0+}-\int_0^\infty E_+\frac{d\zeta}{\kappa}+\int_0^\infty M_+\frac{d\zeta}{\kappa}\text{ at }\zeta=\infty, \tag{24}$$

$$R_{n+}-a\theta_{0+}^4-H_+-bE_+-S_+=0\text{ at }\zeta=0, \tag{25}$$

and

$$q_{0+}=q_s(\theta_{0+})\text{ at }\zeta=0. \tag{26}$$

Here, $\lambda_+=h/\lambda$ is approximately equal to 10 (Rossby and Montgomery 1935), $\zeta_0=z_0/h$, $\theta_{0+}=k\theta_0/T_*$, $q_{0+}=kq_0/q_*$, $R_{n+}=R_n/H_*$, $S_+=S/H_*$, $a=\epsilon\sigma T_*^4/(k^4H_*)$, $b=1/(0.61c_pT_0)$, $H_+=H/H_*$, $E_+=E/E_*$, $R_+=R/H_*$, $M_+=M/E_*$, and

$$Q=-\left\{\frac{k^2g\gamma(H-R)}{u_*^2f\rho c_pT_0}+\frac{0.61k^2\gamma c_pT_0(E-M)}{u_*^2f\rho c_pT_0}\right\}=H_+-R_++E_+-M_+ \tag{27a}$$

is a function of ζ because R and M depend on the height from the surface. In the present study, R will be assumed compensated for by M so that Q is a constant; that is,

$$Q=H_++E_+=\frac{h}{L_*}\left(1+\frac{E}{bH}\right). \tag{27b}$$

In eq (27b), L_* is the Monin-Obukhov length divided by γ (Yamamoto 1959).

Equations (15)–(18) with boundary conditions (19) and (20) can be decoupled from eq (21) and (22) with boundary conditions (23)–(26) if Q in eq (17) is considered as a known parameter. Therefore, for given external parameters, f, G, θ_g, q_g, R_n, S, z_0, and ρ, and constants c_p, k, a_h, a_v, ϵ, σ, and L, the solutions of the systems [eq (15)–(26)] will consist of the following procedures:

1. Assume Q in eq (17) is known.
2. Solve the decoupled eq (15)–(18) with boundary conditions given in eq (19) and (20) to obtain κ, u_+, and v_+. At this stage, the characteristic quantities, H_*, E_*, q_*, and T_* can be calculated with the aid of eq (48).
3. With κ obtained in step 2, solve for θ_{0+}, q_{0+}, H_+, and E_+ from eq (23)–(26).
4. Substitute H_+ and E_+ obtained from step 3 into the right side of eq (27). If the resulting value equals the assumed Q in step 1, then proceed to step 5. Otherwise, repeat steps 1–4 until a correct Q is obtained.
5. With θ_{0+}, q_{0+}, H_+, and E_+ obtained from steps 1–4, the profiles of the potential temperature and specific humidity are then given by eq (21) and (22). Thus, it is seen that the solution of the present problem hinges on the solution of the decoupled system given by eq (15)–(20) with Q as a parameter. Hence, attention will be given to the solutions of this decoupled system with Q as a free parameter in the remainder of this paper.

Let

$$\phi=\kappa\frac{du_+}{d\zeta}$$

and (28)

$$\psi=\kappa\frac{dv_+}{d\zeta}.$$

Differentiating eq (15) and (16) with respect to ζ and making use of eq (28), one obtains the following from eq (15)–(19):

$$\frac{d^2\phi}{d\zeta^2}+\frac{\psi}{\kappa}=0, \tag{29}$$

$$\frac{d^2\psi}{d\zeta^2}-\frac{\phi}{\kappa}=0, \tag{30}$$

$$\frac{\phi^2+\psi^2}{\kappa}-\frac{\kappa^3}{\iota^4}-Q=0, \tag{31}$$

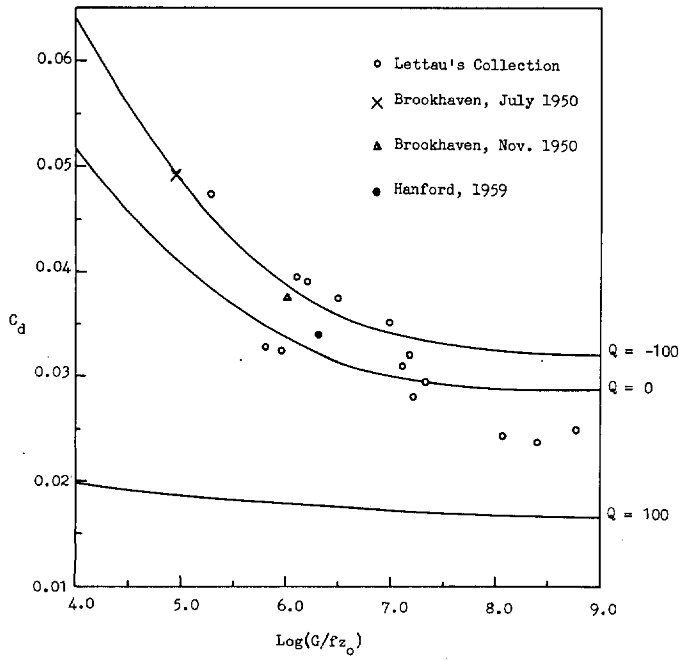

FIGURE 1.—The variation of the geostrophic drag coefficient, C_d, versus the Rossby number, R_o, for $Q=-100$, 0, and 100.

$$\iota=\frac{\zeta+\zeta_0}{1+10(\zeta+\zeta_0)},\tag{32}$$

and

$$\phi=1 \text{ and } \psi=0 \text{ at } \zeta=0.\tag{33}$$

Boundary conditions given in eq (20) require a little further examination. Equations (15) and (16) relate the balance of three kinds of forces; namely, the frictional force (the first term), the Coriolis force (the second term), and the pressure force (the third term). In the meantime, eq (20) says that, at $\zeta=\infty$, the Coriolis force equals the pressure force so that the frictional force is zero; that is,

$$\phi=0 \text{ and } \psi=0 \text{ at } \zeta=\infty.\tag{34}$$

An expanding grid is used in the planetary boundary in which each grid length is 1.25 times the size of the one below it, thus providing greater resolution in the lower layers (Luther 1970). The minimum grid size is 10^{-5}. This is equivalent to about 1 cm in the physical plane if $u_*=25$ cm/s. The expanding scheme presents a problem when it comes to writing differential equations in finite-difference form. The usual method of expressing derivatives as the difference between values of a variable at certain gridpoints results in expressions that are not centered on gridpoints when an expanding grid is used. Most finite-difference methods are not applicable to expanding grids without sacrificing accuracy. To get around this problem, we introduce the transformation

$$\zeta(i)=4\times10^{-5}[(1.25)^i-1]\tag{35}$$

where i is an integer ranging from 1 to ∞. For an evenly spaced grid in i, there corresponds an expanding grid in ζ as desired. Now, to express the derivatives in finite-difference form in i space, we consider i as a continuous variable temporarily; then, for any function F,

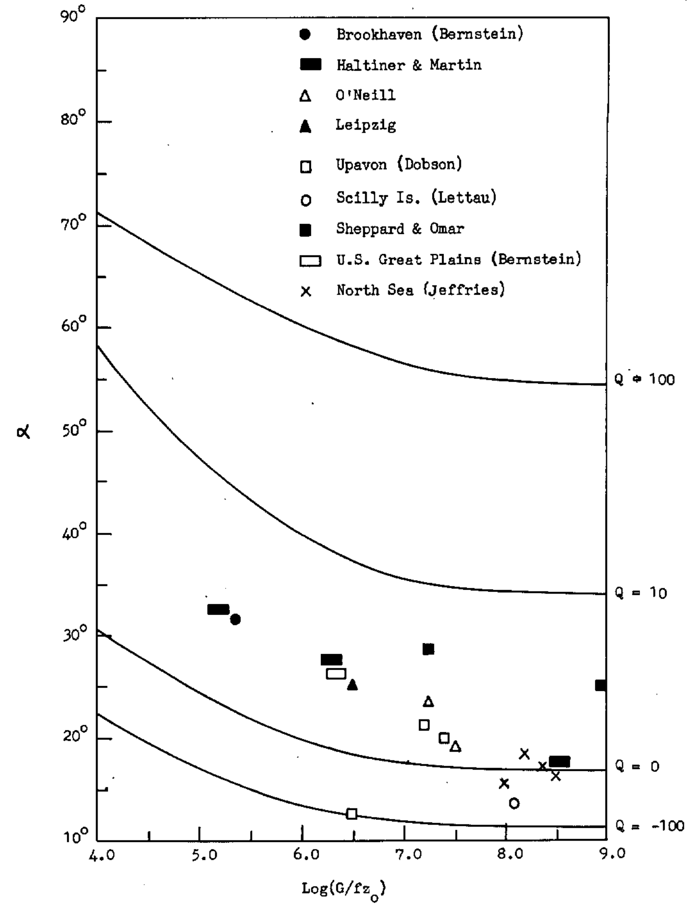

FIGURE 2.—The variation of the angle, α, between the geostrophic wind and the surface shear stress for $Q=-100$, 0, and 100.

$$\frac{dF}{d\zeta}=\frac{dF}{di}\frac{di}{d\zeta}.\tag{36}$$

Solving i from eq (35), one obtains

$$i=\frac{\ln\left(1+\frac{\zeta}{4\times10^{-5}}\right)}{\ln (1.25)}.\tag{37}$$

Taking the derivative of eq (37) with respect to ζ and substituting eq (35) into the result, one has

$$\frac{di}{d\zeta}=\frac{1}{4\times10^{-5}\ln (1.25)(1.25)^i}.\tag{38}$$

Approximating the derivative of F by

$$\frac{dF}{di}=\frac{\Delta F}{\Delta i}\tag{39}$$

and substituting eq (38) and (39) into eq (36), we get

$$\left(\frac{dF}{d\zeta}\right)_{i+1/2}=\frac{F(i+1)-F(i)}{4\times10^{-5}\ln (1.25)(1.25)^{i+1/2}}.\tag{40}$$

The second derivative evaluated at the ith gridpoint

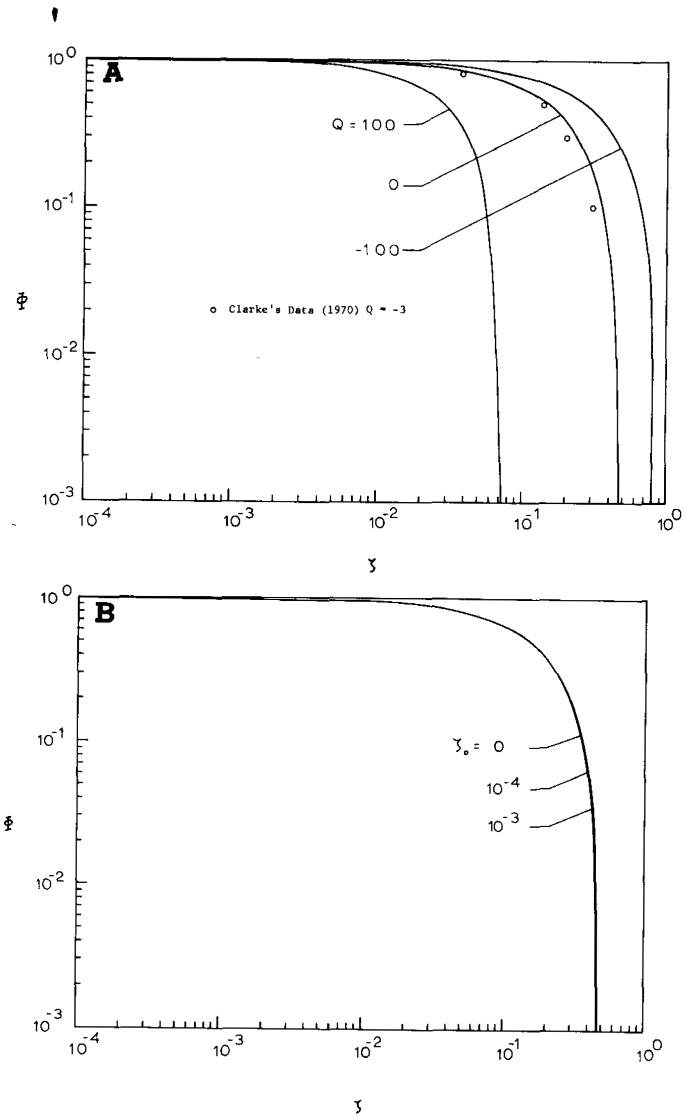

FIGURE 3.—The vertical profile for the x component shear stress, Φ, for (A) $Q=-100$, 0, and 100 with $\zeta_0=0$, and (B) $\zeta_0=0$, 10^{-4}, and 10^{-3} with $Q=0$.

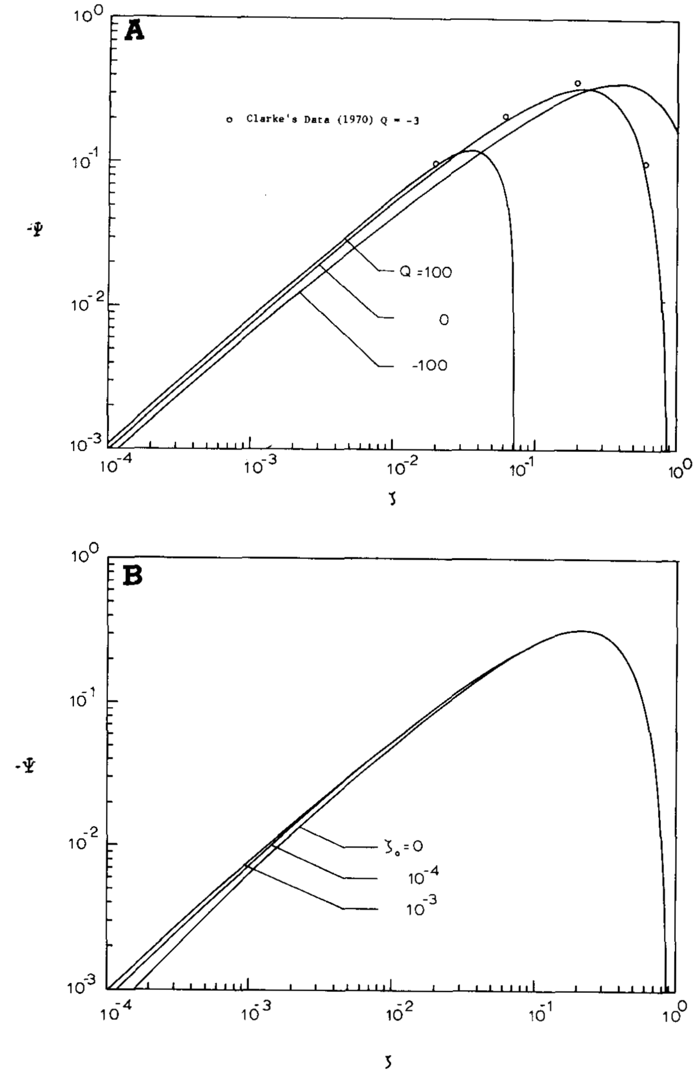

FIGURE 4.—The vertical profile for the y component shear stress, Ψ, for (A) $Q=-100$, 0, and 100 with $\zeta_0=0$ and (B) $\zeta_0=0$, 10^{-4}, and 10^{-3} with $Q=0$.

is approximated by

$$\left(\frac{d^2F}{d\zeta^2}\right)_i=\frac{di}{d\zeta}\frac{d}{di}\left(\frac{dF}{d\zeta}\right)$$

$$=\frac{1}{4\times10^{-5}\ln(1.25)(1.25)^i}\,\frac{\left(\frac{dF}{d\zeta}\right)_{i+1/2}-\left(\frac{dF}{d\zeta}\right)_{i-1/2}}{\left(i+\frac{1}{2}\right)-\left(i-\frac{1}{2}\right)}. \quad (41)$$

Substituting for the first derivatives in eq (41), one has

$$\left(\frac{d^2F}{d\zeta^2}\right)_i=\frac{F(i+1)-2.25F(i)+1.25F(i-1)}{(4\times10^{-5})^2(\ln 1.25)^2(1.25)^{2i+1/2}}. \quad (42)$$

Using eq (42) for the second derivatives in eq (29) and (30), we obtain a system of nonlinear equations for eq (29)–(34). The Gauss-Seidel successive iterative method (e.g., Conte 1965) is applied to solve the systems of equations. The computer program was written in such a manner that Q need not be a constant but a reasonably behaved function of ζ.

The solutions of the system given by eq (29)–(34), depending on a roughness parameter ζ_0 and a stratification parameter Q, will be denoted by the functions

$$\phi=\Phi(\zeta;\,\zeta_0,\,Q), \quad (43)$$

$$\psi=\Psi(\zeta;\,\zeta_0,\,Q), \quad (44)$$

and

$$\kappa=K(\zeta;\,\zeta_0,\,Q). \quad (45)$$

The velocity profiles and other characteristic quantities can then be expressed in terms of these universal functions. From eq (15) and (16), one obtains for the velocity profiles,

$$\frac{k}{u_*}(u-u_g)=\frac{d\Psi}{d\zeta}=\Psi'(\zeta;\,\zeta_0,\,Q) \quad (46)$$

and

$$\frac{k}{u_*}(v-v_g)=-\frac{d\Phi}{d\zeta}=-\Phi'(\zeta;\,\zeta_0,\,Q). \quad (47)$$

The geostrophic coefficient, defined as $C_d=u_*/G$, is calculated from eq (15), (16), and (19); it is

$$C_d=\frac{u_*}{G}=k\sqrt{[\Phi'(0;\,\zeta_0,\,Q)]^2+[\Psi'(0;\,\zeta_0,\,Q)]^2} \quad (48)$$

where G is the speed of the geostrophic wind, and the tangent of the angle, α, between the geostrophic wind and the shear stress at the surface is

$$\tan \alpha = \frac{v_g}{u_g} = -\frac{\Phi'(0;\zeta_0, Q)}{\Psi'(0;\zeta_0, Q)}. \tag{49}$$

3. RESULTS AND DISCUSSION

The variations of C_d and α with the Rossby number, $R_o = G/(fz_0)$, are shown in figures 1 and 2. Various sources of measurements (e.g., Blackadar 1962) are reproduced here for comparison. The scatter of the data may be partly due to the range of atmospheric stratification. For example, the Brookhaven data for C_d are in good agreement with the present modeling with $Q=-100$. It is thus worthwhile to note that a good estimation of C_d and α is conditioned on good knowledge of the stratification conditions. When the atmosphere is thermally slightly stable; that is, when H has a small negative value, the presence of the evaporation, E, will cause Q in eq (27) to become negative (i.e., an unstable atmosphere prevails instead). In such case, C_d and α will have a great difference from the one without evaporation. This situation must be handled very carefully.

Figure 3 shows vertical profiles of the dimensionless x component shear stress, Φ. Figure 3A illustrates its variation with the stratification parameter, Q. Note that Φ approaches zero when ζ is about 0.45 under neutral conditions. This means that the geostrophic wind velocity is reached approximately at $z=0.18u_*/f$. Deardorff (1972) obtained a value of $0.2u_*/f$. It is also seen that the greater the instability, the higher the level that the geostrophic wind velocity can reach and vice versa under stable conditions. On the other hand, Deardorff (1972) has shown that Φ approaches zero at $z=z_i$ (z_i is the height of the inversion base) regardless of the instability. This should imply and allow us to make a conclusion that $z_i f/u_*$ is a function of Q determined from figure 3. Figure 3B shows that the surface roughness has practically no effect on Φ.

The vertical profile of y component shear stress, Ψ, is shown in figure 4. All the curves increase nearly linearly for some distance from the surface and then drop sharply to zero. This indicates that the free atmosphere is attained quickly in the upper part of the planetary boundary layer. Figure 4A shows that the greater the instability, the higher the maximum point can reach. Figure 4B shows that the effect of the roughness parameter, ζ_0, is limited only to a very small fraction of the boundary layer. Clarke's (1970) measurements for the shear stress components, Φ and Ψ, are also plotted in figures 4A and 3A for the purpose of comparison. They show the validity of the present analysis.

The vertical distribution of the eddy diffusivity, K, is shown in figure 5. Note that K tends to be constant under unstable conditions. This is easily seen from eq (31),

$$\kappa = \iota^{4/3} Q^{1/3} \text{ at } \zeta=\infty, \tag{50}$$

which is in agreement formularily with Kazansky and Monin (1958) for the surface layer under free convection.

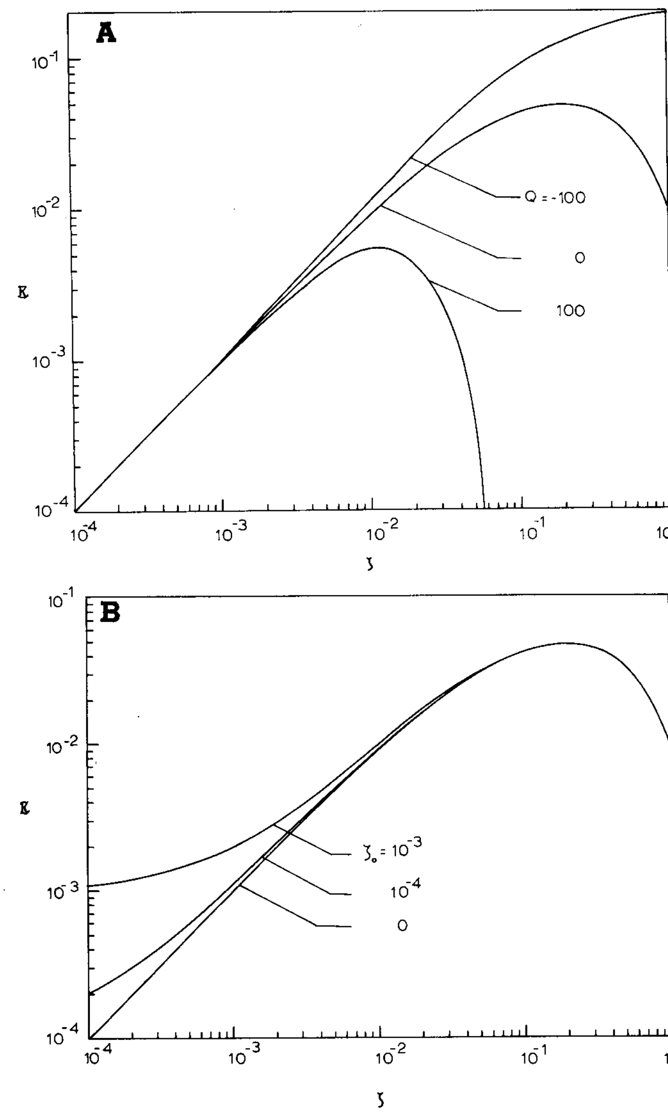

FIGURE 5.—The vertical profile for the eddy viscosity, K, for (A) $Q=-100$, 0, and 100 with $\zeta_0=0$ and (B) $\zeta_0=0$, 10^{-4}, and 10^{-3} with $Q=0$.

If the mixing length varies linearly with z, which is the case for the subsurface layer, the turbulent diffusivity, K, varies with $z^{4/3}$ as can be seen from eq (50). However, since the former tends to constant as z approaches infinity [eq(32)], so does K other than increasing with $z^{4/3}$ as in the study of subsurface layers (Kazansky and Monin 1958). On the other hand, K approaches zero under neutral and stable conditions due to the damping effect of the thermal stratification and evaporation. Figure 5A shows that K increases nearly linearly up to $\zeta=0.025$ under neutral conditions. This is equivalent to $z=0.01u_*/f$, which was the value Blackadar and Tennekes (1968) obtained. Figure 5B shows that the effect of roughness on K is again limited to the surface layer.

4. CONCLUSION AND REMARK

The fields of the velocity, eddy viscosity, potential temperature, and specific humidity in the planetary boundary layer are decoupled by introducing a free parameter, Q, which combines the effects of the thermal and humidity stratification. Solutions of the whole system can be done by the method of trial and error on Q. The decoupled system for the velocity field and eddy viscosity with Q as parameter is solved numerically. Results show good agreement when both the thermal and humidity stratification are taken into consideration.

In practical applications, θ_g and q_g are usually unknown. On the other hand, the equivalent blackbody temperature of the water surface can be found easily using a thermal radiometer or thermal scanner in the 10-to 12-μm window. Due to the high emissivity of the water surface in this region (Wolfe 1965), the kinetic temperature, θ_0, may be assumed to be equal to the equivalent blackbody temperature in the topmost 10 μm of the water surface; q_0 may then be estimated by assuming that the air in contact with the water surface is saturated. Under this circumstance, with θ_0 and q_0 given, one should be able to solve for θ_g and q_g in addition to the heat flux, H, and evaporation rate, E, by using eq (23)–(26) with the universal function K.

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