A

TREATISE

OF

OPTICS:

CONTAINING

ELEMENTS OF THE SCIENCE;

IN TWO BOOKS.

Rev. A. Singer
T.C.D. 1808

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THE Author of the following Work, in the year 1742 first proposed printing *a Treatise upon Microscopes* only. The plan laid down was entirely new; and to render it more compleat, a prefatory discourse was intended, to introduce such general principles as were necessary for understanding the subject scientifically. This discourse increased so fast, under the Author's pen, that it was soon thought adviseable to enlarge the original design, into a general *Treatise of Optics*. Two books, containing the Elementary parts of this science, being finished with all convenient expedition, were immediately printed off---both Plates and Letter-press---with uncommon care and correctness. And upon this foundation, it was proposed to explain the Theory and Mechanism of Optical Instruments, in a third Book, under the general head of Telescopes and Microscopes. A Task of this kind, required a multitude of accurate experiments; and perhaps nobody could be better qualified for making or reasoning upon them, than Mr. Harris. But, while this business was forwarding with unusual celerity, our Author was called from the pursuit, by an extraordinary demand of duty in His Majesty's Mint: And from that time, a variety of avocations, for the publick service, with a series of bad health, the consequence of laborious application and study, prevented any regular renewal of the subject for several years. Mr. Harris however, did not lose sight of his favourite object, but employed most opportunities for the advancement of it; till at length, finding himself once more at leisure, he resumed his application to this work with an earnestness that probably hastened his dissolution: For, in the actual prosecution of several interesting experiments he was taken sick, and, unfortunately for the Publick, expired 26th September 1764.

After Mr Harris's decease, a numerous collection of Manuscripts were submitted to the consideration of some Gentlemen, well acquainted with the Science of Optics, in hopes of compleating so valuable

valuable a work. But it appeared, upon examination, that though there ſeemed to be ample materials for the purpoſe, and many of them prepared for the preſs; yet the greater part, having never been digeſted, was incapable of any arrangement, without aſſiſtance of the Author---no longer to be had---or taking up the ſubject *de novo*, and purſuing it through an almoſt infinite variety of experiments--A Labour, neither to be expected or deſired of a Stranger!

Under theſe circumſtances, the whole Treatiſe has long lain dormant, againſt the importunities of many friends, who have frequently recommended to publiſh this Elementary Part alone. They have repeatedly alledged---That the Elements of Science are always the ſame---That theſe Elements, being quite diſtinct, are compleat in themſelves---That very little has been written on the ſubject---And That the following ſheets, containing much new matter, would be a valuable acquiſition to the ſcience. Such have been the reaſons advanced for this publication, by Gentlemen whoſe names might do honour to any Book: And they have at laſt prevailed over every ſcruple of the Proprietor. But as nothing here is intended to bias the favour of the Public, the true hiſtory of this work is fairly related, and the performance ſubmitted, upon its own merit, to the candid judgment of the impartial Reader.

THE

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A

TREATISE OF OPTICS.

BOOK I.

The Elementary Part of Optics.

THE object or design of OPTICS is to explain the *Theory of Vision*, as it is founded upon known and eſtabliſhed properties of light. Theſe properties may be conſidered in the abſtract, that is, without an immediate application to Viſion; and this properly makes the elementary part of the ſcience: for upon theſe are founded the ſurpriſing effects of all ſorts of optical inſtruments; the principal of which are *Microſcopes* and *Teleſcopes*. Both theſe enlarge the apparent magnitudes of objects: the Microſcope being adapted for magnifying ſmall objects that are near at hand; whilſt the Teleſcope is applied to remote objects.

Optics is diſtinguiſhed into two principal heads: *Catoptrics*, which treats of the reflection of light by ſmooth or poliſh'd ſurfaces of given figures; and *Dioptrics*, whoſe object is the progreſs of light through tranſparent bodies, terminated alſo by given ſurfaces. But of all the kinds of ſurfaces, the moſt uſeful in Optics are the *Plain* and *Spherical*, and to conſider others is ſeldom neceſſary: nor is it requiſite to make a formal diſtinction as to Catoptrics and Dioptrics, it being often more advantageous to blend them together.

SECTION I.

Of the chief Properties of Light, with an Explanation, as they occur, of ſome general Terms.

Art. 1. IT is manifeſt that rays of light are inceſſantly propagated in all poſſible directions, from every phyſical point of a luminous body; and when they enlighten other bodies upon which they fall, Light in continual motion.

 they

they are alſo reflected and diſperſed from every part of theſe bodies. For opaque bodies ſo enlightened are * viſible to the eye, as well as thoſe that are luminous.

Ray of Light.

2. By *a Ray of light* is meant, the leaſt light, or part of light, which may be ſtopt alone without the reſt of the light, or propagated alone, or do or ſuffer any thing alone, which the reſt of the light doth not or ſuffers not †.

Light conſiſts of ſmall material Particles.

3. It is very probable that rays of light are not continued filaments or threads; but conſiſt of ſmall and diſtinct particles of matter ||, which following in a cloſe ſucceſſion do as it were make one continued thread.

Light not inſtantaneous, but its Velocity exceeding great.

4. That light ſhould be propagated in time, and not inſtantaneouſly, is very reaſonable to ſuppoſe: And that its velocity is much quicker than that of ſound, we know, becauſe the flaſh of a gun fired at a conſiderable diſtance will be ſeen ſome time before the report is heard **. The *Aſtronomers* have diſcovered the velocity of light ‡ to be ſuch, as to reach from the ſun to us in about 8 minutes of time.

Medium.

5. *Definition.* Any *tranſparent Body*, as *Air*, *Glaſs*, *Water*, &c. alſo *empty Space*, is called a Medium: And mediums are ſaid to be *denſer* or *rarer*, as they are ſpecifically heavier or lighter. *Note*, By *a Medium is always underſtood a denſe Medium*, unleſs it be otherwiſe expreſſed.

Object, Naked Viſion, Radiant.

6. *Defin.* All bodies ſeen by rays propagated directly from them to the eye, without being at all bent out of their courſe in their paſſage, are called *Objects*. And the viſion of objects in this manner is called *Naked Viſion*. A ſmall object, or a phyſical point of an object, conſidered as propagating light towards a certain part, is ſometimes called a *Radiant* or *Radiating Point*.

7. The

* It cannot be doubted, but that light by its impulſe upon the proper organ is the mediate cauſe of viſion: and therefore wherever bodies are viſible, light muſt be propagated from them to the eye. A continued uninterrupted motion is eſſential to the very being of light, according to what ideas we can form of it; for the moment it ſtops, it ceaſes to be light to us.

† Sir *Iſaac Newton*'s Defin. Optics, page 2. Light is ſo ſubtile and fine, that it is probable the ſmalleſt part of it which we can ſtop, may yet conſiſt of ſeveral rays.

|| How exquiſitely minute muſt theſe particles be, that myriads of them can move promiſcuouſly all manner of ways, without impinging one another! And that this is the caſe, we need not doubt, ſince different bodies and different parts of the ſame body are diſtinctly viſible at the ſame time. And how curious muſt the eye be framed, to be ſenſible of theſe ſmall impulſes, and to diſtinguiſh at the ſame time thoſe from different objects!

** It is for the ſame cauſe that a clap of thunder is not heard till ſome time after the flaſh of lightening has been ſeen.

‡ This diſcovery was firſt made by *Monſieur Romer*, who having obſerved that the eclipſes of the *Satellites of Jupiter* appeared ſooner or later than according to the theory, as the earth was nearer to or farther from *Jupiter*, concluded from thence that the progreſs of light is not inſtantaneous: And by obſerving the different times of the appearances of theſe eclipſes, according to the different diſtances of the earth from *Jupiter*, the time the ſun's light takes in coming from him to us, has been diſcovered. This hypotheſis, that light is not inſtantaneouſly propagated thro' all ſpace, is now univerſally agreed to; and is farther confirmed by a notable diſcovery of an apparent motion of the fixed ſtars, by the *Reverend* and ingenious *Dr. James Bradley*, *Reg. Profeſſor of Aſtronomy*, who has elegantly accounted for that phænomenon, by the theory of the light's motion.

7. The progreſs or path of light, whilſt it continues uninterrupted in a medium of the ſame uniform denſity, is in * ſtrait lines. For all bodies ceaſe to be viſible, when any opaque body is interpoſed between them and the ſpectator.

Thus from a radiating point Q of an object PQR, are propagated inceſſantly in ſtrait lines, and in all poſſible directions the rays Q*q*, Q*q*, Q*q*, &c. And in like manner rays of light are inceſſantly propagated from every other phyſical point of the object, as well as from Q; otherwiſe the whole object would not be viſible, and that all at once, and to all poſitions of the eye. Fig. 1.

8. A ray of light, as AC, meeting obliquely with a ſurface RS, if it be turned back into the medium from whence it came, it will take ſome new direction, as CB: And if it enters the medium RG, inſtead of proceeding directly forward in the ſtrait line ACG, it will be bent at C into ſome † new direction CD, in which it will proceed ſtrait || whilſt it continues in that medium. The diſpoſition of the rays of light to be turned back into the medium from whence they came, is called *Reflexibility:* as is that to be bent at their entrance into another medium, called *Refrangibility*. The changes which the rays undergo in theſe two caſes, are called *Reflection* and *Refraction*. The original or primary ray AC, is called the *incident Ray*; C *the Point of Incidence*; CB, *the reflected Ray*; and CD, *the refracted Ray*. Fig. 2. Reflection and Refraction &c.

In like manner, if BC or DC be an incident ray, which at C is bent into CA; the *Ray* CA is accordingly called the *reflected or refracted Ray*.

9. Let PCO be a ſtrait line perpendicular to the ſurface RS at the point of incidence C; the angle ACP which the incident ray makes with this perpendicular, is called the *Angle of Incidence*; BCP the *Angle of Reflection*; and DCF the *Angle of Refraction*. From the point of incidence C, with any radius deſcribe a circle cutting the incident and refracted rays in A and D; and from the points A, D upon the line PO let fall the perpendiculars AE, DF. Now the line AE is the Sine of the angle of incidence; and DF is the ſine of the angle of refraction. The line AE for ſhortneſs of expreſſion is called *the Sine of Incidence*, and DF *the Sine of Refraction*. Sines of Incidence and Refraction.

 The

* The propagation of light is in this reſpect different from that of ſound, which ſeems to ſuffer no obſtruction by the crookedneſs of its paſſage; whereas no object can be ſeen through a crooked pipe. Sound is undoubtedly propagated to the ear by an undulating motion of the air, (like that of water after a ſtone has been thrown into it) cauſed by the tremor or vibrating motion of the ſounding body; and not by any particles of matter emitted from that body.

† The advantages we derive from the refrangibility of light are inestimable: without this property in light, we ſhould have figured glaſſes in vain, and Teleſcopes and Microſcopes would not have exiſted.

|| It ſeems that light is not reſiſted, at leaſt ſenſibly, by any medium through which it paſſeth; for if it was, its motion would be curvilineal, and not ſtrait.

The following *Axioms* about Light, are by repeated experiments found to be general without any exception; and are by ſome called the *Laws of Reflection and Refraction.*

10. Axiom I. *The Angles of Reflection and Refraction lie in one and the ſame Plane with the Angle of Incidence.* That is, AC, CB, CD and the perpendicular PO are all in the ſame plane.

11. Axiom II. *The Angle of Reflection* (BCP, or *b*CP) *is* * *equal to the Angle of Incidence.* (ACP or *a*CP.)

12. *Corol.* Hence it follows, that when the incident ray is perpendicular to the reflecting ſurface, it ſhall be reflected directly back along the ſame perpendicular. For in that caſe the Angles ACP, BCP both vaniſh, and AC, BC coincide with PC.

13. Axiom III. *If the Reflected or Refracted Ray* (BC or DC) *be returned directly back to the Point of Incidence* (C); *it ſhall be reflected or refracted into the ſame Line* (CA) *before deſcribed by the incident Ray.*

14. Axiom IV. *Refraction out of the rarer Medium into the denſer, is made towards the Perpendicular; that is, ſo that the Angle of Refraction be leſs than the Angle of Incidence. And the converſe.* That is, the angle of incidence ACP from a rare medium, as air, into a denſer RSO, is greater than the angle of refraction DCO. And if DC be an incident ray, the angle of incidence DCO ſhall be leſs than the angle of refraction ACP.

15. Axiom V. *The Sine of Incidence is either accurately, or very nearly, in a given Ratio to the Sine of Refraction* †. That is, any other incident ray *a*C will be ſo refracted into C*d*, that whatever ratio there be between

* This ſeems to agree with the law of reflection of hard bodies of a ſenſible bulk.

† The refraction of light at the ſurfaces of tranſparent bodies was taken notice of by the Ancients. *Ariſtotle* has a problem concerning the apparent curvity of an oar in water: And *Archimedes* is ſaid to have written a book about the appearance of a ring or circle under water. *Alhazen* the *Arabian* and *Vitellio* thought, that the angles of incidence and refraction were in a given ratio; but this proportion being found erroneous in large angles, the Moderns began to examine the matter more ſtrictly. *Kepler*, among the reſt, made ſeveral experiments about it; and tho' he miſs'd his aim, his attempts and conjectures became uſeful to others. After the invention of Teleſcopes, the ſubject of this enquiry being thought more valuable than before, was farther purſued; and *Willebrardus Snellius* (a *Dutchman*) found out the truth: but ſtill he did not throughly comprehend his own invention. This ſhort hiſtory is extracted from *M. Huygens*'s *Dioptricks*, pag. 1, 2. To which may be added, that *Des Cartes*, ſome time after it had been diſcovered by *Snellius*, was the firſt who publiſhed, that *the Sines of Incidence and Refraction* (and not their angles) *are in a given ratio.*

The cauſe of refraction ſeems to be, an attraction of light by the ſurfaces of mediums of greater denſities: Upon this ſuppoſition, the reaſon

tween the ſines AE, DF; there will be the ſame ratio between the ſines *ae*, *df*: i.e. AE : DF :: *ae* : *df*.

16. *Corol.* 1. Hence it follows, that if the angles of incidence be increaſed or diminiſhed, the angles of refraction will be alſo increaſed or diminiſhed; but not in the ſame proportion, becauſe angles and their ſines have not the ſame ratio; ſmaller angles and their ſines being nearer to a ratio of equality, than larger angles and their ſines. Alſo if two angles of incidence be equal to each other, the angles of refraction ſhall be likewiſe equal to each other.

17. *Corol.* 2. If the incident ray is perpendicular to the refracting ſurface, as PC, it will then undergo no refraction, but proceed directly in the ſtrait line PCO. For the angle of incidence being now vaniſhed, the angle of refraction muſt alſo vaniſh.

18. *Corol.* 3. If the ratio between the ſines of incidence and refraction at the ſurface of a given medium, be known, and an angle of incidence ACE be given; the angle of refraction FCD may be thus found: About the point of incidence C, with any radius, deſcribe a circle cutting the incident ray AC ſomewhere in A; and from A upon the perpendicular PCO to the refracting ſurface, let fall the perpendicular AEB; then make AE to EH, as the ſine of incidence to the ſine of refraction; and through H draw HD parallel to PO, and where this line cuts the circle as in D, is a point through which the refracted ray CD will paſs. Or, the angle FCD may be found by a table of ſines, thus: *Sine of Incidence* : *Sine of Refraction* :: *Sine* ∠ ACE : *Sine* ∠FCD.

19. *Scholium.* A ray of light as AC falling upon a ſpherical ſurface ECF, either convex as *Fig.* 3. or concave as *Fig.* 4. will be reflected or refracted after the ſame manner, as it would be by a plane RS touching the ſpherical ſurface at the point of incidence C. For in that caſe a line PO, which is perpendicular to the ſpherical ſurface ECF, will be alſo perpendicular to the plane RS; and therefore in both caſes, the angle of incidence ACP being the ſame, the angles of reflection and refraction will be the ſame alſo. Fig. 3, 4.

20. It is found by experience, that if the refraction be made out of air into water, the ſine of incidence (AE) is to the ſine of refraction (FD) as 4 to 3: If out of air into glaſs, the ſines are as 3 to 2 nearly; or more accurately as 17 to 11, or 14 to 9. And reverſely, if the refraction

ſon is manifeſt why light at its entrance into a denſe medium is refracted nearer to the perpendicular, and farther from the perpendicular at its entrance into a rarer medium. And from the ſame principle Sir *Isaac Newton* hath ſhewed, why the refractions are in the ratio of the ſines. *Newton's Optics*, 3d Edition, pag. 68 to 70.

refraction be made from water into air, the ſines will be as 3 to 4: if from glaſs into air, as 2 to 3, or 11 to 17 *.

Fig. 2. 21. *Corol.* Hence if the refraction be from water or glaſs into air, and the angle of incidence GCF be greater than about $48\frac{1}{2}$ degrees in water, or greater than about 40 deg. in glaſs; the ray GC will not be refracted into air, but will be reflected into the line C*g*, making the angle of reflection *g*CF = GCF the angle of incidence: Becauſe the ſines of $48\frac{1}{2}$ deg. and 40 deg. are to the radius (C*k*) as 3 to 4, and as 11 to 17 nearly; and therefore when the ſine G*h* has a greater proportion to the radius than as above, the ray will not be refracted.

22. *Scholium.* The ſurfaces of bodies, as well thoſe that are tranſparent, as opaque, being for the moſt part uneven; it neceſſarily follows that abundance of light is diſſipated by bodies even the moſt tranſparent: Some being reflected and ſome refracted towards different parts by their uneven ſurfaces, whilſt ſome is reflected and ſome refracted pretty uniformly. This accounts, why a perſon can ſee through water, and his image reflected by it at the ſame time: Why, in the duſk, the furniture in a room may be ſeen by the reflection of a window glaſs, whilſt objects that are without are ſeen through it.

[22.] Sir *Iſaac Newton* † hath ſhewed that the reflection and refraction of light are probably both owing to the ſame power or cauſe differently exerciſed in different circumſtances. And that this power exerts itſelf at ſome diſtance from the reflecting ſurface; ſo that light is not reflected by impinging on the ſolid parts of bodies, but by ſome principle, which is uniformly diffuſed to a certain diſtance from the body. And this accounts why poliſh'd ſurfaces reflect light ſo uniformly as they do, which otherwiſe would be inexplicable. For the beſt poliſhed ſurface is full of ſcratches or fiſſures made by the powder wherewith it was poliſhed, and therefore utterly incapable of reflecting light uniformly, if reflection was made by the impulſes of light upon the ſolid parts of bodies. Nevertheleſs we find that rough or uneven ſurfaces reflect light all manner of ways; and that the ſmoother they are, the more uniform will be their reflections. The reaſon of which ſeems to be, that the reflective power reaches every where to the ſame diſtance from the body nearly; and that when the depths of the fiſſures are leſs than ſome aſſignable quantity, the reflective power may be ſo extended as to form as it were a film or ſurface round the body that ſhall be perfectly ſmooth. And hence perhaps there may be a degree of ſmoothneſs of bodies attainable by human art, beyond which no optical advantages are to be expected.

* The rays of light are not all equally refrangible, but this inequality need not in this place be conſidered. Alſo the numbers 2, 3, for expreſſing the ratio of refraction between glaſs and air, are ſufficiently accurate for our preſent purpoſe. How to find, by experiments, the refractive power of all ſorts of mediums both fluid and ſolid, is ſhewed in Sir *Iſaac Newton*'s Optical Lectures, §. 2.

† Vid. *Newton*'s Opticks, p. 237, to 244.

expected. What has been said as to the reflection of bodies, may be also applied to their refractions.

23. *Defin.* Any parcel of rays, as QAC, QBC, or QAB considered apart from the rest, is called a *Pencil of Rays.* When the base of the pencil is a circle, as the pupil of the eye, an optic glass, *&c.* or considered as being a circle, it is also called a *Cone of Rays.* That ray as QA, *fig.* 5, 9, which falls perpendicular upon any surface, is called the *Axis* of the pencil or cone; also the *Principal Ray:* [a]And this ray undergoes no change either by reflection or refraction. Fig. 1,5,9. Pencil or Cone of Rays. Incident and reflected or refracted Rays. [a] 12&17. I.

The rays of any pencil before they have undergone any change by reflection or refraction, are called *Incident Rays:* afterwards they are called *Reflected* or *Refracted Rays*, according to the change they underwent. Both the incident and reflected or refracted rays, are considered as making one pencil. Fig. 5, 9.

24. *Defin.* Rays proceeding from a certain point as Q, and as they go on, spread or recede farther asunder, are called *Diverging Rays.* And rays, which, on the contrary, convene towards a certain point at which they would at last unite, if not intercepted in their passage, are called *Converging Rays.* *Rays* are said to be *parallel* when their inclination, if they have any, is insensible: or, when the distance of the point from, or towards, which they diverge, or converge, is so great, that the breadth of the pencil bears no sensible proportion thereto. Thus, the solar rays are reckoned parallel, because their inclination at the distance we are from the sun, is altogether insensible. In like manner, the rays which enter the eye from an object at a mile or even less distance, may be reckoned parallel, because the breadth of the eye's pupil bears no sensible proportion to that distance. Diverging, converging, and parallel Rays.

25. *Defin.* The point from which rays diverge, or towards which they converge (tho' they never arrive at it) is called their *Focus.* Every pencil of rays after reflection or refraction has two focus's, the one as Q being the *Focus of incident Rays*; and the other as *q*, the *Focus of the reflected or refracted Rays.* These considered as being the focus's of the same pencil, are called *Conjugate Focus's.* Parallel rays are considered as having focus's at an infinite distance. The Line Q*q* joining the conjugate focus's, is called the *Axis of the Pencil.* The point Q from whence the rays diverge, or in which they would unite after reflection or refraction, if *q* be the focus of incident rays converging, is called a *real Focus.* But *q* in the three figures referred to, is called a *virtual* or *imaginary Focus*; because the rays are not really there united *. Fig. 5,9,10. Focus, real, imaginary: conjugate Focus's. Geometrical Focus. Aberration, proper Focus

All

* These definitions are placed here, because they are general; but a beginner need not dwell upon them, their meaning will be more obvious after a perusal of part of the next section.

All the rays in a pencil will not be always reflected or refracted into one point, as will be shewed hereafter; but only those rays that are equidistant from the axis on different sides, all those that have different inclinations having different focus's. And, strictly speaking, by the focus of a pencil is meant the focus of those rays that are nearest to the axis: this is also called the *Geometrical Focus*. And the distance between this focus and the *proper or respective Focus* of any ray, is called the *Aberration* of that ray. Thus, *fig.* 13, 14, if Q be the focus of incident rays, and q be the focus of those rays that are next the axis after refraction, x the focus of any particular ray CD; then is q called the *Geometrical Focus*, x the *respective* or *proper Focus* of the ray CD, and qx its *Aberration*.

Image. 26. *Defin.* A physical point Q (*fig.* 5, 9, 10.) being considered as an object, the corresponding point q from or towards which the rays diverge or converge after reflection or refraction, is called the *Image* of the said point Q. In like manner the images of all the several points of an object PQR, (*fig.* 6, 7, 8, 11, 12.) being taken collectively or as one image, as pqr, is called the *Image* of the said *Object* PQR.

27. Light the farther it is propagated from any radiating point, becomes continually rarer: And if it met no obstruction in its course, the decrease of its density would be as the square of the distance from the radiating point. This is manifest from the principles of geometry. Fig. 5. Let BQC be a pencil of light propagated from the point Q, and let Qa, Qb be equal respectively to half QB, QC. Now a circle whose diameter is BC will be four times as big as a circle described upon ab; but the quantity of light contained in these two circles is equal; and therefore the light at the distance Qa is four times denser than at the distance QB. In like manner, if Qe, Qf be one fourth of QB, QC, the light at ef will be 16 times denser than at BC. For a circle described upon ef contains all the light that can come into the circle described upon BC, and these two circles are to one another as 1 to 16: That is, *Light the farther it is propagated decreases in its density in a duplicate Ratio of the distance.*

28. *Corol.* Hence luminous bodies, or those that shine with their own light, are vastly brighter than opaque bodies illuminated by them. For opaque bodies disperse the light falling upon them, also all manner of ways; whence, supposing that all the light was reflected, the quantity of it received by the eye from the opaque body, compared to that which it receives from the luminous body, is only as the visible illuminated surface of the opaque body, to the surface of an hemisphere whose radius is the distance of the opaque body from the eye; supposing the breadth of the pupil to be the same in both cases, and that the sum of the distances of the opaque body from the eye and from the luminous body, differs

differs insensibly from the direct distance of the luminous body from the eye.

29. Hence again, the light of the full moon at a medium is about 100,000 times fainter or rarer than the sun's light when at the same height above the horizon, all other circumstances being the same also; as will appear by comparing the moon's apparent disk to the surface of a hemisphere. Whence it is easy to conceive, that since we can bear the sun's heat, why we are not sensible of any from the moon; and much less can we be sensible of any heat from any of the other planets.

30. We do not know any medium that is perfectly transparent. Even the air, the most rare and transparent medium we know of, is full of opaque particles that impede light; this is manifest from the phænomenon of a beam of light let in through a small hole into a darkened chamber, which will be visible like a *luminous Cone* from every part of the room: and this shews that the whole light does not go forward in its rectilineal course, but that at every point of the medium through which it passeth, some part of it is reflected every way; for the visibility of the luminous cone is caused by this reflection. The greater faintness of the sun and moon when near the horizon, than when elevated higher up, shews also that their light is more obstructed by the longer tract of air and vapours in the one case, than in the other. The loss of light in passing through glass is still greater.

No Medium perfectly transparent.

31. Hence the quantity of light contained in any pencil is continually diminished, the greater the distance from the radiant: and this diminution is more or less, according to the more or less opacity of the medium through which it passes. This makes it impossible to assign exactly the different proportions of the density of light, at different distances from the radiant; but in all cases, the decrease of the density of light is greater than as the squares of the said distances. And this is the cause why objects appear less bright the farther they are from the spectator. For if none of the rays were stopt in their passage, the degree of brightness of the picture of an object upon the retina, would be the same at all distances between the eye and object, supposing the aperture of the eye's pupil to remain the same. For in that case the magnitude of the picture upon the retina, and the density of the light would increase or decrease together in the same proportion, *viz.* reciprocally as the squares of the distances of the eye from the object; and therefore the density of the light upon the retina would be invariable at all distances *.

Hence Light decreases in its density in a greater proportion than as the squares of the distances.

C Luminous

* For example; when the eye approaches as near again to the object, the picture upon the retina becomes quadruple; and the quantity of light received from the object thro' the same aperture

Luminous bodies ſhining in the dark, as a *lamp* or *torch*, &c. emit to very great diſtances much more light than in that caſe is neceſſary for viſion. And tho' their light ſuffers a continual diminution by the heterogeneity of the medium, the farther it goes; yet there being ſtill more left than is requiſite for viſion, their ſplendour is not ſenſibly increaſed or diminiſhed, by leſſening or increaſing their diſtances.

Light not all reflected.

32. Light is not only diverted out of its courſe by the reflection of bodies it meets with, but alſo a great part of it is frequently ſuffocated, or as it were deſtroyed by them. For it is manifeſt, that black or dark bodies reflect much leſs light than others of darker colours; and it is probable, that no bodies whatever reflect all the light they receive *. Hence the ſurface of the moon having in it ſeveral large dark ſpots, the quantity of light we receive from her muſt be leſs than according to the computation in *Art.* 29.

33. Hence the power of the eye to diſcern objects without inconvenience, by different quantities of light, is vaſtly extenſive. For not only objects of different colours placed in the ſame light, are ſeen with equal eaſe; but even objects ſo ſmall as the letters of a book may be diſtinguiſhed and read by a clear moon-light. And admitting the ſurface of the eye's pupil to be ten times greater in a weak light than in a ſtrong light, yet the proportion of the weakeſt light to the ſtrongeſt by which the eye can conveniently ſee objects, perhaps does not exceed that of 1 to 10,000 †. How exquiſite are our faculties!

Speculum. Plane and ſpherical Speculums.

34. *Defin.* Any ſmooth or poliſhed ſurface, reflecting light, is called a *Speculum*. Reflecting ſurfaces that are *flat*, as a *Looking-glaſs*, are called ‖ *Plane Speculums*; and thoſe that are *Segments of a Sphere*, are accordingly called *Spherical Speculums*; and as they are convex or concave as ECF, *fig.* 3, or *fig.* 4. they are called *convex* or *concave Speculums*.

35. *N. B.* There will be frequent occaſion to conſider the ſines of incidence and refraction; and therefore both for the ſake of brevity and perſpicuity,

aperture of the pupil at half the diſtance, is alſo quadruple: and this being equally ſpread over four times the ſurface of the retina, the light is juſt as denſe as before when the object was at twice the diſtance.

* Light, whatever afterwards become of its component particles, ceaſes to be light as ſoon as its motion is ſtopt. Whether there be a decreaſe in the velocity of light, is impoſſible for us to know from any experiment. But it cannot well be doubted, that ſince light is continually propagated from the ſun and other ſelf-ſhining bodies, ſo it muſt alſo ſuffer a continual waſte and decay, to make room for theſe continual ſupplies.

† By a well-contrived experiment M. *Bouguer* found full moon-light at a medium to be about 300,000 times weaker than the ſun's light. Vide the learned Dr. *Smith*'s Remark 96. upon his compleat ſyſtem of Optics. But it is to be obſerved, that ſky-light, which properly is day-light, is always much weaker than the direct ſun's light, and the degrees of it do not only differ conſiderably, at different times of the year, but alſo frequently in the ſame day; and thoſe different degrees are not eaſily meaſured, ſo as to be reduced to numbers. However, it is manifeſt that the difference of the degrees of light ſuitable for viſion, is very conſiderable.

‖ Theſe are alſo called *Mirrours*, but Opticians generally make uſe only of the word *Speculum*.

perſpicuity, we ſhall often abbreviate theſe expreſſions, by calling the *Sine of Incidence*, I; and the *Sine of Refraction*, R. And when the ratio between theſe ſines is the only thing under conſideration, without regarding which is the ſine of incidence, or which the ſine of refraction, we ſhall call the *greater* of theſe ſines, m; and the *leſſer* of them, n. Hence, and from *Art.* 14. if the incidence is from a rare medium into a denſe, it will be $I : R :: m : n$; and when the incidence is from a denſe medium into a rare, it will be $I : R :: n : m$. If the refraction be betwixt air and glaſs, $m : n ::$ [a] $3 : 2$; if betwixt air and water, $m : n ::$ [a] $4 : 3$. [a] Art. 20.

36. *Lemma.* Small arches and their ſines are very near in a ratio of equality *, and therefore have the ſame ratio to each other, as their ſines nearly; that is, if the Arches DO, dO, are but ſmall, it will be DO : dO :: DF : df nearly. And the ſmaller are the arches, the leſs will be the error in ſuppoſing them equal to their ſines. Thus, if the arch DO does not exceed 20 *min.* and the radius CD or CO be divided into 10,000,000 of equal parts; the exceſs of the arch DO above its ſine DF will be leſs than one of thoſe parts. Fig. 2.

Alſo the verſed ſines OF, Of of very ſmall arches OD, Od are as the ſquares of the ſines FD, fd of thoſe arches nearly. For OF : FD :: FD : FP; and Of : fd :: fd : fP; but when the arches OD, Od are very ſmall, FP and fP will be nearly in a ratio of equality, and therefore OF : Of :: FD^2 : fd^2 nearly.

SECTION II.

Of the Reflection and Refraction of a Pencil of Rays at a plane Surface.

§. 1. *Of Reflection at a plane Surface.*

PROPOSITION I.

37. A *Pencil of rays diverging from a point* Q, *will after reflection by a plane ſpeculum* BAC, *diverge in ſuch a manner, that if the reflected rays were produced backwards, they would all meet in a point* q, *taken in the perpendicular* QA *to that plane produced, as far below* A *as* Q *is above it; that is,* QAq *is a ſtrait line, and* qA=AQ. Fig. 5.

 Let

* Mathematicians will at firſt ſight underſtand this lemma; and this is not a proper place to prove propoſitions purely geometrical. And tho' the lemma is never ſtrictly true till the arch vaniſhes, yet as by diminiſhing the arch the error may be reduced to a leſs quantity than any aſſignable one, no objection can lie againſt it.

Let QB be any incident ray, and let QA, BF be each perpendicular to the plane or ſtrait line BAC, and make the angle DBF equal to
[a] 11. I. the angle QBF; then will BD be the [a]reflected ray. Produce DB backwards until it meets QA produced in q; I ſay q is the focus of all the rays diverging from Q upon BAC, and that $Aq = AQ$. For BF,
[b] Conſtruct. Qq being [b]parallel, the angle $QBF = {}^{c}\angle BQq = {}^{b}FBD = {}^{c}BqQ$;
[c] 29. I *Eucl.* wherefore the right-angled triangles BAQ, BAq having beſides the right angles at A, the ∠s BQA, BqA equal, and the ſide BA common;
[d] 26. I *Eucl.* the ſides AQ, Aq are alſo [d]equal. The ſame argument holds good with reſpect to any other incident ray, as QC; and therefore all the rays diverging from Q, will after reflection by the plane BC, belong to the focus q. *Q. E. D.*

38. *Corol.* 1. Hence, and from *Art.* 13. if q be the focus of incident rays, Q will be the focus of the reflected ones; that is, if rays DB, EC, *&c.* converge towards the focus q, they will all meet in Q after reflection.

39. *Corol.* 2. If the ſpeculum BC be inclined a little, as into the poſition kCK, ſo that CH ſhall be a perpendicular thereto at the point C; the incident ray QC ſhall be reflected into CI, ſo that the ∠ ECI contained between the two reflected rays CE, CI at both poſitions of the ſpeculum ſhall be double the angle GCH, the inclination given the ſpeculum. And hence, *Any error in the figure of a ſpeculum, as well as any inclination given it, produces a double error in its effect.*

PROP. II.

40. *Images formed by the reflection of a plane ſurface, are equal and ſimilar to the objects; and are at the ſame diſtance from, and have a like ſituation with reſpect to the backſide of the plane; that the objects are from, and have with reſpect to the foreſide.*

Fig. 6, 7, 8. For the images p, q, r of the radiating points P, Q, R [a]being in the perpendiculars PBp, QAq, RCr to the reflecting plane BC, and at
[a] 37. I. the ſame diſtance from one ſide of the plane, [a]that the radiants are from the other; that is, ſo that Bp, Aq, Cr are equal reſpectively to BP, AQ, CR: And this being true of the images of every point of the object PQR, it is manifeſt that the whole image p, q, r is equal and ſimilar to the object PQR; and has a like ſituation with reſpect to one ſide of the plane, that the object has with reſpect to the other. *Q. E. D.*

41. *Corol.* 1. If the poſition of the object PQR, and of the eye e, be given, ſuppoſing the pupil of the eye to be only a point *, the parts of

* In the figures the breadth of the pupil is taken in, and it is manifeſt that it receives light from a larger part of the ſpeculum than it would do if it was no bigger than a point. And in the next

of the ſpeculum a,b which reflect to the eye the rays proceeding from any points P, R of the object, may be found by drawing from the correſponding points p,r of the image the ſtrait lines pae, rbe. For the rays diverging from the points P, R upon the parts a,b, do after reflection belong to the [a] focus's p,r. [a] 37. I.

Or the point a of the ſpeculum which reflects to the eye at e, the rays diverging from P, may be found without having the place of the image p: Thus, upon the ſpeculum let fall the perpendiculars PB, eu. Now if a be a point from whence the reflected ray comes to the eye, the angles PaB, eau will be equal. Wherefore PB : Ba :: eu : au; and therefore PB + eu : Ba + au (Bu) :: eu : au. After the ſame manner may be found the point b, which reflects to the eye a ray which came from R, &c.

42. *Corol.* 2. The eye at e will ſee the whole object PQR by rays reflected from the part of the ſpeculum contained between a,b, and from no other part; all the reſt of the ſpeculum in this caſe being uſeleſs. But if any part of ab be covered, ſome part of the image will be accordingly loſt. If either the eye or the object change their poſitions, the rays that come to the eye will be reflected from a different part of the ſpeculum, which part may be found as in the preceding *Article*.

43. *Corol.* 3. The diſtance of any point of the image from the eye, is equal to the incident and reflected rays taken both together; that is, pe, or re, is equal to Pa+ae, or Rb+be. And tho' with reſpect to the different ſides of the ſpeculum, the object and image have a like ſituation; yet to a ſpectator who firſt looks at one and then at the other, the caſe will be different.

44. *Corol.* 4. If ab be the breadth of the ſpeculum, and e the place of the eye; the breadth of the images that can be ſeen by the eye at e, will be contained between the lines ea, eb produced indefinitely. And therefore having the diſtances and ſituations of objects before the ſpeculum given, the extent of thoſe that are viſible by the reflection of a ſpeculum having a given breadth, may be found from the extent of the viſible image.

45. *Corol.* 5. The nearer is the eye, and the leſs oblique to the ſpeculum, the greater will be the field of viſible images.

46. *Corol.* 6. If an object PR be placed parallel to a plane ſpeculum BC, and two lines as Pp, Pr be drawn from any point P thereof to the extremities of its image pr; the part BA of the ſpeculum included. Fig. 6.

next article, the eye will ſee the object by rays reflected from a ſmaller part of the ſpeculum, than it could do if its pupil was only a point. The differences in both caſes is manifeſt from the figures.

cluded between these two lines will be equal to half the length of the object or of its image *pr*. For the triangles PBA, P*pr* being similar, and PB being equal to half P*p*, it will be BA $= \frac{1}{2} pr = \frac{1}{2}$ PR. Whence,

47. *Corol.* 7. A person viewing himself in a plane looking-glass placed upright, will see his image compleat in a part of the glass, whose length and breadth is equal to half the length and breadth of the corresponding parts of his own body. And this will be always the case at whatever distance he stands from the glass.

48. *Corol.* 8. *A spectator will see his own image as far beyond the speculum, as he is before it; and as he moves to or from the speculum, his image will at the same time, move towards or from him on the other side, but apparently with a double velocity, because the two motions are equal, and contrarily. In like manner, if while the spectator is at rest, an object be in motion, its image behind the speculum will be seen to move at the same rate. And if the spectator moves, the images of objects that are at rest will appear to approach or recede from him, after the same manner as when he moves towards real objects.

49. *Corol.* 9. Whatever has been said of the image of any object, or primary radiant, holds also true of the image of another image, *&c.* whence by two † plane speculums duly placed, a multiplication of images may be made; and the distance of any image will be equal to the length of a ray propagated from the primary radiant through all the intermediate reflections to the eye.

§. 2. *Of Refraction at a plane Surface.*

PROP. III.

Fig. 9, 10. 50. *If* Q *be the focus of incident rays upon a refracting plane* BAC, *and a point* q *be taken in the perpendicular* QA *(produced if need be) to that plane, so that* Aq *shall be to* AQ, *as the sine of incidence to the sine of refraction; or if it be made* R : I :: AQ : Aq, *the point* q *will be the focus of those rays that are nearest to the perpendicular* QA *after refraction: that is, if the points of incidence* B, C *of the rays* QB, QC *are not remote from* A; *the refracted rays* Be, Cf *produced backwards would meet in* q *nearly.*

Let QC be any incident ray, and C*f* the refracted ray, which produce backwards until it meets the perpendicular AQ somewhere, as in *q*. Through the point of incidence C draw EH parallel to QA; then will
[a] 29. I. *Eucl.* QCE = [a] AQC be the angle of incidence, and HC*f* = [a] A*q*C be the angle

* It is here presupposed that images formed by plane speculums, appear in their true places; and this supposition is confirmed by universal experience. *Vide* Book II. Sect. 3.

† *Vide* Ibid.

angle of refraction. And in the triangle QqC, it will be *As the sine of* the $\angle CQq$ (AQC), is *to the sine of the* $\angle CqQ$ (AqC*) *so* Cq *to* CQ: that is, $I : R :: {}^{b}Cq : CQ$. But when the angle of incidence AQC is very small, it will be $Cq : CQ ::$ † $Aq : AQ$ nearly; that is, $I : R :: Aq : AQ$ nearly. And the nearer any incident ray as Qb, or Qa is to the perpendicular QA, the nearer will the sides qb, Qb, or qa, Qa of the $\triangle$s Qqb or Qqa approach to a † ratio of equality with qA, QA until at length they coincide. And therefore the point q found as above, is without sensible error, the focues of the refracted rays. *Q. E. D.* [b] Construct.

51. *Corol.* 1. If q be the focus of converging rays (eB, fC, &c.) Q will be their focus after refraction.

52. *Corol.* 2. Because $Aq : AQ ::$ [a] $I : R$; it will be, [a] 50. I.
Qq ($Aq \backsim AQ$) : $Aq :: m - n$ ($I \backsim R$) : m or n (I) according as the incidence is into a rare or dense medium. And if the point of incidence of a ray QC is so near to A, that the angles and their sines may be reckoned proportional, it will be

$\left\{ {m \atop n} \right\}$ (I) : $m - n ::$ $\angle$ *Incidence* AQC : *refracted* $\angle Qcq = hCf$.

53. *Corol.* 3. The image q of a small object Q placed under water is one fourth nearer to the surface than the object. And hence the bottom of a pond of water is one third part deeper than it appears to a spectator above. For Aq being [a] $\frac{3}{4}$ of AQ; qQ is $\frac{1}{3}$ of Aq. This may be an useful caution to persons who are not swimmers, to prevent their plunging out of their depths unwarily ‖. Fig. 10. Water is $\frac{1}{3}$ deeper than it appears. [a] 20 & 52 I.

Hence we have the reason of the common phænomenon of a shilling or other object placed in an empty vessel, appearing to be elevated higher and higher as the vessel is filling with water. Suppose the vessel empty, Bk its side, and Q the object at the bottom: If the eye be at e, the object will be hid by the side Bk; but by filling the vessel with water, it will become visible, and be seen at q, the ray QB being refracted into Be. And if the eye be so placed as to see the object at Q when the vessel was empty, while it is filling the object will appear to rise gradually in the line Qq. Why an object at the bottom of an empty vessel appears elevated as the vessel is filling with water.

54. *Corol.* 4. If the refracting medium is glass, and the angles of incidence and refraction as AQC, AqC be so small as to be reckoned proportional to their sines; the refracted angle $QCq = fch$, that is, the angle contained between the incident ray produced and the refracted ray, will be [a] $\frac{1}{3}$ of the greatest of those angles, and [a] $\frac{1}{4}$ of the least of them. And hence if the angle of incidence be increased or diminished, the refracted angle will be increased or diminished by $\frac{1}{3}$ of that augmented or Fig. 9, 10. [a] 20 & 52 I.

* By the known rules of Trigonometry.

† This is manifest from an inspection of the figures.

‖ If a strait stick be immersed perpendicularly into water, until the part immersed appears of equal length with the part above; these two parts being measured, will be found to be to one another, about as 4 to 3.

or diminiſhed part, if the incidence is from air to glaſs; and by $\frac{1}{2}$ if the incidence is from glaſs to air.

55. *Corol.* 5. Hence, and from *Art.* 39. the aberration ariſing from a fault in the figure of glaſs, if a ray be refracted, and the refraction be from air to glaſs, is but $\frac{1}{6}$ of the aberration of a reflected ray: And the aberration of refraction from glaſs to air is $\frac{1}{4}$ of the aberration of reflection; the angle of incidence in both the caſes of reflection and refraction being the ſame. And if the medium is water, the aberration of refraction will be $\frac{1}{8}$ and $\frac{1}{6}$ of the aberration of reflection ariſing from a fault in the ſurface.

Fig. 13, 14. 56. Scholium. *When an incident ray as* QC *is oblique to the plane refracting ſurface* AC; *the proper focus* x *of the refracted ray* CD, *will be farther from* Q *than the geometrical focus* q; that is, Qx *is always greater than* Qq. *And the more oblique is the incidence*, that is, *the greater is the angle* AQC, *the greater will be the aberration* qx *of the point* x *from* q, *the focus of thoſe rays that are next to the axis or perpendicular* QA. In each figure ſuppoſe a ſtrait line drawn from q to C. Now if q be ſuppoſed to be the focus of the refracted ray CD, it ought to be, AQ : Aq :: [a]CQ : Cq :: [a]I : R. But it is manifeſt that theſe ratio's are not equal. For if QA, and qA be made radius's, CQ, and Cq will be the ſecants of the unequal angles AQC, AqC. And becauſe it muſt be AQ : Aq :: [a]CQ : Cx, it is manifeſt that in *fig.* 13. Cx is greater than Cq; and in *fig.* 14. Cx muſt be leſs than Cq; and in both caſes Qx is greater than Qq. And it is manifeſt alſo, that the greater is the angle AQC, the greater will be qx. Therefore, *&c. Q. E. D.*

[a] 50. I.

PROP. IV.

Fig. 13, 14. 57. *The focus* Q *of any incident ray as* QC *upon a refracting plane* AC, *being given; to find the point* x *where the refracted ray* CD *produced backwards would cut the axis or perpendicular* QA *(produced if need be) to the refracting plane.*

In the perpendicular AQ, take the point q ſuch that AQ : Aq :: R : I; and through the point q parallel to the refracting plane AC, draw the indefinite line qt. Let t be the point where CQ (produced if need be) cuts the line qt: From the center C with the radius Ct deſcribe the arch tx, and the point x, where this arch meets the perpendicular AQ, is the focus of the refracted ray CD. For the right-angled triangles QAC, Qqt being ſimilar, it will be CQ : Qt :: [a]AQ : Qq; and therefore Ct (CQ ± Qt) = [b]Cx : CQ :: Aq (AQ ± Qq) : AQ :: [b]I : R, and conſequently the ∠ AxC = HCD is the angle of refraction *. Therefore, *&c. Q. E. I.*

[a] 4. VI *Euc.*
[b] Conſtruct.

Or,

* The author of this elegant conſtruction was Dr. *Barrow*.

Or the proper focus x of the refracted ray CD, may be otherwise found, thus: say,

I : R :: *Sine* ∠ AQC : *Sine* ∠ AxC; or R : I :: *Co-secant* AQC : *Cosec.* AxC. then, *Co-tangent* AQC : *Co-tangent* AxC :: AQ : Ax.

58. *Lemma.* The versed sines of very small arches of unequal circles, having the same right sine common to them all, are reciprocally proportional to the diameters of these circles, very nearly. Fig. 13, 14.

From the centers Q, x describe the arches Ca, Cb; then will Qa = QC, and xb = xC. I say the versed sines Aa, Ab of the unequal arches Ca, Cb having the same right sine CA, are reciprocally as the radii Qa, xb nearly.

For Aa : AC :: [a] AC : 2AQ + Aa; and, [a] 13. VI. *Eu.*

Ab : AC :: [a] AC : 2Ax + Ab; therefore,

Aa : Ab :: 2Ax + Ab : 2AQ + Aa.

But when Aa and Ab are but small in proportion to the diameters 2Ax + 2Ab, 2AQ + 2Aa, it will be without sensible error; Aa : Ab :: 2Ax + 2Ab : 2AQ + 2Aa. And the two last terms being divided by 2, it will be Aa : Ab :: xb : Qa nearly. *Q. E. D.*

59. *Corol.* Aa : Ab :: I : R nearly; and therefore Aa = $\frac{I}{R}$ Ab; and Ab = $\frac{R}{I}$ Aa. In *fig.* 13. Aa = $\frac{m}{n}$ Ab; and in *fig.* 14 Aa = $\frac{n}{m}$ Ab.

60. *Schol.* The aberration qx is always in the same ratio to Aa the versed sine of the angle of incidence nearly. And $qx = \frac{m}{n}$ Aa − $\frac{n}{m}$ Aa nearly. For,

CASE 1. *When the incidence is into a dense medium; it will be,* Fig. 13.

$n : m$ :: [a] AQ : Aq :: AQ + Aa (CQ) : Aq + † $\frac{m}{n}$ Aa = [a] Cx = [a] bx = Ax + Ab. Wherefore Ax = Aq + $\frac{m}{n}$ Aa − Ab; and consequently $qx = \frac{m}{n}$ Aa − Ab = [b] $\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa. *Q. E. D.* [a] Construct. & 35. I. [b] 59. I.

CASE 2. *When the incidence is into a rare medium; it will be,* Fig. 14.

$m : n$ :: [a] AQ : Aq :: AQ + Aa (CQ) : Aq + $\frac{n}{m}$ Aa = [a] Cx = [a] bx = Ax + Ab; wherefore Aq = Ax + Ab − $\frac{n}{m}$ Aa; and therefore qx = Ab − $\frac{n}{m}$ Aa = [b] $\frac{m}{n}$ Aa − $\frac{n}{m}$ Aa. *Q. E. D.*

61. *Corol.* Hence, and from *Art.* 20. when the refraction is between glass and air, the aberration qx is $\frac{5}{6}$ of Aa, the versed sine of the angle of incidence; for $\frac{m}{n} - \frac{n}{m} = \frac{3}{2} - \frac{2}{3} = \frac{9}{6} - \frac{4}{6} = \frac{5}{6}$. In like manner when the refraction is between air and water, $qx = \frac{7}{12}$ Aa.

D PROP.

† For $n : m$ :: Aa : $\frac{m}{n}$ Aa.

PROP. V.

62. *Images formed by refractions at plane surfaces, are erect, or alike situated with the objects, and on the same side of the refracting planes.*

Fig. 11, 12. From each point P, R, &c. of an object PQR radiating upon the refracting plane BC; upon the said plane let fall the perpendiculars PB, RC, &c. Now if in the said perpendiculars, the points *p*, *r*, &c. be so taken that R : I :: BP : B*p* :: CR : C*r*, &c. the points *p*, *r*, &c. so found will be the [a] images of the points P, R, &c. and therefore all these points connected together, will give the *image p q r* of the whole *object* PQR. Whence the whole of the proposition is self-evident.

[a] 50. I.

63. *Corol.* 1. If the object be rectilinear, its image will be so too; and the corresponding parts of both will be similar. All other objects and their images, will be similar, if they lie parallel to the refracting plane; otherwise not. For if they be oblique, the image of a square may be an oblong; and the image of an oblong parallellogram, may be a square; as is manifest from the preceding, and the inspection of the figures.

64. *Corol.* 2. If *e* be the pupil of the eye, the points P, R of the object will be seen by rays *ae*, *be* diverging from the focus's *p*, *r* nearly. When the eye is so situated that the axis of the pencil entering the pupil is perpendicular to the refracting plane; the focus of that pencil is [a] accurately the corresponding point of the image. But when the rays of the said pencil are oblique to the refracting plane, those rays if produced backwards would unite in a point [b] farther from the object than the corresponding point of the image; and the distance of that point from the image will be more or less, according to the greater or lesser obliquity of the rays to the refracting surface. And hence it is that objects in a different medium from that wherein is the spectator, generally appear a little distorted.

[a] 50. I.

[b] 56. I.

65. *Corol.* 3. Objects under water, to a spectator above, appear one fourth higher than they really are; and also somewhat diminished, and distorted, if they are not parallel to the surface. And to a spectator under water, objects that are above appear one third higher than they really are: And if they are oblique to the surface of the water, they'll appear somewhat magnified and distorted.

Fig. 10. Why a strait stick immersed in water appears bent.

66. *Corol.* 4. From what has been above delivered, we have also the reason why a strait stick STQ immersed obliquely into water, appears bent at the surface: the end Q appearing in *q*, and every other part by the refraction of the rays at the surface, being elevated in proportion, the whole stick appears under the form ST*q*; that is, the part under water

water appears alſo *ſtrait, as T*q*, the only bending being at the ſurface T.

§. 3. *Of the Progreſs of Light through a Medium terminated by plane Surfaces.*

PROP. VI.

67. *A ray of light as* QA, *falling obliquely upon a medium terminated by two parallel plane ſurfaces* RS, TV, *will after its ſecond refraction at* B, *emerge in a line* BC *parallel to the incident ray* QA. Fig. 15.

Produce the incident ray QA to D, and the emergent ray BC to E; and through the points A, B draw the perpendiculars PAO, FBG to the ſides RS, TV. Now becauſe the ſides RS, TV are parallel, the ray AB within the medium is equally inclined to them both; that is, the ∠ OAB = [a]FBA; wherefore if theſe be conſidered as angles of incidence, that is, if the ray be conceived to go both ways along AB, the ∠s PAQ, GBC will be [b]equal; and conſequently their oppoſite ∠s OAD, FBE are [c]equal; and taking away from theſe the equal angles OAB, FBA, the alternate angles DAB, EBA will be equal; and therefore QAD is [d]parallel to CBE. *Q. E. D.*

[a] 29. I. *Euc.*
[b] 13 & 16. I.
[c] 15. I. *Euc.*
[d] 27. I. *Euc.*

68. *Schol.* It is manifeſt that the leſs is the thickneſs of the medium RV, or the leſs is the obliquity of the incident ray, the leſs will be the diſtance between the parallel lines AD, BC: And when RV is very thin, or the angle QAP is very ſmall, the ſaid diſtance will be ſo inconſiderable, that the ray QABC may without ſenſible error be reckoned as one ſtrait line.

PROP. VII.

69. *If a ray* QABC *be tranſmitted through a medium* RST *terminated by two plane ſides* RS, TS *inclined to one another in an angle* RST; *the emergent ray* BC, *will incline from the incident ray* QA, *towards the thickeſt part* RT *of the medium: And if the angle* S, *and the angles of incidence and refraction be ſo ſmall, as to be reckoned proportionable to their ſines; the angle of deviation* QEq, *contained between the incident ray* QAE, *and the emergent ray* CBE*q*, *produced till they meet in* E, *will be to the refracting angle* RST, *as the difference of the ſines of incidence and refraction is to the leſſer of them; that is,* ∠ QE*q* : ∠ RST :: *m*—*n* : *n*. Fig. 16.

Let A*a*, B*b* be perpendicular to the ſides RS, TS reſpectively, and produce them till they meet in D: Alſo produce the incident and emergent rays QA, CB till they meet in E. Now if it be made, *m* : *n* :: Sine ∠ *a*AE : Sine ∠ *a*AB; and *n* : *m* :: Sine ∠ *b*BA : *b*B*q*; *q*B produced will be the emergent ray: whence it is manifeſt,

1. That

* For if perpendiculars be drawn from every point of the ſtrait object TQ to the plane ſurface TA, the images of theſe points will divide all the ſaid perpendiculars proportionably. Whence from the 2 of the VI. of *Euclid*, it is eaſy to ſhew, that all theſe images lie in a ſtrait line.

1. That the emergent ray BC, is more inclined towards the thickeſt part RT of the medium, than the incident ray QA: that is, the emergent ray CB produced backwards, lies between QA and S.

[a] Hyp. 2. Becauſe $\angle b\mathrm{BA} : \angle b\mathrm{B}q :: {}^{a}n : m :: {}^{a}\angle\mathrm{DAB} : \angle\mathrm{DAE}$; it will alſo be, $\angle b\mathrm{BA} : \angle\mathrm{AB}q\ (\angle b\mathrm{B}q - \angle b\mathrm{BA}) :: n : m-n :: \angle\mathrm{DAB} : \angle\mathrm{BAE}$ ($\angle\mathrm{DAE} - \angle\mathrm{DAB}$). Wherefore $\angle b\mathrm{BA} \pm \angle\mathrm{DAB}$ ($= {}^{b}b\mathrm{DA} = {}^{c}\mathrm{RST}$) : $\angle\mathrm{AB}q \pm \angle\mathrm{BAE}$ ($= {}^{b}\mathrm{AE}q$) $::{}^{d}\ n : m-n$; that is,

[b] 32. I. *Euc.* [c] Conſtruct. [d] 12. V. *Euc.*

$$\mathrm{AE}q : \mathrm{RST} :: m-n : n. \quad \textit{Q. E. D.}$$

70. *Corol.* 1. Hence the angle of deviation AEq is invariable in all poſitions of the ray; that is, whether the incident and emergent rays are equally inclined to the refracting ſurfaces, as in *Examp.* 1. or, whether one of them is perpendicular to one of the refracting ſurfaces, and therefore the whole refraction is at the other ſurface, as in *Examp.* 2. or, whether they incline to different ſides of the perpendicular, as in *Examp.* 3. The angle of deviation AEq in all theſe caſes will be the ſame, if the angles of incidence and refraction are ſo ſmall, as to be reckoned proportionable to their ſines.

Fig. 16. 71. *Schol.* Becauſe the angle AEq is leſs than S, and arches have a leſs proportion to their ſines, the ſmaller they are; the error ariſing from the ſuppoſition of the arches and their ſines being proportional, will make the angle AEq have a greater ratio to the angle S, than that of $m-n$ to n. And the leſs is the greateſt obliquity of the ray to either of the refracting ſides of the medium, as in *Examp.* 1. where the ray is equally inclined to each of the ſides RS, TS; the nearer will the ratio of the angle AEq to S, be to an equality with the ratio of $m-n$ to n. And the converſe. But in moſt caſes the inequality of theſe ratio's need not be conſidered.

Fig. 18. 72. *Corol.* 2. When the angles of incidence and refraction are ſo ſmall, as to be reckoned proportional to their ſines, the angle Q contained between two incident rays QA, QD upon a triangular priſm RST, will be equal to the angle q contained between the emergent rays BC, EF produced backwards till they meet in q. For in the triangles Qob, qoe, the angles Qbq, qeQ contained between the incident rays QA, QD produced, and the emergent rays BC, EF produced backwards, being [a] equal, and the vertical angles at o being [b] equal, the angles Q, q muſt be equal.

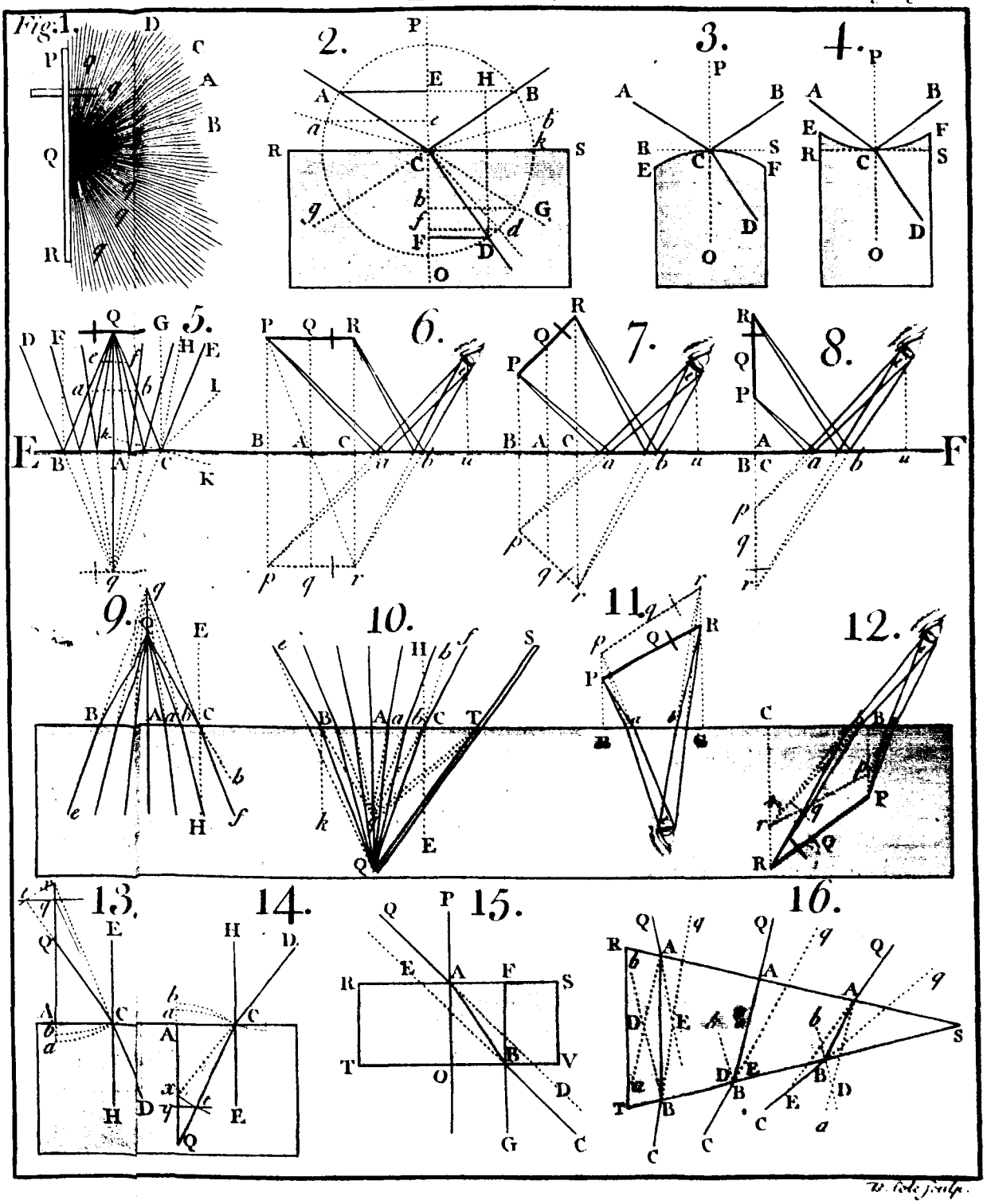
[a] 70. I. [b] 15. I. *Euc.*

PROP. VIII.

Fig. 17. 73. *If* Q *be the focus of rays diverging upon a medium* AE, *terminated by two plane and parallel ſides* AD, BE; *the focus* q *of the emergent rays will be between* q *and the medium: and* Qq *the diſtance be-*

 tween

Facing Page 20.



W. Cole Sculp.

tween the two focus's will be to DE *the thickness of the medium, as* m—n *to* m.

Let QDE be perpendicular to the refracting surfaces AD, BE, and let QA be any incident ray not very remote from the axis QD. Now, if in the axis DQ produced, the point a be taken such, that QD : Da :: [a]n : m; a will be the focus of the refracted ray AB. And if the point q be taken such, that aE : qE :: [a]m : n; q will be the focus of the emergent ray BC. Wherefore, aQ (aD—QD) : aD :: m—n : m :: aq (aE—qE) : aE :: aQ+Qq : aD+DE (aE.) Whence aq is greater than aQ; and Qq : DE :: [b]m—n : m. *Q. E. D.*

[a] 35 & 50. I.

[b] 19. V. *Euc.*

74. *Corol.* When the refracting medium is glass, it will be Qq = $\frac{1}{3}$DE nearly.

75. Schol. *The aberrations at the different surfaces, are contrary ways, and almost destroy each other.*

Let x be the proper focus of the refracted ray AB; y the geometrical focus of rays within the medium, whose focus of incidence is x, after their emersion at the side BE; that is, xE : yE :: m : n; and r the proper focus of the emergent ray BC. Now the aberrations xa, yr are in the ratio of the sines nearly; that is, xa : yr :: [a]m : n nearly. But m : n :: [b]xE : yE. Wherefore,

xE—xa (aE) : yE—yr (rE) :: m : n :: [b]aE : qE. And therefore the point r coincides with q nearly. *Q. E. D.*

[a] 58, 59, & 60. I.

[b] Construct.

Otherwise thus: produce QA to s, and draw At parallel to QE, and tu parallel to QA; then will Qu = At = DE. Now the ray CB produced backwards to q is [c]parallel to AQ; and st : Bt :: [d]m : n, nearly. But Qu : qu :: [e]st : Bt; and therefore Qq : Qu :: m—n : m; that is, the point q found as in the *Prop.* is without sensible error the focus of the emergent ray BC. *Q. E. D.*

[c] 67. I.

[d] 35. I.

[e] 2. VI. *Euc.*

PROP. IX.

76. *Having the focus of incident rays, upon a medium terminated by two plane sides inclined to one another in a given angle, to find the focus of the emergent rays.*

Let Q be the focus of incident rays, QA, QD upon the triangular prism RST. Upon the side SR (produced if need be) let fall the perpendicular QG, and in this perpendicular take the point a such that GQ : Ga :: n : m, then will a [a]be the geometrical focus of the pencil of refracted rays AB, DE within the prism. Again in the perpendicular aH to the side TS (produced if need be) take the point q such, that Ha : Hq :: m : n; and [a]q will be the geometrical focus of the pencil of emergent rays BC, EF. *Q. E. I.*

Fig. 18.

[a] 50 & 35. I.

77.

77. *Schol.* The aberrations of the rays AB, DE; and of BC, EF from the focus's a, q may be found, as in *Art.* 57: The proper foci of the ſaid rays, will be in lines without, and parallel to, the line aH; and the more oblique is the incidence of any ray QA or QD, the more remote will the focus's of the refracted rays be from the line aH.

78. *Corol.* The line Qq joining the focus's Q, q is parallel to the line GH, joining the points G, H where the perpendiculars meet the refracting ſides, RS, TS.

PROP. X.

Fig. 19, 20. 79. *The image* pqr *of an object* PQR, *formed by the refraction of a medium terminated by two plane and parallel ſides* BC, bc; *is erect, equal and parallel to the object, and on the ſame ſide of the medium, but nearer to it by* $\frac{m-n}{m}$ *part of the thickneſs of the medium.*

Draw the perpendiculars PBb, QAa, RCc, *&c.* from each point of the object to the refracting ſurfaces, then will the diſtances of the ſeveral correſponding focus's Pp, Qq, Rr, *&c.* be each to the thickneſs of the medium Aa or Bb, *&c.* as [a] $m-n$ to m; that is Qq = $\frac{Aa \times \overline{m-n}}{m}$. *Q. E. D.*

[a] 73. I.

80. *Corol.* When the refracting medium is glaſs, the diſtance between the object and the image will be $\frac{1}{3}$ of the glaſs's thickneſs.

Fig. 21. 81. *Schol.* 1. The image pq of an object PQ formed by rays refracted by a triangular priſm RST, may be determined by finding the images of each point of the object, after the manner ſhewed in *Art.* 76. Thus, from each point Q, P, *&c.* of the object upon the refracting ſide SR (produced if need be) let fall the perpendiculars QG, PI, *&c.* and in theſe perpendiculars take the points a, b, *&c.* ſuch that $n : m ::$ GQ : Ga :: IP : Ib, *&c.* From the points a, b, *&c.* thus found, upon the ſide TS produced, let fall the perpendiculars aH, bK, *&c.* and in theſe perpendiculars take the points q, p, *&c.* ſuch, that $m : n ::$ Ha : Hq :: Kb : Kp, *&c.* and the points q, p, *&c.* thus found will be the places of the images of the points Q, P, *&c.*

82. *Schol.* 2. From the conſtruction of the figure, it appears that the image is not exactly parallel to the object; nor are the object and image exactly equal as to length, and conſequently if they have any breadth they will not be exactly ſimilar; for there will be no inequality betwixt thoſe ſides and their images, that are parallel to the axis of the priſm. But becauſe of the great obliquity of the rays to the refracting ſides, the apparent images, or the images of objects ſeen by rays tranſmitted through two ſides of a triangular priſm, will, in many caſes, be very obſcure and imperfect; and differ greatly as to likeneſs, ſituation, *&c.* from thoſe found by theory.

SECTION

SECTION III.

Of the Reflection and Refraction of Light at a Spherical Surface.

§. 1. *Of Reflected Rays.*

83. *Defin.* LET O be the center of the ſpherical reflecting ſurface or ſpeculum BAC, then will any line as OA drawn from O to the ſpeculum be perpendicular thereto; and therefore be the axis of a pencil of rays whoſe focus is any where in the ſaid perpendicular. Any line OA drawn from the center to the ſpeculum, is alſo called the *Axis of the Speculum*; but more particularly that line, as OA, is called the *Axis* which is drawn from the center to the middle of the ſpeculum. Let F be the geometrical focus of rays parallel to the axis, after reflection, or the focus of thoſe rays that are neareſt to it; then is F called the *Focus of the Speculum*; alſo the *principal Focus*, to diſtinguiſh it from the focus of diverging or converging rays; alſo F in *fig.* 22. is called the *burning Focus*, becauſe it is the focus of the ſun's rays. Fig. 22, 23.

Axis, Focus of a Speculum.

PROP. XI.

84. *When parallel rays are incident upon a ſpherical ſpeculum* BAC, *the geometrical focus* F, *of the reflected rays, will biſect that radius* OA, *which is parallel to the incident rays.* Fig. 22, 23.

Let DB be an incident ray parallel to the radius OA, and from the center O to the point of incidence B, draw the perpendicular OB, and produce it *fig.* 23. to *o*; alſo in that figure produce DB to *d*, and let *f*BF be a ſtrait line. Now the angle of reflection OBF or *o*B*f* = [a] angle of incidence DBO or DB*o* = [b] angle BOF, therefore is OF = [c] FB. But when the triangle OFB is vaniſhing, that is when the point B is very near to A, FB will be = FA nearly, and therefore OF will be equal FA nearly. Therefore, *&c.* *Q. E. D.*

[a] 11. I.
[b] 29. I. *Euc.*
[c] 6. I. *Euc.*

85. *Corol.* 1. If F be the focus of incident rays, the geometrical focus of the reflected rays will be at an infinite diſtance; that is, rays diverging from the principal focus of a concave ſpeculum, or converging towards the ſaid focus of a convex ſpeculum, will have the reflected rays next the axis, parallel thereto.

86. *Corol.* 2. When the angle of incidence DBO (= BOA) is conſiderable, that is, when the point of incidence B is at a ſenſible diſtance from A, the reflected ray will interſect the axis in ſome point *x*, between Fig. 24.

tween F and the vertex A of the ſpeculum; and the greater is the angle BOA, the greater will be the *aberration* Fx from the geometrical focus
[a] 84. I. F. For Ox = [a] xB, and Ox + xB is greater than OB or OA; and therefore Ox is greater than ($\frac{1}{2}$OA) OF. And it is manifeſt that the greater is the angle O, the more will Ox exceed OF.

When the ∠ O is 45°, the ∠ OxB will be 90°; that is, the ray will be reflected along the right line BV of the angle O. If the angle O is 60°, the triangle OxB will be equilateral, and the point x will coincide with A; that is, the ray will be reflected into the vertex A of the ſpeculum.

Fig. 22, 23. 87. *Corol.* 3. Hence, if F be the focus of incident rays, the rays that are remote from the axis, will after reflection by a concave ſpeculum *converge*; and *diverge* after reflection by a convex ſpeculum.

Fig. 24. Longitudinal and lateral Aberration. 88. *Defin.* Let FA be the axis of a pencil to which the ray Bx belongs; F the geometrical focus of that pencil, and x the proper focus of the ray Bx. At the point F upon the axis FA raiſe the perpendicular FG, and produce it till it meets Bx produced in G. Now Fx is called the *longitudinal Aberration*, and FG the *lateral Aberration*, of the ray Bx.

PROP. XII.

Fig. 24. 89. *Let* DB *be an incident ray parallel to the axis* OA; BT *a tangent at* B *to* AB; *then will* OT *be the ſecant of the angle* O = DBO *the angle of incidence. I ſay, the longitudinal aberration* Fx *of the reflected ray* Bx, *is equal to half* AT *the exceſs of the ſecant* OT *of the angle of incidence above the radius* OA *or* OB.

[a] Conſtruct. [b] 32. I. *Eucl.* [c] 6. I. *Eucl.* For the ∠ xBT = [a] 90° — ∠O (= OBx) and alſo the ∠T = [b] 90° — ∠O; and therefore xT = [c] xB = xO. Wherefore (becauſe OT = 2Ox = 2OF + AT) Ox = OF + $\frac{1}{2}$AT; that is, Fx = $\frac{1}{2}$ AT. *Q. E. D.*

And thus having the angle of incidence given, by a table of ſecants we have Fx; or, in the triangle OxB, we have OB and all the angles, to find Ox.

90. *Corol.* 1. When the angle of incidence is not very large, the la-
[d] 13. VI. *Eu.* teral aberration FG = $\frac{VB^3}{2OA^2}$ nearly. For VA or AT = [d] $\frac{VB^2}{2OA}$ nearly, and ſo Fx = $\frac{VB^2}{4OA}$ nearly. But FG : Fx $\left(\frac{VB^2}{4OA}\right)$ :: VB : xV = $\frac{1}{2}$ OA nearly. Therefore, *&c.*

91. *Schol.* 1. When the angles of incidence are not very large, the *longitudinal aberrations* will be *as the ſquares*, and *the lateral ones as the cubes, of the ſines of theſe angles*, nearly.

Draw

Draw the right line BV. Now the right-angled triangles OVB, BVT being similar, it will be,

OV (OA—VA) : VB :: VB : VT; But when the arch AB is very small, VA will be incomparably less than the invariable quantity OA; and therefore a small augmentation or diminution of the arch AB, will not sensibly affect the first term OV; whence VT will be as VB^2, nearly. Also in this case, VA is nearly equal to AT; for VA (OB—OV) : AT (OT — OB) :: OV : OA; and therefore Fx ($= \frac{1}{4}$ VT nearly) will be as VB^2 nearly. *Q. E. D.*

Again, If whilst the $\angle$ O is increasing or diminishing, Fx be supposed to remain invariable; FG will be as VB, for the $\triangle$s xFG, xVB are similar: And if during the augmentation of the $\angle$ O, the $\angle$ x be supposed to remain invariable, or that BG moves parallel to itself; FG will be as Fx, that is, as VB^2. Whence it is manifest, that since Fx and the angle x are together both increased or both diminished, FG will be as VB x VB^2; that is, as VB^3 nearly. *Q. E. D.*

92. *Schol.* 2. When the arch AB has a sensible magnitude, the *increase* or *diminution* of VT will be greater than in the ratio of VB^2, and *that* of VA will be less than *that* of AT (which is always as Fx), and therefore *that* of VT will be less than *that* of Fx: And consequently the ratio of Fx is still greater than that of VB^2; and therefore also the ratio of FG (which with respect to Fx increases or diminishes as VB^3 to VB^2) is greater than that of VB^3. However in most cases, VB^2 and VB^3 are sufficiently accurate for estimating the ratio's of the aberrations, of rays reflected from different distances from the axis.

93. *Corol.* 2. The rays that are nearest the axis, are reflected much closer together, than those that are remote: And therefore the burning focus of the speculum, or the point in the axis where most rays are collected together, will be in F, or very near to F.

It is also easy to conceive, that the reflected rays are denser or closer together between F and the speculum, than they are at the same distance from F on the contrary side.

PROP. XIII.

94. *Let* QA *be perpendicular to the spherical reflecting surface* BA, *whose center is* O, *and principal focus* F; *and let* Q *be the focus of incident rays: Now if it be made* FQ : FO :: FO : Fq; *and the point* q *be placed the same way from* F *that* Q *is from* F, *the point* q *will be the geometrical focus of the reflected rays.* Fig. 25 *to* 28.

Let B be a point of incidence not remote from A, to which from the center O draw the perpendicular OB, and parallel to the incident and reflected rays QB, Bq, draw the lines Oa, Ob; then will the tri-

[a] 29. I *Eucl.* angles QOb, Oqa be [a] ſimilar, and therefore $Qb : Ob$ :: [b] $Oa : qa$.
[b] 4. VI. *Euc.* Now the triangles ObB, OaB having the common baſe OB, are equi-
[c] 11. I. crural and alſo equal to each other; (for the $\angle bBO =$ [c] OBa, and the alternate $\angle aOB =$ [a] bBO, alſo $bOB =$ [a] aBO) and therefore when theſe triangles are vaniſhing, that is, when B comes to A, the ſides Ob, Oa will be each $= \frac{1}{2}$ OB $=$ OF, and the points b, a will coincide with F: Wherefore it will be $QF : OF :: OF : qF$. *Q. E. D.*

The figures ſerve for the caſes of a *convex* as well as for a concave ſurface, by ſuppoſing the incident and reflected rays to be produced backwards through that ſurface.

[94.] *Corol.* 1. QO ($QF \mp OF$ or $OF \mp QF$) : OF :: qO ($OF \mp qF$ or $qF \mp OF$) : qF. Alſo $QO : Qq$ ($QO \pm qO$ or $qO \pm QO$) :: OF (AF) : Aq ($AF \pm qF$ or $qF \pm AF$.)

95. *Corol.* 2. If q be the focus of incident rays, Q will be the focus of the reflected ones. Compare together *fig.* 25, 27; and alſo *fig.* 26, 28.

96. *Corol.* 3. Concave and convex ſpeculums (that are ſegments of the ſame ſphere) are juſt oppoſite in their effects; that is, rays falling upon them the juſt contrary ways, will have the ſame focus after reflection.

Fig. 25, 27. 97. *Corol.* 4. Both the focus's are united in O; that is, when one is in O the other will be alſo in O, and their motion from O is the contrary way, but with a very different velocity; the one being continually retarded or accelerated, as the other moves uniformly from or towards F: So that when one is in F, the other will be at an infinite diſtance (for when QF or qF is infinitely little, the other muſt be infinitely great.) And when *(fig.* 25.*)* $QO = OF$, Oq will be $= qF$ (for then $QF = 2OF$; and therefore $qF = \frac{1}{2}OF$). So that an image moves as far, whilſt the object moves back from the center of a ſpeculum half the length of the radius of the ſpeculum, as it would afterwards, if the object ſhould proceed to an infinite diſtance.

Fig. 26, 28. 98. *Corol.* 5. When one of the focus's is between F and A, the other will be on the contrary ſide of the ſpeculum: Alſo their motions will be contrary, as before; that is, as one moves towards or from the ſpeculum on the one ſide, the other will alſo move towards or from the ſpeculum on the other ſide; but with a different velocity: For when one is in F, the other will be at an infinite diſtance, and yet they'll both arrive at A, which is the place where they ultimately unite, in the ſame time. When one is half way between F and A, the other will be within twice that diſtance of A on the other ſide. The motions of the two focus's on each ſide of A, being exactly like to their motions on each ſide of O, in the preceding Corollary.

99.

99. *Corol.* 6. If the focus of incident rays, diverging upon a concave speculum, or converging upon a convex speculum, be farther from the speculum than its principal focus F; the focus of the reflected rays will be also on that same side of the speculum; that is, the rays reflected by the concave speculum will have a real focus, and those reflected by the convex speculum will diverge from an imaginary focus: And the contrary happens, when the focus of incident rays is between F and the speculum; that is, the focus of the reflected rays will be on the other side of the speculum; the rays reflected from the concave speculum diverging from, and those reflected by the convex speculum converging towards, that said focus. Fig.25,27. Fig.26,28.

100. *Corol.* 7. When the focus's Q, q, lie on the same side of the reflecting surface; if the incident rays diverge from Q, the reflected ones will converge towards q; and if the incident rays converge towards Q, the reflected ones will diverge from q. But when the focus's Q, q, are on contrary sides of the surface: If the incident rays diverge from, or converge towards Q, the reflected rays will accordingly diverge from or converge towards q. Fig.25,27. Fig.26,28.

101. *Schol.* When Q is in O, all the rays reflected by the speculum will either accurately converge to O, or diverge from O, without any aberration. But in all other cases, the focus q, found as by the Proposition, will be the focus of those rays only that are nearest the axis; all others after reflection, intersecting the axis at a lesser or greater distance from q, as their incidence is less or more oblique.

PROP. XIV.

102. *Having the focus* q *and the point of incidence* B, *of an incident ray* qB, *together with the geometrical focus* Q *of the reflected rays; to find the proper focus* Y, *of the ray* qB *after reflection, it will be* Fig 27,28.

$\overline{qO \pm \tfrac{1}{2}\,secant\ of\ AB} \times \overline{\tfrac{1}{2}\,Radius\ OA} : \overline{qO \pm \tfrac{1}{2}\,Rad.} \times \overline{\tfrac{1}{2}\,sec.\ of\ AB} :: OQ : OY.$

Parallel to the incident and reflected rays qB, BY, draw the lines Ob, Oa: draw also the tangent Bt, and parallel thereto through the point a or b, draw the line ade; then will ad bisect OB at right angles, For the △ OaB or ObB [a] is equicrural; and therefore Oe = [b] et = $\tfrac{1}{2}$ secant of the arch AB. Now, [a] Construct. [b] 2. VI *Euc.*

$qY : OY ::$ [b] $qB : (Ob)\ aB ::$ [b] $qt : et$; and therefore,
$qe\,(qt \mp et) : Oe\ (= et) :: qO\ (qY \mp OY) : OY$; Also
$OF : qF ::$ [c] $QO : qO$; Wherefore, [c] [94.] I.
$qe \times OF : Oe \times qF :: (qO \times QO : OY \times qO ::)\ OQ : OY$; that is,
$\overline{qO \pm \tfrac{1}{2}\,secant} \times \overline{\tfrac{1}{2}\,Rad.} : \overline{qO \pm \tfrac{1}{2}\,Rad.} \times \overline{\tfrac{1}{2}\,secant} :: OQ : OY.$ Q.E.D.

Or,

Or, the point Y may be found by Trigonometry, thus: In the $\triangle$ qOB, are given the ſides qO, OB, and the $\angle$ included qOB, to find the $\angle$ qBO: then in the $\triangle$ OBY will be given the ſide OB, and all the angles, to find OY.

103. *Corol.* 1. OQ is to QY, as the firſt term is to the difference of the ſecond and firſt; that is, OQ : QY :: $qe \times$ OF : O$e \times q$F — $qe \times$ OF.

104. *Corol.* 2. If YB is an incident ray, q will be its focus after reflection. And if QB is an incident ray, its focus y after reflection will be ſuch, that the $\angle$ qBy = QBY : and therefore the aberration QY being found, the aberration qy is alſo eaſily found. For in the $\triangle$ qBy, we may have the ſide qB and all the $\angle$s, to find qy.

105. *Corol.* 3. OQ is leſs than OY. And therefore when both the
Fig. 27. focus's Q,q, are on the ſame ſide of the ſpeculum, the proper focus Y or y, of a ray qB or QB after reflection, will be nearer the ſpeculum than the geometrical focus Q or q. Alſo when the focus's Q,q are on different ſides; if the focus q of the incident ray be of the convex-ſide of the ſpeculum, the proper focus Y of the reflected ray, will be between the
Fig. 28. geometrical focus Q and the ſpeculum: But if the focus Q of the incident ray, be between F and the ſpeculum; the proper focus y of the reflected ray, will be farther from the ſpeculum than the geometrical focus q.

106. *Corol.* 4. When q (the focus of incidence) is in O, the points Q,Y, will both coincide with O: And the arch AB remaining the ſame; the greater is Oq, if the focus's are both on the ſame ſide, otherwiſe the leſſer is Oq, the greater will be the aberration QY; and when OQ is infinite, QY becomes = Fx (*fig.* 24.) the aberration of parallel rays. And Oq remaining the ſame, the greater is the arch AB, the greater will be the aberration QY.

107. *Corol.* 5. The arch AB remaining the ſame, the aberration QY continually increaſes, as the point Q or Y is farther removed from O towards A: And when it is greateſt of all, that is, when Q, and conſequently q is in A, it will be equal to twice the exceſs of the ſecant of the arch AB above the radius, nearly.

Fig. 28. Draw the chord AB, then will the $\angle$ ABt be the complement of the angle of incidence, and conſequently the reflected ray will belong to the focus S; ſo that the $\angle$ SBt = ABt. And therefore AS = 2At nearly, is the aberration.

[a] 102. I. 108. *Corol.* 6. Becauſe $qe \times$ OF : qF $\times$ Oe :: [a]OQ : OY; And
[b] [94.] I. qF : OF :: [b]qO : OQ; it will alſo be, qe : Oe :: qO : OY; which is a very eaſy rule for finding the point Y.

Y. That is, $qO \pm \frac{1}{2}$ *sec.* : $\frac{1}{2}$ *sec.* :: qO : OY. Also $qO \pm \frac{1}{2}$ *Rad.* : $\frac{1}{2}$ *Rad.* :: [b] qO : OQ.

109. *Corol.* 7. To find the lateral aberration QG; having drawn the sine BV, it will be YV : VB :: QY : QG. Whence, the nearer the point Y is to V, the greater is the proportion of QG to QY. And when Y is in V, QG will be infinite. In like manner when the focus of the reflected rays, is farther from the speculum than the center O, the farther it is from O, the less proportion will the lateral aberration bear to the longitudinal.

§. 2. *Of Images formed by rays reflected at a spherical Surface.*

PROP. XV.

110. *Having the position of an object* PQR, *with respect to a given spherical speculum* BAC; *to find the image* pqr. Fig. 29 to 32.

Through the center O of the speculum, and as many points P, Q, R, *&c.* of the object as are necessary, draw the lines POB, QOA, ROC, *&c.* and bisect the perpendiculars OB, OA, OC, *&c.* in E, F, G, *&c.* Then in these perpendiculars (produced if need be) find the points *p, q, r, &c.* by saying PE : OE :: OE : E*p*; and QF : OF :: OF : F*q*; also RG : OG :: OG : G*r*, *&c.* and lay these points on the same side of the arch EFG, that the object is of; then will the points *p, q, r, &c.* thus found [a] be the *geometrical Foci* after reflection, of those pencils whose foci of incidence were the points P, Q, R, *&c.* And by connecting these points orderly together, we shall have the *Image* pqr *of the whole Object* PQR. *Q. E. I.* [a] 94. I.

After the same manner as the images of physical lines are found, by dividing them into physical points; so also may the images of surfaces be found by dividing them into physical lines, and connecting together the images of those lines.

In the figures, to prevent too great a confusion of lines, there are drawn lines only from the extreams and middle part of the object; but every other point of the object as well as these is understood to propagate rays of light to the speculum. Also in the pencils besides the axes, POB, QOA, ROC there are drawn only two rays P*b*, P*b*; Q*a*, Q*a*; R*c*, R*c*; one on each side the respective axes at a small distance from them, the rays that are between these contributing mostly towards forming the image; the others, whose incidence are more oblique, having their proper foci after reflection, at [b] some distance from the geometrical *Foci p, q, r.* [b] 105. I.

111. *Corol.* 1. The lengths of the object and image are to each other in the ratio of their distances from the center O of the speculum; that is, PQR : *pqr* :: OQ : O*q*. And ∠POR = ∠PO*r*; that is, the object and its image form both the same angle, at the center O of the speculum.

112. *Corol.* 2. When the object and its image are on different sides of the center, as in *fig.* 29, 30. the image will be inverted: But when they are on the same side, as in *fig.* 31, 32. the image will be erect.

113. *Corol.* 3. A convex speculum, *fig.* 32. cannot form a real image; nor a concave one, when the object is betwixt it and its principal focus, as in *fig.* 31. the reflected rays represented by *bb*, *bb*; *aa*, *aa*; *cc*, *cc*; in these two cases diverging as from the points *p*, *q*, *r* on the other side of the speculum: And these also are the only two cases where the image is erect. A real image, as in *fig.* 29, 30. being always inverted.

114. *Corol.* 4. If the object PQR be an arch of a circle, concentric to the spherical surface BAC, its image *pqr* will be also a similar concentric arch. And so will also a spherical segment, formed by the rotation of the arch PQR round the axis OA, have a similar concentric segment for its image.

115. *Corol.* 5. In all other cases, excepting as in the last article, the object and its image will not be exactly similar. But as a small arch differs insensibly from a strait line; so a small strait object at a good distance from the speculum, may be reckoned to have a strait image very nearly.

116. *Schol.* 1. The image of a strait object is curvilineal, and its curvity is the same way with that of the speculum *. And the nearer the object is to the principal focus F, *fig.* 30, 31. the greater will be the curvity of its image.

Fig. 29 *to* 32.

Because QF : OF :: [a]OF : F*q*; it will also be,

QF : QO (QF ∓ OF) :: OF : O*q* (OF ∓ F*q*).

[a] 110. I.

Also PE : PO :: OE : O*p*. Wherefore, OF : QF :: O*q* : OQ; and OF (OE) : PE :: O*p* : OP. Whence it is manifest, that

In *fig.* 29, 31. the ratio of O*q* to OQ is greater than the ratio of O*p* to OP; And in *fig.* 30, 32. the ratio of O*q* to OQ is less than the ratio of O*p* to OP. And so will also the ratio of O*q* to OQ be greater or less, as the case is, than the ratio of the distance of any point of the image from O, to the distance of the corresponding point of the

* The learned Dr. *Barrow*, in his XVIIth Optical Lecture, hath shewed that *the image of a rectilinear object is one or other of the Conic Sections* nearly. But it would be needless here to pursue the speculation of images any farther.

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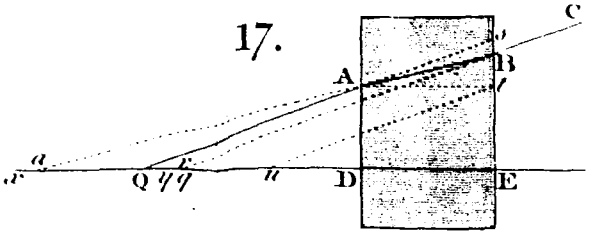

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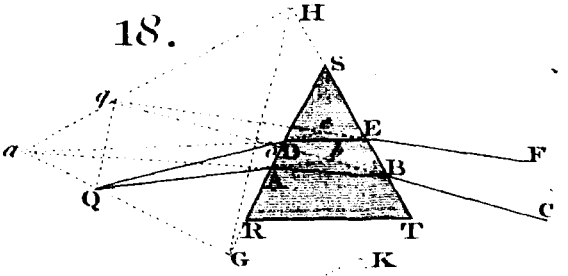

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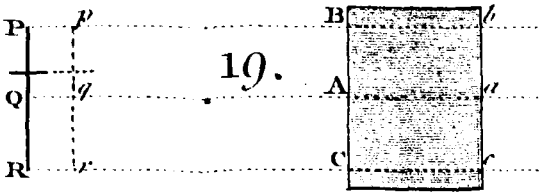

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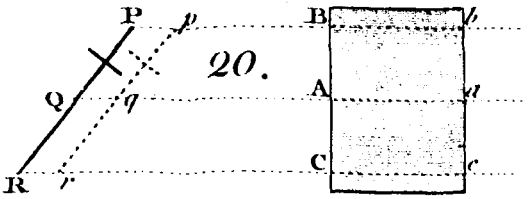

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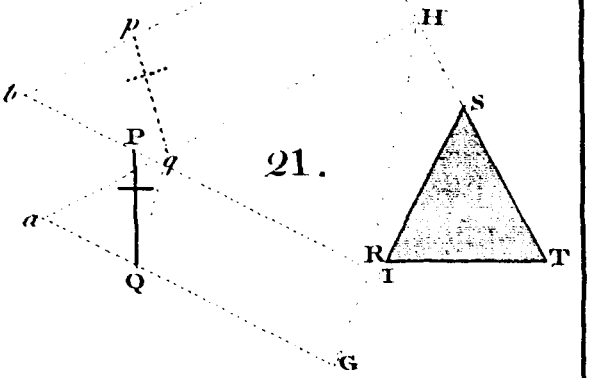

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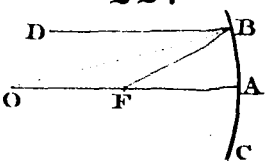

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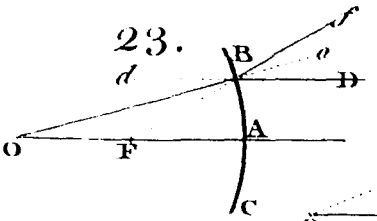

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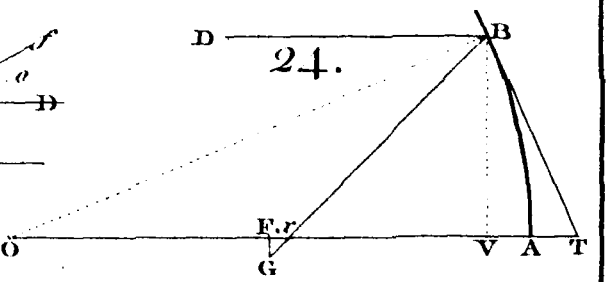

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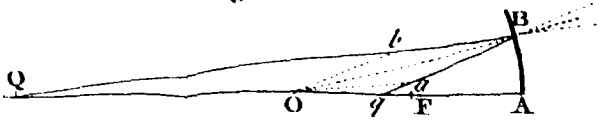

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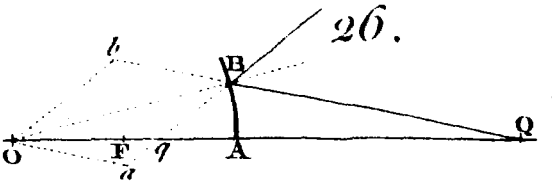

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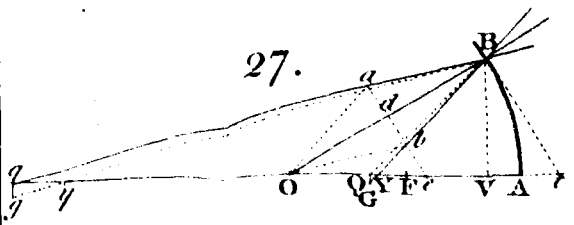

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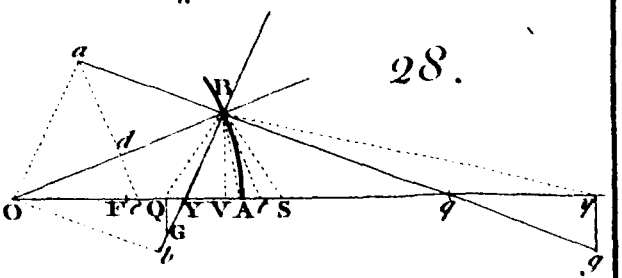


the object from O: And consequently *the image* qp *is a curve, and its curvity is the same way with that of the speculum* AB. And it is manifest also, that *the nearer the point* Q *is to* F, fig. 30, 31. *the greater will be the curvity of the image.* Therefore, *&c.* *Q. E. D.*

117. *Schol.* 2. Since an object placed at any distance, and its image, form both [a] the same angle at the center of a speculum; the angle under which any very remote object appears, being given, it will be easy to find the diameter of its image. [a] 111. I.

Let O be the center, and q the principal focus of the concave speculum A; and let the angle POR = *por* be that under which the sun, or any other very remote object, appears to the naked eye. Bisect the angle pOr with the line Oq; then in the right-angled triangle Oqr, or Oqp, we shall have the side Oq and all the angles, to find qr or qp = the semi-diameter of the image. Fig. 33.

118. *Schol.* 3. The theory of the focus's of rays reflected by a spherical speculum may be thus illustrated. Through the center O of the speculum A, draw the indefinite line BD, and bisect OA in F; from the points O and A, divide the lines OB, AD into parts, each equal to OF or FA, marked by the figures 1, 2, 3, 4, 5, *&c.* And from F take on each side F 1, F $\frac{1}{2}$, F $\frac{1}{3}$, F $\frac{1}{4}$, F $\frac{1}{5}$, *&c.* each equal to the whole or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* of FO or FA. Now if in the line OB, the point 1 or 2, 3, 4, 5, *&c.* be the focus of incident rays; the corresponding point 1 or $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* in the line OF will accordingly be the focus of the reflected rays. And, *vice versâ*, if these last be the foci of incidence, the others will be the foci after reflection. In like manner, if 1, 2, 3, 4, 5, *&c.* in the line AD be the foci of incidence; 1, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, *&c.* in the line AF will be the foci of the reflected rays. And, *vice versâ*. So that the foci of incidence and reflection unite in O and A: And their motions from O and A towards B and F in the one case, and towards D and F in the other case, are exactly similar; F being the focus of parallel rays in both cases. Fig. 34.

119. If in a dark room a lighted candle be held farther from a concave speculum than its principal focus F, as suppose at 2; its image will be seen distinct, but inverted, upon a white paper held at the corresponding point $\frac{1}{2}$. And if as the candle is moved towards or from the speculum, the paper be also moved so as to retain the distinct image of the candle, we shall have an ocular proof of the theory of the *Foci* of incident and reflected rays. When the candle is at O, the paper will be there also. But because the image of an object placed between F and a concave speculum, or any where before a convex speculum, is on the other side of the speculum; in either case the experiment of the candle and paper cannot take place.

PROP.

PROP. XVI.

120. *The ſituation of an object, and the place of the eye, being given; to find the viſible area, or how much of the object can be ſeen at one view, by rays reflected from a given portion or aperture of a given ſpeculum.*

Fig. 35 to 38. Let PR be an indefinite object, E the place of the eye; BC the given aperture of the ſpeculum, and O its center. From E to the extremities B, C of the aperture of the ſpeculum, draw the lines EB, EC; draw alſo the perpendiculars OB, OC; and to the indefinite object PR, draw the lines BP, CR by making the angles OBP, OCR equal reſpectively to the angles OBE, OCE; then will PR be the viſible area required. For it is manifeſt that the rays PB, RC will be reflected to E; and alſo that no rays diverging upon the ſpeculum BC, from points that are farther from the axis than P, R will be reflected to E; and conſequently that PR is the largeſt object that can be ſeen in the aperture BC from the point E. *Q. E. I.*

Or, PR may be otherwiſe thus found: Draw the indefinite image *pr*, and having drawn the lines EB, EC, produce them, if need be, till they cut the image in the points *p*, *r*; then lines drawn from O to the points *p*, *r*, and produced as the caſe requires, will give the limits of PR.

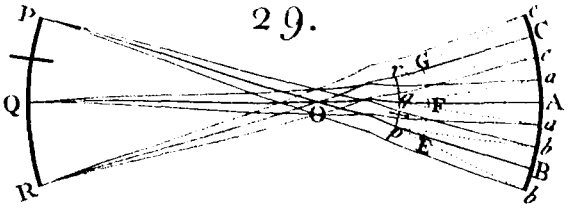
Fig. 35, 36. 121. *Corol.* 1. When the object is farther from a concave ſpeculum than its principal focus; if E is farther from the ſpeculum than the image *pr*, the farther E is from *pr*, the greater will be PR; for then the angles OBP, OCR will be greater; and the object will appear inverted. And when E is in *pr*, PR will be diminiſhed to nothing. If E be between *pr* and the ſpeculum; the nearer E is to the ſpeculum, the greater will be PR; and PR will appear erect.

Fig. 37, 38. 122. *Corol.* 2. When the object is nearer a concave ſpeculum than its principal focus, or any where before a convex ſpeculum, it will appear erect, wherever E is placed: And the nearer E is to the ſpeculum, the greater will be PR.

123. *Schol.* In finding PR, no allowance was made for the breadth of the eye's pupil; but that is eaſily done by drawing lines from the extreams of the pupil to the points B, C; and this would make PR ſomewhat greater. But that nicety would be ſuperfluous; for in ſome caſes the phenomena do not exactly agree with this theory, for reaſons that will be given in the next Book.

§. 3.

29.



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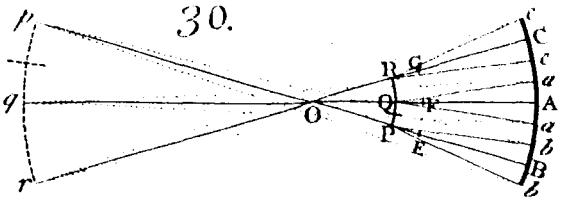

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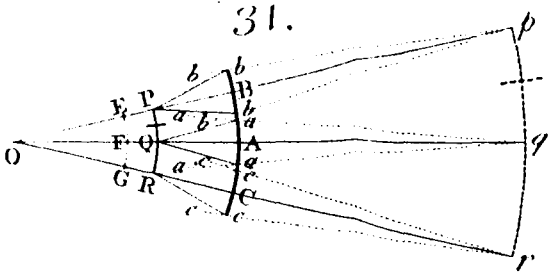

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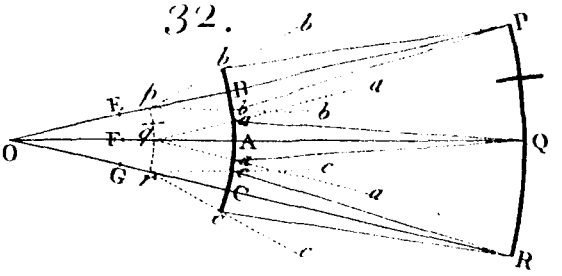

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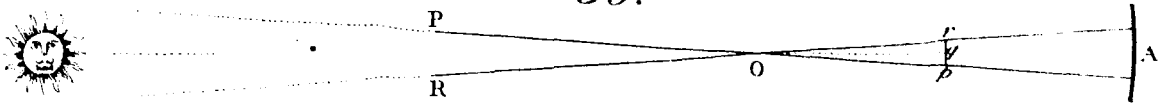

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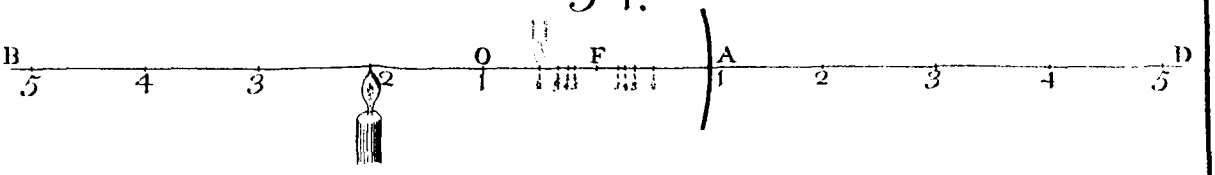

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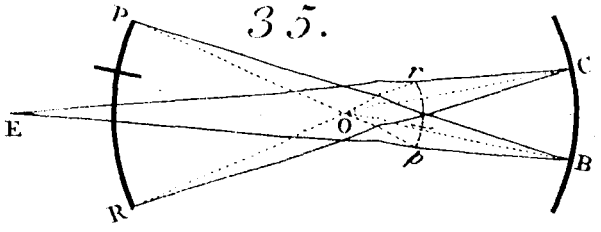

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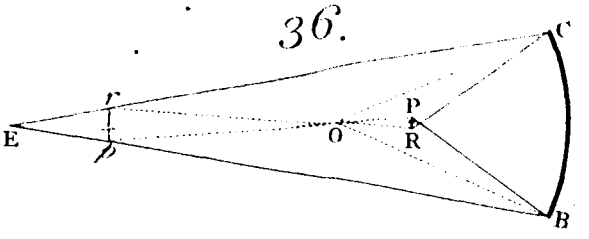

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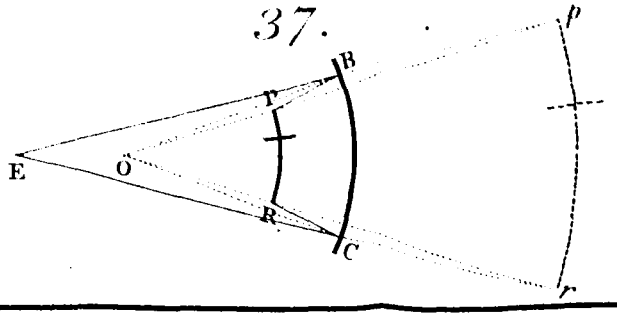

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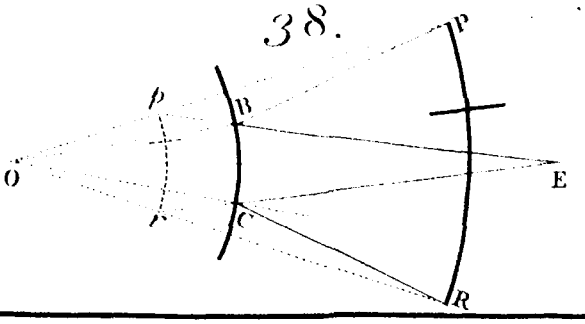


§. 3. *Of the Refraction of Light at a Spherical Surface.*

PROP. XVII.

124. *To find the geometrical focus of parallel rays, after refraction at a spherical surface, of any given refracting medium.*

Let BAC be a spherical refracting surface, whose center is O, and let the perpendicular KAOF to the refracting surface, be the axis of a pencil of parallel rays. Now if in the said axis the point F be taken such, that AF : OF :: I : R; and the point F be taken on the same side of A that O is of, if the incidence is out of a rare medium; otherwise on the contrary side, the point F will be the focus required. Fig. 39 *to* 42.

Let DB be any incident ray, parallel to the axis KOF, and through the point of incidence B, draw the perpendicular OBE; then will the ∠ DBE, *fig.* 39, 42. or its supplement DBO, *fig.* 40, 41. be the angle of incidence: And if BF (or its continuation BL, *fig.* 40, 42.) be the refracted ray; the ∠ OBF, *fig.* 39, 40. or its supplement EBF, *fig.* 41, 42. will be the angle of refraction. Wherefore in the triangle OBF, it will be BF : OF ::[a] I : R; And AF : OF ::[b] I : R. But when the arch AB is vanishing, BF will be equal to AF, and therefore the point F is the geometrical focus of the refracted rays. *Q. E. I.*

[a] Trigonom.
[b] Hypoth.

125. *Corol.* 1. AO : AF :: $m-n$: I; and AO : OF :: $m-n$: R.

126. *Corol.* 2. When the incidence is into a dense medium, AO : OF :: $m-n : n$; and AO : AF :: $m-n : m$. Also OF : AF :: $n : m$. Fig. 39, 40.

127. *Corol.* 3. When the incidence is into a rare medium; AO : OF :: $m-n : m$; and AO : AF :: $m-n : n$. Also OF : AF :: $m : n$. Fig. 41, 42.

From *Art.* 13, 20, and 124, we have also the following Corollaries.

128. *Corol.* 4. Rays in Air, falling upon the {convex / concave} surface of glass BAC, parallel to the axis KOF; will after refraction {converge to / diverge from} a focus F, whose distance FA from the vertex A, is three times the radius OA, of the arch BAC. Fig. 39, 40.

129. *Corol.* 5. Rays in Glass, {diverging from / converging towards} a focus F, whose distance from the vertex A, is three times the radius OA; do after refraction by the {concave / convex} surface of Air BAC, become parallel to the axis FOK. Fig. 39, 40.

Fig.41,42. 130. *Corol.* 6. Rays in glaſs, parallel to the axis OAF; do after refraction by the {concave / convex} ſurface of air BAC, {converge to / diverge from} a focus F, whoſe diſtance from the vertex A is twice the radius OA.

Fig.41,42. 131. *Corol.* 7. Rays in air, {diverging from / converging towards} a focus F, whoſe diſtance from the vertex A, is twice the radius OA; do after refraction by the {convex / concave} ſurface of glaſs BAC, become parallel to the axis OAF.

132. *Corol.* 8. Convex and concave ſurfaces, are juſt oppoſite in their effects: that is a pencil of parallel rays refracted by either, will have its focus at the ſame diſtance from the vertex, but on contrary ſides; as appears by comparing the figures 39, 40; and 41,42. together.

133. *Corol.* 9. Parallel rays, refracted by a {convex / concave} ſurface of a {denſe / rare} medium, will have a real focus, as in *fig.* 39, 41: But being refracted by a {concave / convex} ſurface of a {denſe / rare} medium, as in *fig.* 40, 42. the focus will be only imaginary.

Fig. 39 *to* 42. 134. *Schol.* The point F is the focus of thoſe rays only, that are neareſt to the axis AOF; all other rays as *db*, whoſe incidence are more oblique, will have their focus at ſome point *x*, between F and A; and the more oblique is the incidence, the greater will be the aberration F*x*.

[a] 124. I. [b] Trigonom. For AF : OF ::[a] I : R ::[b] *bx* : O*x*; Now in *fig.* 39, 40. AF (= OF + O*b*) is greater than *b*F; And in *fig.* 41, 42. AF (= OF − O*b*) is leſs than *b*F; and therefore in both caſes, OF is greater than O*x*; that is, the point *x* is ſomewhere between F and A. And it is manifeſt, that the greater is the arch A*b*, the greater will be the aberration F*x*. Therefore, *&c. Q. E. D.*

PROP. XVIII.

Fig. 39 *to* 42. 135. *The ratio of the ſines of incidence and refraction, and the angle of incidence* = AOb, *of any ray* db *upon the given ſurface* BAC, *being given; to find the point* x, *where the refracted ray cuts the axis* AOF; *And conſequently to find the aberration* Fx.

Make, Sine, ∠ AO*b* : Sine ∠ O*bx* (or *ebx*) :: I : R; and draw *bx*: then in the △ O*bx*, will be given the ſide O*b* and all the angles, to find O*x*. And OF − O*x* = F*x*. *Q. E. I.*

PROP.

PROP. XIX.

136. *The longitudinal aberration* Fx, *of a parallel ray incident upon the spherical surface of a dense medium*, fig. 39, 40. *is to* vA *the the versed sine of the angle of incidence; as* nn *to* mm—mn *nearly. And when the incidence of the parallel ray is upon a spherical surface of a rare medium*, fig. 41, 42. Fx : vA :: mm : mn—nn *nearly.*

From the center x describe the arch ba, and let fall the perpendicular bv.

Now, 1. $Ox : bx$ :: [a]OF : AF. Wherefore, [a] Construc.

2. $Fx\,(OF—Ox) : OF :: (AF—bx \text{ or } ax)\,Fx \pm Aa : AF$; therefore

3. $Fx : Aa\ (\overline{Fx \pm Aa} \backsim Fx) :: OF : OA\ (AF \backsim OF) ::$ [b] $\left\{\begin{matrix} n \\ m \end{matrix}\right\} : m—n.$ [b] 126 & 127. I.

4. $va : vA$ :: [c]OA : xa nearly: But in *fig.* 39, 40. xa is nearly †equal to xA; and in *fig.* 41, 42. xa is nearly equal to †FA. Wherefore substituting these equals, it will be [c] 58 I.

5. $Aa\ (vA \mp va) : vA :: \begin{matrix} Ox \\ OF \end{matrix} \begin{pmatrix} Ax—OA \\ FA+OA \end{pmatrix} : \begin{matrix} xb \\ AF \end{matrix} :: \begin{matrix} {}^{b}n \\ m \end{matrix} : \begin{matrix} m. \\ n. \end{matrix}$

Wherefore compounding the 3d and 5th steps, we shall have,

Case 1. $Fx : vA :: nn : mm—mn$ nearly; and Fig. 39, 40.

Case 2. $Fx : vA :: mm : mn—nn$ nearly. *Q. E. D.* Fig. 41, 42.

137. *Corol.* 1. The angle of incidence being the same, the aberration Fx in *fig.* 39, 40. will be the same; and so it will also be in *fig.* 41, 42. but the aberration in the two last figures will be greater than in the former.

138. *Corol.* 2. If instead of m, n, we put the numbers 3, 2, which express their ratio; we shall have in *Case* 1. $Fx = \frac{4}{3} vA$; and in *Case* 2. $Fx = \frac{2}{3} vA$: And therefore Fx in *Case* 1. is to Fx in *Case* 2. as 8 to 27.

139. *Corol.* 3. Hence and from *Art.* 89, 91. the longitudinal aberration of a parallel ray incident upon a spherical reflecting surface, is to the longitudinal aberration of a parallel ray of a like incidence upon a spherical refracting surface, whose radius is the same with that of the reflecting surface; in *Case* 1, as 3 to 8; and in *Case* 2, as 1 to 9; nearly.

† When the arch AB is very small xa, xA, *fig.* 39, 40. and xa, FA, *fig.* 41, 42. are pretty near in a ratio of equality; but as the arch AB increases, the quantities assumed for xa are greater than it, and therefore Fx bears a greater proportion to vA than is expressed above.

PROP. XX.

140. *The longitudinal aberrations are as the squares, and the lateral ones as the cubes, of the sines of the angles of incidence nearly.*

[a] 136. I. [b] 91. I. For, 1. The longitudinal aberration Fx is almost in a constant [a]ratio to the versed sine vA; and vA is as [b]bv^2 nearly.

Again, 2. The lateral aberration FG, whilst the angle of incidence remains the same, is in some certain ratio to Fx, that is, in a certain ratio to bv^2; but as the angle of incidence increases or diminishes; FG does moreover increase or diminish as the sine of the ∠ FxG, that is, as bv; and therefore the whole increase or decrease of FG is as bv^3 nearly. Therefore, &c. *Q. E. D.*

PROP. XXI.

141. *The focus of incident rays, either diverging or converging upon a spherical refracting surface, being given; to find the geometrical focus of the refracted rays.*

Fig. 43 *to* 48. *Construction.* Let BAC be a spherical refracting surface whose center is O, and let Q be the focus of incident rays, (either diverging as QB, *fig.* 43, 45, 48. or converging as MB, *fig.* 44, 46, 47): through the points Q, O, draw the indefinite line QOq, and let KBk be a strait line parallel thereto. In the line QO produced, let f be the geometrical focus of rays parallel to the axis QOB, as kB; (that is, let it be $m—n : n ::$ AO : Of); and parallel to the incident ray QB or MB, through the center O draw the perpendicular aOb, and take Ob = Of. Through the point b draw the line Bb, and produce it till it meets the axis QO produced in q; and q will be the focus required.

The focus must be somewhere in the line QOq, for this line being perpendicular to the surface, is the axis of the pencil whose focus of incidence is Q; the refracted rays also pass through b, for b is the focus of rays parallel to the axis Ob. Therefore, &c. *Q. E. D.*

142. *To find the focus* q *by calculation:* Take AF = Of, (or let F be the geometrical focus of rays parallel to the axis QOq, as KB), and say QF : QA :: QO : Qq; and lay the point q on the same side of Q that F is of; and q will be the geometrical focus required.

[a] 4. VI *Eu.* [b] Construct. For Ob being parallel to QB, the triangles QBq, Obq are similar, and therefore QB : Ob :: [a]Qq : qO. But when these triangles are vanishing, QB will be = QA, and Ob = Of = [b]AF. Wherefore substituting these equals, it will be

QA : AF :: Qq : qO; and therefore

$$\text{Fig.}\begin{cases} 43, 44; & QA - AF : QA :: Qq - qO : Qq; \\ 45, 46; & AF - QA : QA :: qO - Qq : Qq; \\ 47, 48; & QA + AF : QA :: Qq + qO : Qq; \end{cases}$$

that is, in each case, QF : QA :: QO : Qq. *Q.E.D.*

143. *Corol.* 1. QF : AF (QA ∓ QF) :: QO : Oq (Qq ∓ QO).

144. *Corol.* 2. QF : OF or Af (QO ∓ QF) :: QA : Aq (Qq ∓ QA) :: Of or AF (QA ∓ QF) : fq (Aq ∓ Af. i.e. QF : OF (Af) :: QA : Aq :: Of (AF) : fq.

145. *Corol.* 3. If q be the focus of incident rays, Q will be the focus of the refracted ones.

146. *Schol.* 1. This proposition may be distinguished into the six following cases.

Case 1. When the focus $\left\{\begin{matrix} Q \\ q \end{matrix}\right\}$, of a ray $\left\{\begin{matrix} QB \\ qB \end{matrix}\right\}$ diverging upon a $\left\{\begin{matrix} \text{convex} \\ \text{concave} \end{matrix}\right\}$ surface of a $\left\{\begin{matrix} \text{dense} \\ \text{rare} \end{matrix}\right\}$ medium, is beyond $\left\{\begin{matrix} F \\ f \end{matrix}\right\}$. Fig. 43.

Case 2. When the focus $\left\{\begin{matrix} Q \\ q \end{matrix}\right\}$, of a ray $\left\{\begin{matrix} MB \\ NB \end{matrix}\right\}$ converging upon a $\left\{\begin{matrix} \text{concave} \\ \text{convex} \end{matrix}\right\}$ surface of a $\left\{\begin{matrix} \text{dense} \\ \text{rare} \end{matrix}\right\}$ medium, is beyond $\left\{\begin{matrix} F \\ f \end{matrix}\right\}$. Fig. 44.

Case $\left\{\begin{matrix} 3. \\ 4. \end{matrix}\right\}$ When the focus Q, of a ray $\left\{\begin{matrix} QB \text{ diverging} \\ MB \text{ converging} \end{matrix}\right\}$ upon a $\left\{\begin{matrix} \text{convex} \\ \text{concave} \end{matrix}\right\}$ surface of a dense medium, is between F and A: Or, when a ray $\left\{\begin{matrix} bB \text{ converges} \\ qB \text{ diverges} \end{matrix}\right\}$ upon a $\left\{\begin{matrix} \text{concave} \\ \text{convex} \end{matrix}\right\}$ surface, of a rare medium. Fig. 45. 46.

Case $\left\{\begin{matrix} 5. \\ 6. \end{matrix}\right\}$ When a ray $\left\{\begin{matrix} MB \text{ converges} \\ QB \text{ diverges} \end{matrix}\right\}$ upon a $\left\{\begin{matrix} \text{convex} \\ \text{concave} \end{matrix}\right\}$ surface of a dense medium: Or, when the focus q, of a ray $\left\{\begin{matrix} qB \text{ diverging} \\ NB \text{ converging} \end{matrix}\right\}$ upon a $\left\{\begin{matrix} \text{concave} \\ \text{convex} \end{matrix}\right\}$ surface of a rare medium, is between f and A. Fig. 47. 48.

147. *Schol.* 2. The situations of the focus's Q, q, in *Case* 1, 2. in *Case* 3, 4. and in *Case* 5, 6. are alike; that is,

148. *Case* 1, and 2. The focus's Q, q are on opposite sides of the surface BAC; and when Q is in F, q is at an infinite distance beyond f; and, as Q recedes beyond F, q approaches towards f, and at last coincides with f, when Q is at an infinite distance. Fig. 43, 44.

149.

Fig. 45, 46. 149. *Case* 3, 4. Q and *q* are both on the same side, and the whole motion of Q is from A to F; but *q* may be either between A and F, or beyond F. If Q is in A, *q* will be also in A; and as Q recedes from A, *q* also recedes from it the same way, but with an accelerated velocity; so that when Q is got very near to F, *q* will be got at almost an infinite distance beyond F.

Fig. 47, 48. 150. *Case* 5, 6. Q and *q* are here also both on the same side, and the whole motion of *q* is properly from O to *f*; but Q may be either between O and *f*, or beyond *f*: When Q is in O, *q* will be also in O. And if Q be between O and A, *q* will be between O and Q.

151. *Schol.* 3. In all these cases, the focus's Q, *q* move both the same way. And the surfaces of the mediums remaining the same, *Case* 1, 3, 5, and also *Case* 2, 4, 6. pass from one into the other.

152. *Schol.* 4. Convex and concave surfaces of like mediums; as also dense and rare mediums with like surfaces; are just opposite in their effects.

153. *Schol.* 5. Whatever has been shewed as to the focus's Q, *q*; is also applicable to objects and their images. And since the axis's of all pencils do evidently intersect in O, it is manifest also, that the breadth of the object will be to that of its image, * as OQ to O*q*.

[153.] *Schol.* 6. If an object be an arch of a circle concentric to the spherical refracting surface, its image will be also a similar concentric arch. And so will also a concentric spherical segment, have a similar concentric segment for its image.

Fig. 43, 45. 48. But a strait object PQ, will have a curvilineal image *pq*, whose curvity is the contrary way to that of the refracting surface BAC; as will be manifest by comparing the † rules for finding the focus's *q*, *p*.

Also a strait object (*pq*) within the dense medium, *fig.* 43, 46, 47. will have a curvilineal image (PQ); but its curvity will be the same way with that of the refracting surface.

PROP. XXII.

Fig. 43 to 48. 154. *Having the focus* Q, *of any incident ray* QB *or* MB, *and the distance of the point of incidence* B *from the vertex* A, *or* ∠ AOB; *to find the proper focus* y, *of the refracted ray.*

Having drawn QB, and the perpendicular OBG; with the sides QO, OB, and the angle QOB, find the angle of incidence QBG, or QBO: Then say,

I : R :: Sine ∠ GBQ or QBO : Sine ∠ OB*y* or GB*y*; and draw B*y*, which will be the refracted ray.

Now

* Vide Art. 110, 111. I.

† Vid. 116, & 143. I.

Now in the triangle OBy will be given the side OB, and all the angles; to find Oy, which gives the point y. *Q. E. I.*

155. *Schol.* Let b be the geometrical focus after refraction of rays parallel to the axis aOB, and x the point where the refracted ray By cuts that axis: Upon the axis's Ob, Oq at the points b,q raise the perpendiculars bs, qt. Now bs is the lateral aberration, of the parallel ray QB after refraction, from the geometrical focus b; and qt is the lateral aberration, of the diverging or converging ray QB or MB, from the geometrical focus q. I say $bs : qt ::$ Af : Aq nearly.

For in the $\triangle$s Bbs,Bqt, having the same very acute angle at B, the lines bs, qt do not sensibly differ from perpendiculars at the points b,q upon the line bq; in which case the $\triangle$s Bbs, Bqt, would be exactly similar; and then it would be accurately, Bb : Bq :: bs : qt. But as the point B approaches nearer to A, the $\triangle$s Bbs, Bqt approach nearer to a similarity; and the lines Bb, Bq approach also nearer to an equality with the lines Af; Aq; and therefore it will be nearly Af : Aq :: bs : qt. *Q. E. D.*

156. *Corol.* 1. Draw the right line Bv. And it will be, Bv : qt :: vy : yq. Wherefore also B$v \pm qt$: qt :: vq ($vy \pm yq$) : yq. Also Bv : B$v \pm qt$ :: vy : vq ($vy \pm yq$).

157. *Corol.* 2. Hence, having either the lateral or longitudinal aberration of any ray, the other is easily found. And from *Art.* 155. having the lateral aberration of a parallel ray, the lateral aberration of any other ray of a like incidence, is also readily found somewhat nearly. But both are accurately found by *Art.* 142, 154.

158. *Corol.* 3. When AQ is $\left\{\begin{matrix}\text{greater}\\ \text{lesser}\end{matrix}\right\}$ than AF, Aq is $\left\{\begin{matrix}\text{greater}\\ \text{lesser}\end{matrix}\right\}$ than Ay. And *cæteris paribus* the more oblique is the incidence, the more will the ray be refracted, or the greater is qy. And hence an object may be so placed, that the rays next the axis shall diverge after refraction by a convex surface, when the more remote rays shall have a real focus.

I shall conclude this section, with the two following constructions, for finding the proper focus of a ray refracted by a single surface *.

159. *To find the proper focus, of a ray parallel to the axis of a refracting surface, and at a given distance from it.*

Let O be the center of the spherical refracting surface BAC, and DB an incident ray parallel to the axis AO; to find the proper focus x of the refracted ray. In AO produced, find the geometrical focus F, by taking AF : OF :: [a] I : R; and divide OF in E, by taking EF : EO :: I : R. From the center E with the radius EF describe a circle EFG; and through the given points B, O, draw a line BOa cutting this circle Fig. 49, 50. [a] 124. I.

* Dr. *Barrow*, I think, was the Author of these Constructions. Vid. *Bar.* Lect. XI. XIII.

circle in a; then in the axis OF take $Ox = Oa$, and drawing Bx it will be the refracted ray. Join Ea and Aa, and since AF : OF :: [b](I : R ::[b]) EF : EO; it will be,

[b] Construct.

AE (AF—EF) : EF :: EF (OF—EO) : EO. But EF = [b]Ea; and therefore since the legs of the △s AEa, aEO having the common ∠ E, are proportional, it will also be, Aa : aE :: [c]aO : OE; and therefore Aa : aO :: EF (aE) : OE :: [b]I : R. But in the △s AOa, BOx, we have AO = [b]BO, Oa = [b]Ox and the ∠ AOa = [d]∠ BOx; and consequently Bx = [e]Aa: Therefore Bx : Ox :: (Aa : aO ::) I : R. But the ∠ BOx is equal to the angle of incidence or to its supplement, and Bx is as the sine of that angle; and therefore OBx is the angle of refraction or its supplement; that is, Bx is the refracted ray. *Q. E. I.*

[c] 6. VI. *Euc.*

[d] 15. I. *Euc.*

[e] 4. I. *Euc.*

160. *Corol.* As the point B approaches to A, the point a approaches to F; and therefore every two contiguous rays will intersect each other before they cross the axis OF.

Fig. 51 *to* 55.

161. *Schol. Fig.* 49. corresponds to *fig.* 39, and the same construction will serve to find the point x, *fig.* 40. *Fig.* 50. corresponds to 41, and will also serve to find x, *fig.* 42.

162. *Having the focus* Q, *and the point of incidence* B, *of a ray either diverging or converging upon a given spherical surface* BAC; *to find the proper focus* y *of the refracted ray.*

From Q through the center O of the refracting surface BAC, draw the axis QO, and produce it both ways, if need be; and to the point of incidence B draw the perpendicular OB which produce as the case requires. In the axis QO take the point R such, that OQ : OR :: I : R; and with the radius QB, from the center R describe an arch intersecting OB in D; parallel to RD draw By, and By will be the refracted ray.

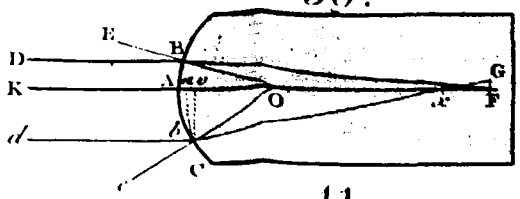
Upon the axis QO, the incident ray QB (produced if need be), and the refracted ray By, let fall the perpendiculars Bv, Oa, Ob. Now because By is parallel to RD, the triangles BOy, DOR are similar; and so are also the right-angled △s ybO, yvb; as are likewise the right-angled △s QvB, QaO. Wherefore OR : QB (RD) :: Oy : By :: Ob : Bv. Therefore OR : Ob :: QB : Bv :: QO : Oa; and consequently OQ : OR (:: I : R) :: Oa : Ob. But Oa is the sine of incidence, and therefore Ob is the sine of refraction; that is, By is the refracted ray. *Q. E. I.*

The figures here referred to, correspond to those in the preceding plate; as *fig.* 51, to 43. and the same kind of construction will serve for 44. *Fig.* 52. corresponds to 45, as doth also 53 to 46, only changing the places of Q, y. *Fig.* 54, 55. correspond to 47, 48.

163. *Corol.* 1. If QB be equal to OR, the point D will coincide with O; and then the refracted ray will be parallel to the axis.

164.

30.



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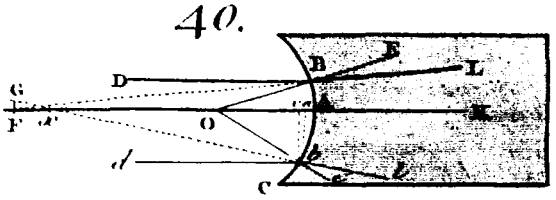

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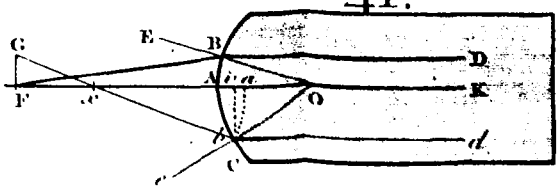

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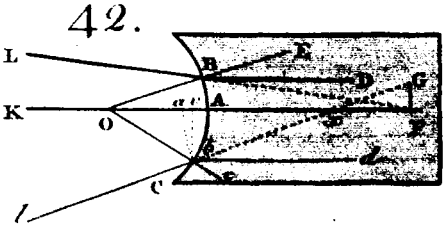

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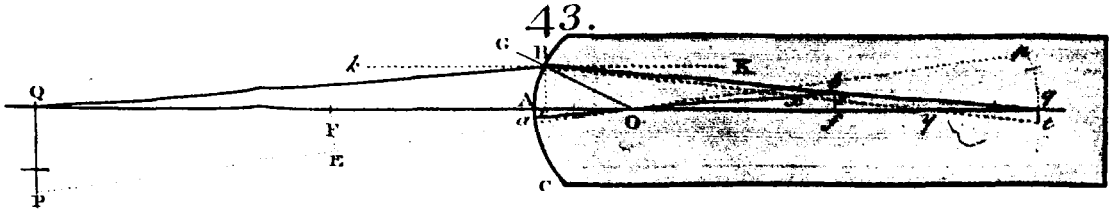
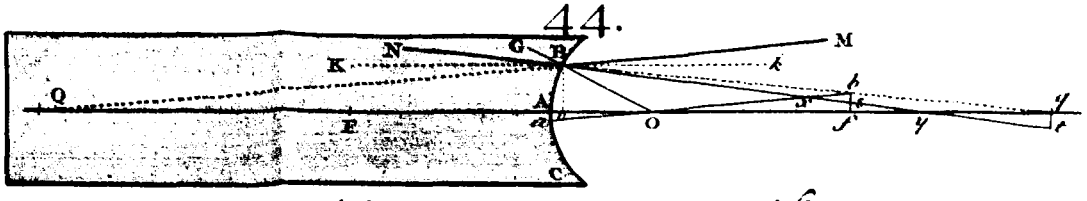


44.



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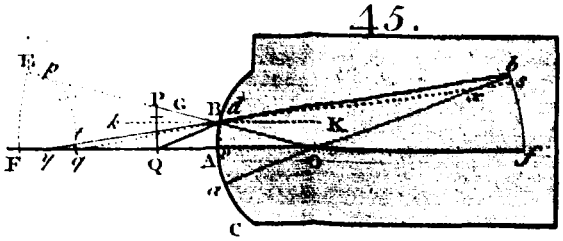

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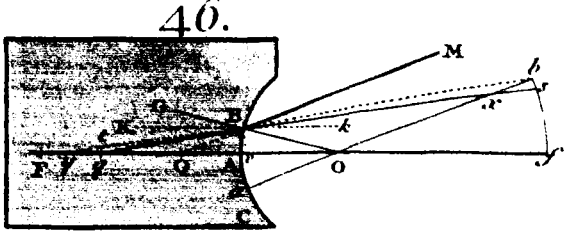
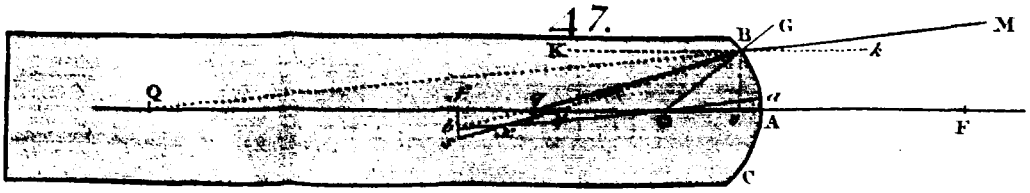
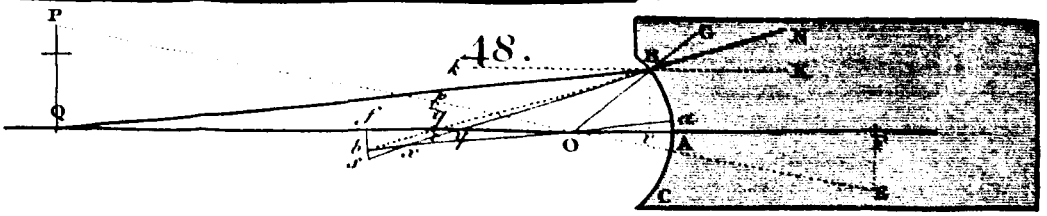


47.



48.



B. Cole sculp.

164. *Corol.* 2. If QB be greater than OR (*fig.* 51, 54, 55.) the focus y will fall on the ſame ſide of the ſurface BAC, as the center O is of: And the focus y will fall on the contrary ſide, if QB be leſs than OR, (*fig.* 52, 53.)

165. *Corol.* 3. If the point D falls on the ſame ſide of the axis as the point B, (*fig.* 52 to 55); the focus's Q, y will be alſo on the ſame ſide of the ſurface BAC: And the ſaid focus's Q, y will be on contrary ſides, when the points D, B are on different ſides of the axis, (*fig.* 51.)

166. *Corol.* 4. DO : OB :: OR : Oy :: RD or QB : By. Whence to find the geometrical focus q, becauſe in that caſe QA = QB = RD, it will be

167. *Corol.* 5. QA ∽ OR (OD) : OA :: OR : Oq; but OR = [a] $\frac{R}{I}$QO. [a] Conſtruct.

Therefore QA ∽ $\frac{R}{I}$QO : OA :: $\frac{R}{I}$QO : Oq :: (RD : Bq ::) QA : Aq. And thus the focus q may be found without having the point R.

SECTION IV.

Of Lens's.

168. *Deſin.* 1. A LENS in Optics (a ſection whereof is repreſented by the ſhaded figures), is a tranſparent body of a different denſity from the ambient medium, and terminated by two ſurfaces*, either both ſpherical, or one plain and the other ſpherical. And becauſe lens's for optical uſes are made of glaſs, it is uſual to call them *glaſſes*, with the addition of the uſe that they are intended for; as a *magnifying-glaſs*, a *ſpectacle-glaſs*; *object-glaſs*, or *eye-glaſs of a Teleſcope*, &c. Fig. 56 to 72. Lens.

169. *Deſin.* 2. A lens having one ſide plain and the other convex, as in *fig.* 57, 59. is called a *plano-convex lens*: So the lens, *fig.* 58, 60. is called a *plano-concave*. A lens terminated by two convex ſides or two concave ſides, is accordingly called a *double convex*, as *fig.* 61. or *double concave lens*, as *fig.* 62. A lens having one ſide concave and the other convex, as *fig.* 63, or 64. is called a *concavo-convex lens*. The lens *fig.* 63. is called a *meniſcus*; as is alſo very often, tho' improperly, the lens, *fig.* 64. Plano-convex, plano-concave, double convex, double concave, concavo-convex, meniſcus, lens's.

170. *Deſin.* 3. When the particular figure is not conſidered, a lens that is thickeſt in the middle, is called a *convex-lens*; and that which is Convex and concave lens's.

* This definition might be made more general; for any tranſparent body ſo figured as to divert the courſe of the rays refracted by it, towards or from a certain focus, is properly an optical lens; but we ſhall have occaſion to conſider no other but thoſe above defined. A tranſparent ſphere may be alſo called a lens.

is thinneſt in the middle, is called a *concave lens*, without farther diſtinction.

Axis of a lens.

171. *Defin.* 4. A ſtrait line drawn perpendicular to both the ſurfaces of any lens, is called the Axis of *the lens*. The axis therefore paſſes through the centers of the ſpherical ſides: And it is manifeſt that no lens, excepting the ſphere, can have but one axis; and therefore all pencils are oblique, excepting thoſe whoſe *foci* are in the axis of the lens.

Focus, or principal, ſolar, or burning focus of a lens.

Focal length of a lens.

172. *Defin.* 5. The geometrical focus of a direct pencil of parallel rays, (or that point in the axis where rays that are parallel and neareſt thereto, are united after refraction, if the lens is thickeſt in the middle; or from whence they diverge, if the lens is thinneſt in the middle) is called the *focus of the lens:* Sometimes the *principal focus*, by way of diſtinction from the focus after refraction, of diverging or converging rays. It is alſo called the *ſolar* or *burning focus*, if the lens is convex, becauſe there the ſun's rays are cloſeſt united. The diſtance of the principal focus from the lens, is called the *focal length of the lens*.

§. 1. *To find the principal focus of any given Lens.*

PROP. XXIII.

173. *To find the principal focus of a ſphere.*

Fig. 56.

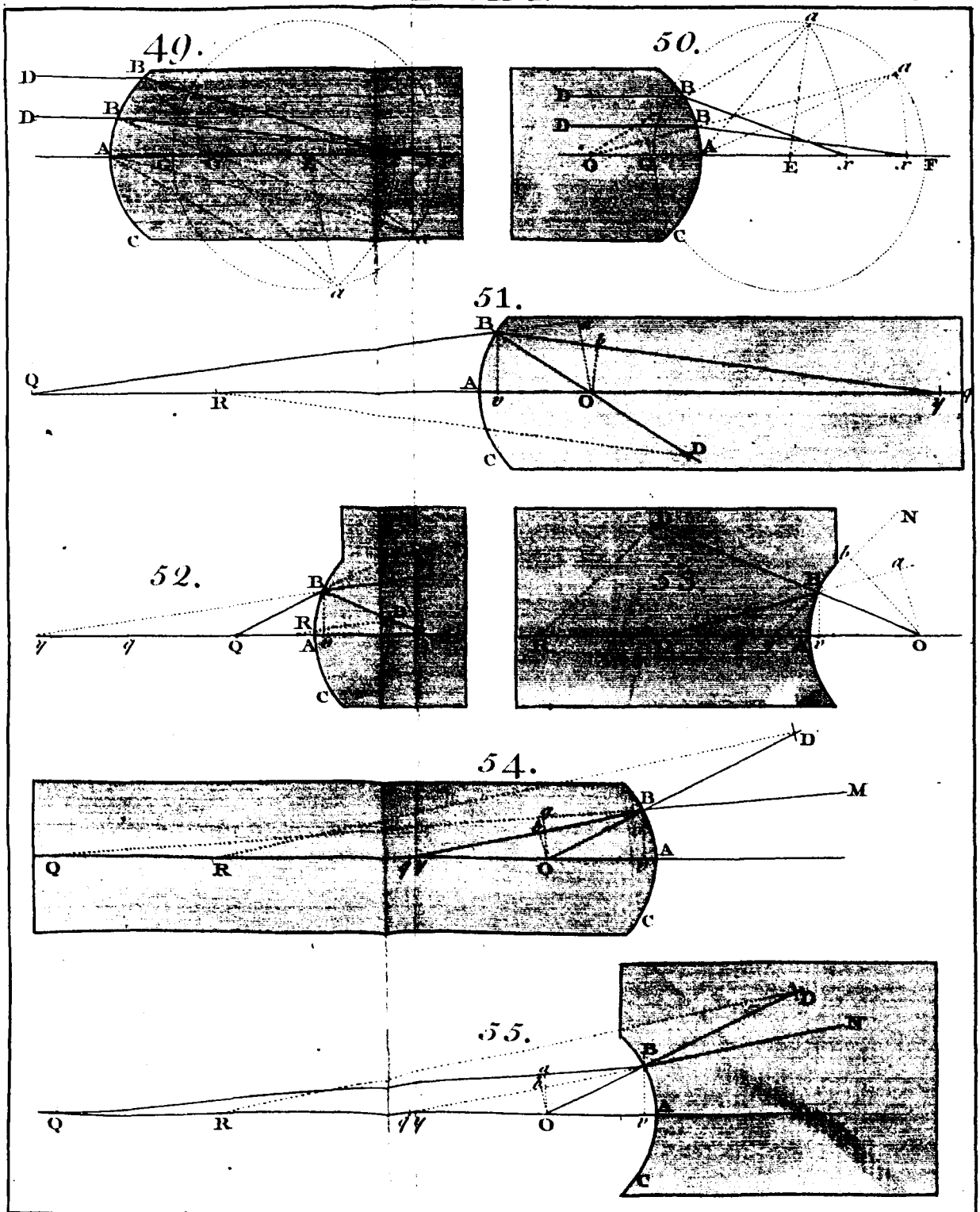
Let DB be an incident ray parallel and very near to the axis ARE*b*, and let *b* be the geometrical focus of the pencil whoſe axis is AR*b*, after the firſt refraction by the ſurface AB. Biſect E*b* in F, and F will be the focus required.

Produce the incident and emergent rays DB, F*a* till they meet in K, and from the center R draw RB, R*a*G. Now becauſe the ∠ *ba*G = [a] R*a*B = [b] RB*a*; and Sine ∠ RB*a* : Sine ∠ KB*a* :: [c] Sine ∠ *ba*G : Sine ∠ F*ab* (:: *n* : *m*—*n*). Therefore the ∠ F*ab* = KB*a* = [d] F*ba*; and conſequently F*a* = [e] F*b*. But when *a* coincides with E, FE will be = F*a*; and therefore the point F, which biſects E*b*, is the focus of thoſe rays that are neareſt to the axis AE. *Q. E. I.*

[a] 15. I. *Eucl.*
[b] 5. I. *Eucl.*
[c] Laws of refraction.
[d] 29. I. *Euc.*
[e] 6. I. *Euc.*

174. *Corol.* Hence the focus F of a glaſs ſphere, is one fourth of its diameter diſtant from the vertex. For AR (RE) = [f] $\frac{1}{2}$ R*b*, and therefore FE = $\frac{1}{2}$ AR = $\frac{1}{4}$ AE.

[f] 128. I.



Blide sculp.

PROP. XXIV.

To find the principal focus of a plano-convex, or a plano-concave lens.

Let RAF be the axis of the lens (or a line drawn perpendicular to both its surfaces), R the center of the spherical surface BAC, and let DB be an incident ray parallel and very near to the axis. Now this proposition admits of two cases, according to which side of the lens is turned to the incident rays. Fig. 57 to 60.

175. *Case* 1. When the incident ray falls upon the plane side, it will pass on directly to B unrefracted. Wherefore, when the point B is very near to A, if it be made RA : AF :: [a] $m-n : n$; the point F so found will be the focus. *Q. E. I.* Fig. 57, 58. [a] 127. I.

[175.] *Case* 2. When the incidence of the parallel rays is upon the spherical side of the lens. Fig. 59, 60.

Let AR : Rb :: $m-n : n$; and b [a] will be the point where the refracted ray Ba produced would meet the axis. [a] 126. I.

Again, take Eb : EF (:: R : I) :: $m : n$; and F [b] will be the focus. *Q. E. I.* [b] 50. I.

176. *Corol.* 1. $AF = \frac{n}{m}Eb \pm AE = \frac{n}{m}Ab \pm \frac{m-n}{m}AE$. Fig 59, 60.

For $\frac{n}{m}Ab = \frac{n}{m}Eb \pm \frac{n}{m}AE$; and $\pm \frac{n}{m}AE$ together with $\pm \frac{m-n}{m}AE = \pm AE$.

177. *Corol.* 2. AF (*fig.* 57.) = EF (*fig.* 59.) $+ \frac{n}{m}AE$. And EF (*fig.* 58.) = AF (*fig.* 60.) $- \frac{n}{m}AE$. And in *fig.* 60. $AF = \frac{n}{m-n}AR - \frac{m-n}{m}AE$.

178. *Schol.* 1. A plano-convex glass, with its plain side towards the incident rays, (*fig.* 57.) hath its focus at the distance of twice the radius of convexity from the glass; that is AF = 2RA. And a plano-concave exposed in like manner, (*fig.* 58.) hath its focus at twice the radius from the concave side. But a plano-convex glass with its convex-side towards the incident rays, (*fig.* 59.) hath its focus nearer to the glass than a diameter of the convexity, in the ratio of bE to bA; or EF $(= \frac{2}{3}Fb) = \frac{2}{3}Ab - \frac{2}{3}AE = 2AR - \frac{1}{3}AE$. And in *fig.* 60. the focal length $FA = 2RA - \frac{1}{3}AE$; and consequently $EF = 2RA + \frac{2}{3}AE$.

179. *Schol.* 2. If the thickness AE, is very small in comparison of the radius RA, then the focal lengths of each of these lens's, may be considered as being equal one to the other; and in glass equal to twice the radius RA.

180. *Schol.* 3. The focus of a glaſs hemiſphere, with its convex ſide expoſed to parallel rays, is about $\frac{2}{3}$ of the diameter, diſtant from the glaſs.

Let *fig.* 59. be ſuppoſed a glaſs hemiſphere; then will E coincide with R, and the focus *b* of the rays after their firſt refraction, will be in the ſame place as before; ſo that $Rb = 2RA$. And to find F the focus of the emergent rays, it will be,

$Rb : RF :: m : n :: 3 : 2$; therefore $RF = \frac{2}{3}Rb = \frac{4}{3}RA$. *Q. E. D.*

But if the parallel rays fall upon the plane ſide of the hemiſphere, as in *fig.* 57. the focal length will be the ſame with that of a thin plano-convex lens in the ſame poſition; for the ray DB will paſs to B unrefracted by the plane ſide.

181. *Schol.* 4. Plano-convex and plano-concave lens's, that are unequal ſegments of the ſame ſphere, with their ſpherical ſides expoſed to parallel rays; the ſmaller the ſegment, or the thinner is the lens, the greater will be the focal diſtance from, the neareſt ſide of, the lens: And the contrary, reckoning from the farthest ſide of the lens. But if parallel rays are incident upon the plain ſides, their focal lengths will be the ſame, how unequal ſoever the lens's are in thickneſs.

PROP. XXV.

182. *To find the principal focus of any lens, whoſe ſurfaces are both ſpherical.*

Fig. 61 *to* 64. 1. Let R be the center of the ſpherical ſide AB, and *r* the center of the ſide *ab*; draw the axis R*r*, and produce it both ways. Let K*b* be an incident ray parallel, and very near, to the axis R*r*. Now take $ar : rq :: m - n : n$; and place *q* the ſame way from *r*, that *r* is from *a*; [a] 126. I. then will *q* [a] be the geometrical focus of the parallel rays incident upon *ab*; or *q* may be conſidered as the focus of the incident ray *b*B upon the ſecond ſurface AB.

2. Let *f*, F be the geometrical foci of rays coming both ways, parallel to the axis, upon the ſurface AB; (*f* the focus of parallel rays coming from without, and F the focus of parallel rays within the lens.) Now to find the focus Q of the emergent rays, ſay,

[b] 144. I. $fq : Rf\ (AF) :: {}^{b}Aq : AQ$; or $fq : Rf\ (AF) :: {}^{b}RF\ (Af) : FQ$; and laying the point Q on the ſame ſide of A that *q* is of, or on the ſame ſide of F that R is of; [b] the point Q thus found will be the focus required *.

183. *Corol.* 1. $fq : Rq\ (fq \backsim fR) :: Aq : Qq\ (Aq \backsim AQ) :: RF : RQ\ (RF \backsim FQ)$. Alſo $fq : fA\ (fq \mp Aq) :: AF : FQ\ (AF \mp AQ)$.

184.

* Compare *fig.* 61 with *fig.* 45; 62 with 46; 63 with 48; and *fig.* 64 with *fig.* 47.

184. *Schol.* 1. If the incident ray comes the contrary way, and it be made $qf : qr :: af : ax$; the point x will be the focus. This will be manifest by supposing f and q to change places, then it will be 1. $m-n : n ::$ [a] $AR : Rq$; 2. $fq : fr ::$ [b] $aq : ax$; that is, transposing the letters q, f back again, as they stand in the figures; $qf : qr :: af : ax$. Q. E. D.

[a] 126. I.
[b] 144. I.

185. *Schol.* 2. The rules $fq : Rf :: Aq : AQ$; and $qf : qr :: af : ax$; for finding the geometrical foci Q, x; contain the following. Let the radii RA, ra be called R, r, and the thickness Aa of the lens's be called t.

1. When the lens is a double convex, it will be, Fig. 61.

$$\frac{mR+mr}{m-n} - t : \frac{nR}{m-n} :: \frac{mr}{m-n} - t : AQ;$$

And $\frac{mR+mr}{m-n} - t : \frac{nr}{m-n} :: \frac{mR}{m-n} - t : ax.$

2. When the lens is a double concave; Fig. 62.

$$\frac{mR+mr}{m-n} + t : \frac{nR}{m-n} :: \frac{mr}{m-n} + t : AQ;$$

And $\frac{mR+mr}{m-n} + t : \frac{nr}{m-n} :: \frac{mR}{m-n} + t : ax.$

3. When the lens is a meniscus, or a convex concavo-convex. Fig. 63.

$$\frac{mR-mr}{m-n} + t : \frac{nR}{m-n} :: \frac{mr}{m-n} - t : AQ;$$

And, $\frac{mR-mr}{m-n} + t : \frac{nr}{m-n} :: \frac{mR}{m-n} + t : ax.$

4. When the lens is a concave concavo-convex; Fig. 64.

$$\frac{mR-mr}{m-n} - t : \frac{nR}{m-n} :: \frac{mr}{m-n} + t : AQ;$$

And $\frac{mR-mr}{m-n} - t : \frac{nr}{m-n} :: \frac{mR}{m-n} - t : ax.$

For $m-n : m :: R : Af = \frac{mR}{m-n}$; and $m-n : m :: r : aq = \frac{mr}{m-n}$; Also $m-n : n :: R : Rf = \frac{nR}{m-n}$; and $m-n : n :: r : rq = \frac{nr}{m-n}$. Whence by adding or subtracting t, as the case requires; the four above cases are sufficiently manifest.

186. *Corol.* 2. In all cases, the less is the ratio of R to r, the sum of both being the same, the greater will be AQ and ax: And AQ and ax are greatest of all, when R and r are equal. For in the above cases, the first term will be the same whatever be the ratio of R to r; and it is manifest that the rectangle under the two middlemost terms, is

The less is $\frac{R}{r}$ the greater is AQ and ax.

is greatest of all, when R and r are equal; and that these rectangles will be the less, the greater is the ratio of R to r.

In a convex lens, the less is t, the greater is AQ and ax. And in a concave lens, the greater is t, the greater is AQ and ax.

187. *Corol.* 3. All other things being the same: In a convex lens (*fig.* 61, 63.) the less is t, the greater will be AQ and ax. In a concave lens (*fig.* 62, 64.) the greater is t, the greater also will be AQ and ax; this will be manifest by comparing together the terms wherein t is (*Art.* 185). In *fig.* 61. the first term is greater than the third; and in both t is subtracted. In *fig.* 63. t is added to the first, and t is either subtracted from the third, or the third term is greater than the first; whence in both figures, the greater is t, the less will be the fourth term; or the greater will be the ratio of the second term, which is an invariable quantity, to the fourth *. In like manner, in *fig.* 62, 64. by comparing the first and third terms together, it will manifestly appear, that the greater is t, the greater also will be AQ and ax. But in a convex lens, the greater is t, the greater will be aQ and Ax; and in a concave lens, the contrary.

188. *Schol.* 3. Rays refracted by a single surface of what kind soever, or by a lens of what figure soever; the less is the ratio of the greater of the sines of incidence and refraction to the lesser; that is, the less is the ratio of m to n, or the nearer they approach to an equality, the greater will be the distance of the focus of the refracted rays, from the refracting surface or lens.

PROP. XXVI.

189. *If rays come both ways, upon a lens whose sides have unequal curvities, parallel to the axis; the focus, after refraction by the lens, of those rays that fall upon the flattest side, will be the farthest from a convex lens, and the nearest to a concave lens.*

This has been already shewed of plano-convex and plano-concave lens's in *Art.* 177. And because in *Art.* 185, in each couplet of proportionals, the first terms, and the divisors to the two middle terms are the same; AQ and ax will be as the rectangles under the corresponding middle terms, (multiplied by the common rectangles $\overline{m-n} \times \overline{m-n}$). that is,

In *Fig.* 61. $mnRr - nRt : mnRr - nrt :: AQ : ax$.
Fig. 62. $mnRr + nRt : mnRr + nrt :: AQ : ax$.
Fig. 63. $mnRr - nRt : mnRr + nrt :: AQ : ax$.
Fig. 64. $mnRr + nRt : mnRr - nrt :: AQ : ax$.

Therefore in *Fig.* 61, 63. ax is greater than AQ; and in *Fig.* 62, 64. ax is less than AQ. *Q. E. D.* 190.

* If to or from two unequal quantities, the same quantity be added or subtracted, the least quantity will be thereby most affected; that is, it will accordingly have a greater or less proportion to the greatest, than it had before. Let the two quantities be 3, 4. Now the ratio of 3 ± 1 $\left\{\begin{matrix}4\\2\end{matrix}\right\}$ to 4 ± 1 $\left\{\begin{matrix}5\\3\end{matrix}\right\}$ is $\left\{\begin{matrix}\text{greater}\\\text{lesser}\end{matrix}\right\}$ than the ratio of 3 to 4.

190. *Schol.* If the lens's be glaſs, the rules in *Art.* 185. are convertible into the following;

1. $3R+3r-t : 2R :: 3r-t : AQ$; Fig. 61.
And $3R+3r-t : 2r :: 3R-t : ax$.

2. $3R+3r+t : 2R :: 3r+t : AQ$; Fig. 62.
And $3R+3r+t : 2r :: 3R+t : ax$.

3. $3R-3r+t : 2R :: 3r-t : AQ$; Fig. 63.
And $3R-3r+t : 2r :: 3R+t : ax$.

4. $3R-3r-t : 2R :: 3r+t : AQ$; Fig. 64.
And $3R-3r-t : 2r :: 3R-t : ax$.

191. *Corol.* 1. If the lens's be glaſs, and their thickneſs be neglected as inconſiderable, the above rules will become $3R \pm 3r : \begin{matrix} 2R \\ 2r \end{matrix} :: \begin{matrix} 3r \\ 3R \end{matrix} : \begin{matrix} AQ \\ ax \end{matrix}$. And dividing the antecedents by 3, we ſhall have the following:

192. *Corol.* 2. $R \pm r : \begin{matrix} 2R \\ 2r \end{matrix} :: \begin{matrix} r \\ R \end{matrix} : \begin{matrix} AQ \\ ax \end{matrix}$; that is, in a double convex or double concave glaſs, whoſe thickneſs is inconſiderable, it will be

As the ſum of the radii, is to one of them;

But in a concavo-convex glaſs, it will be

As the difference of the radii, is to one of them; ſo is double the other to the focal diſtance.

193. *Corol.* 3. Hence in a double convex or double concave glaſs, whoſe thickneſs is inconſiderable, and ſides are ſegments of the ſame ſphere, the focal length is equal to the radius of the ſphere.

194. *Corol.* 4. In a concavo-convex glaſs whoſe thickneſs is inconſiderable, if the radius RA (R) be $= 3ra$ $(3r)$; the focus Q will fall in the center R of the flatteſt ſide. For it will be $2r\ (R-r) : r :: 3r\ (2R) : br = AR$.

And if the proportion of RA to ra, is greater than that of 3 to 1; Q will be between the center R and the lens; but if the ſaid proportion is leſs than that of 3 to 1, Q will be farther than R from the lens.

PROP. XXVII.

195. *If the diſtance* Rr *of the centers* R, r *of the ſides* BA, ba *of a concavo-convex medium, be to* Aa *the thickneſs of the medium, as* n *to* m; *and* r *be to the ſame ſide of* R *that* a *is of* A; *the incident and emergent parts* KB, bM, *of a ray* KB *parallel and very near to the axis* Ar, *will be parallel.* Fig. 65.

Let

Let q be the principal focus of the side A, and f of the side a.

[a] 124. I. [b] Hypoth. Because $Aq : Rq :: {}^{a}m : n :: {}^{a}af : rf :: {}^{b}Aa : Rr$; it will be

$$aq\,(Aq - Aa) : rq\,(Rq - Rr) :: af : rf; \text{ and}$$

$$ar\,(aq - rq) : rq :: ar : (ar - rf) : rf.$$ Therefore the points q, f coincide; and consequently the focus of the refracted ray is at an infinite distance, that is, bM is parallel to KB. *Q. E. D.*

196. *Corol.* 1. If the centers R, r, be moved farther asunder, the points f, q will separate, and the lens will have a real focus, as in *fig.* 63. So that a concavo-convex lens, whose surfaces are segments of the same sphere, has a real focus.

197. *Corol.* 2. If the centers R, r be brought nearer together, f will be nearer to the lens than q, and the lens will have a virtual (and not a real) focus. And this will happen when the centers R, r unite: So that a lens with concentric sides is to be deemed a concave lens; but when the thickness Aa is but small, the divergency of the refracted rays will not be sensible. If the centers R, r change sides, that is, if r gets nearest to the lens, the case will be, as is exemplified in *fig.* 64.

Fig. 66.

Fig. 67, 69, 71. 198. *Defin.* The thickness of a convex lens is a part of the axis (AV, *fig.* 67. or Aa, *fig.* 69, 71.) intercepted between the sides; and its breadth is a line (Ss) drawn between the extreams of the sides (perpendicular to the axis), the sides being continued to a sharp edge.

Fig. 68, 70, 72. 199. *Defin.* A concave lens is understood to have its sides touch in the middle, and its breadth is terminated by two lines (SS, ss) drawn parallel to the axis, and at the same distance from it on either side. The length of (either of) these lines (SS$=ss$) is reckoned the thickness of the lens; and the distance between them (Ss=Ss) is the breadth of the lens.

Or, If the concave lens has some thickness in the middle, the difference between that and the thickness at the ends, is to be reckoned the thickness of the lens.

200. *Lemma.* The thickness, half the breadth, and focal length of a plano-convex or plano-concave glass, are continual proportionals, nearly.

Fig. 67, 68. Let the axis Aa be twice the radius of the convexity SAs, and of the concavity Sas; then will [a] a be one of the principal focus's of the plano-convex glass, *fig.* 67 *. and [a] A the focus of the plano-concave glass, *fig.* 68*. And because SV and Sv are each perpendicular to the diameter Aa of the semicircle ASSa, it will be

[a] 179. I. [b] *Vide* 8. VI. *Euc.* AV : SV :: [b] SV : Va; and av : Sv :: [b] Sv : vA. Therefore, *&c.* *Q. E. D.*

201.

* This is never strictly true till the thickness of the lens has vanished. Vid. *Art.* 178.

201. *Corol.* Let the radius of the convex side SAs be called R, and of the concave side Sas, r; and let SV and Sv be called S: Now if the thickness of the lens be neglected as inconsiderable, in proportion to the focal length; it will be $AV : S :: S : 2R$; and $av : S :: S : 2r$; that is, $AV = \frac{S^2}{2R}$; and $av = \frac{S^2}{2r}$.

PROP. XXVIII.

202. *Half the breadth of any very thin glass lens, is a mean proportional betwixt its thickness and focal length, nearly.*

This has been already shewed of plano-convex and plano-concave glasses, in the preceding lemma.

Let half the breadth of the lens, SV or Sv, be as before called S, and the thickness of the lens $AV \pm av$ be called t. Now $AV =^a \frac{S^2}{2R}$; and $av =^a \frac{S^2}{2r}$. Wherefore, Fig. 69 to 72. [a] 201. I.

Case 1. $t\ (= AV + av) = \frac{S^2}{2R} + \frac{S^2}{2r}$; and by reducing these fractions to a common denominator, and dividing by 2, we shall have $t = \frac{R+r}{2Rr} S^2$; Wherefore, multiplying each Side by $2Rr$, and dividing by $R+r$, we shall have $S^2 = \frac{2Rr}{R+r} t$. Fig. 69, 70.

Case 2. $t\ (= AV - av = \frac{S^2}{2R} \backsim \frac{S^2}{2r}) = \frac{R \backsim r}{2Rr} S^2$; and therefore $S^2 = \frac{2Rr}{R \backsim r} t$. Fig. 71, 72.

Let the focal length be called f, then in *Case* 1. $f =^b \frac{2Rr}{R+r}$; and in *Case* 2. $f =^b \frac{2Rr}{R \backsim r}$; and therefore in both cases $ft = SS$; that is, $t : S :: S : f$. *Q. E. D* *. [b] 192. I.

203. *Corol.* Hence, in all sorts of lens's that are very thin in proportion to their focal lengths: If of these three, *viz.* breadth, thickness, focal length, any two be the same, the third will be the same also; or any two being given, the third may be found.

§. 2.

* This proposition is not to be trusted to, when applied to glasses of very short focus's; because the thickness of such glasses usually bear too great a proportion to their focal lengths.

§. 2. *Of oblique pencils of rays.*

204. The axis of any pencil of rays whatſoever incident upon a refracting ſphere, will paſs thro' the center unrefracted. This is ſelf-evident; for a ray that enters a ſphere perpendicular to the ſurface, emerges out of it alſo perpendicular to the ſurface. But the caſe is different, with reſpect to a lens; for the ſides of it not being concentric, no ray can be perpendicular to both the ſides, but that which coincides with the axis of the lens: And hence all pencils of rays refracted by a lens, excepting that one whoſe axis is perpendicular to both the ſurfaces, are ſaid to be oblique, and called *oblique pencils.*

PROP. XXIX.

205. *In every oblique pencil of rays whatſoever, refracted by any lens, there is one ray whoſe incident and emergent parts are parallel; and this ray is the axis of that pencil, or that ray in it which undergoes the leaſt refraction.*

Fig. 73, *to* 78. In *fig.* 75, to 78. let R, *r* be the centers of the ſpherical ſides A, *a*; and in *fig.* 73, 74. let *r* be alſo the center of the ſpherical ſide *a*. In the* flatteſt ſide of any lens whatſoever aſſume any point B, at which ſuppoſe a ray of light to be refracted, either at its enterance into the lens, or at its emerſion out of it: Draw the perpendicular RB to that ſide, and parallel thereto draw alſo the perpendicular *rb* to the other ſide *a*; join B*b*, then will B*b* be a ray whoſe parts PB, *pb*, without the lens, will be parallel. For the ray B*b* drawn between the two parallels RB, *rb* is [a]equally inclined to them both, and conſequently PB, *bp* are alſo [b]equally inclined to RB, *rb*, and therefore parallel to one another. And after the ſame manner, wherever the point B is aſſumed, we can find a ray B*b*, that ſhall be equally inclined to both the ſurfaces; and therefore in every oblique pencil of rays, there is one ray whoſe incident and emergent parts are parallel. And it is manifeſt alſo, that every other ray paſſing through B, will be more refracted than the ray B*b*; for a plane touching the lens in any other point beſides *b*, will be inclined to a plane touching it in B; and therefore a ray paſſing through B and any other point beſides *b*, [c]will be bent out of its courſe, more or leſs, according as theſe planes are more or leſs inclined. Therefore, *&c. Q. E. D.*

[a] 29. I. *Eucl.*

[b] Laws of refraction.

[c] 69. I.

Q*q* is the axis of an oblique pencil on the contrary ſide of the axis of the lens.

206.

* The only reaſon for chuſing B in the flatteſt ſide, is to prevent *b* falling out of the lens, which otherwiſe might happen, becauſe the ſide *A* is a leſſer ſegment than the ſide *a*.

206. *Corol.* The axis of any oblique pencil whatsoever, is equally inclined to each side of the lens: And the less oblique is the pencil, the nearer will the point B be to the vertex A; and [d] the less will be the distance between the parallel rays PB, *bp*. [d] 68. I.

PROP. XXX.

207. *In every oblique pencil of rays, the part* Bb *of the axis within the lens, produced if need be, will intersect the axis of the lens in the very same point* O; *and the point* O *divides the axis of the lens in such a manner, that* AO : aO :: R : r; *that is,* Fig. 73 to 78.

1. *The point* O *is in the vertex of a plano-convex and plano-concave.* Fig. 73, 74.

2. *In the double convex and double concave, the point* O *divides that part of the axis which is within the lens, in the ratio of the radii of the sides, the shortest part being next that side which has the greatest curvity.* Fig. 75, 76.

3. *The line* Bb *must be produced without a concavo-convex lens, before it will intersect the axis: And the point* O *in the axis, where the said line* Bb *produced intersects it, lies next to that side which has the greatest curvity; and its distance from the sides, is as the radii of these sides respectively.* Fig. 77, 78.

Let the radius of the flattest side A be called R, and the radius of the other side, *r*; and because the axis B*b* of any oblique pencil, [a] is equally inclined to both the sides of the lens, it necessarily follows: [a] 206. I.

Case 1. In a plano-convex and plano-concave lens, the axis B*b* of any oblique pencil passes through the vertex *a*, and therefore the point O is also in *a*. For no perpendicular to the spherical side, can be parallel to a perpendicular to the plane side, but that only which passes thro' the vertex *a*. Again, because R in this case is infinite, and O coincides with *a*; it will be, R : *r* :: AO : *a*O. Therefore, *&c.* *Q. E. D.* Fig. 73, 74.

Case 2, and 3. Because the radii RB, *rb* are [b] parallel, the △s RBO, *rb*O are similar; and therefore RB : *rb* :: RO : *r*O. Also RB ± *rb* : RB or *rb* :: RO ± *r*O (R*r*) : RO or *r*O. But the three first terms are invariable quantities, and therefore so is the fourth likewise; that is, in the same lens the point O is invariable. And from the position of the parallel radii RB, *rb*, it is manifest that the point O must be within the double convex and double concave lens, and without the concavo-convex. Fig. 75 to 78. [b] Hypoth.

Again, because RO : RA (RB) :: *r*O : *ra* (*rb*); it will also be,

AO (RO ∽ RA) : RA :: *a*O (*r*O ∽ *ra*) : *ra*; that is,

AO : *a*O :: R : *r*. Therefore, *&c.* *Q. E. D.*

208. *Corol.* The point O is in the middle of a double convex and double concave, whose sides are segments of the same sphere. And in every lens whatsoever, the greater proportion the greater radius bears to the lesser, the nearer will the point O be to the vertex *a* of that side which has the greatest curvity.

PROP. XXXI.

Fig. 73 to 78. 209. *If the axis* Bb *of a pencil of rays, is not very oblique to the axis of the lens; the points* s, v *where the parts without the lens* PB, pb *produced, of the axis of the oblique pencil, cut the axis of the lens, divide that axis in such a manner, that*

AO : A*s* :: *m* : *n* :: *a*O : *av*; *nearly.*

[a] Construc. The angles RB*s*, RBO or their supplements, [a] are the angles of incidence and refraction at the first side A; and therefore their sines are as [b] 15 & 35. I. [b] *m* to *n*. In *fig.* 73, 74. the ∠ RB*s* = [c] A*s*B; and RBO = [c] AOB. [c] 29. I. *Euc.* And in *fig.* 75 to 78, the difference between the ∠s RB*s*, RBO, or their supplements, and the respective ∠s A*s*B, AOB is equal to the ∠ ARB; as will be manifest by drawing *d*B parallel to the axis RA. And therefore the sines of these respective angles, are pretty nearly in the same ratio; that is, Sine ∠ A*s*B : Sine ∠ AOB :: *m* : *n*; nearly. [d] Trigonom. But the sines of the ∠s A*s*B, AOB [d] are as the sides BO, B*s*; and when the point B is not very remote from A, BO and B*s* will be as AO and A*s*, nearly. And therefore when the point B is the nearest of all to A, it will be, AO : A*s* :: *m* : *n*. *Q. E. D.*

In *fig.* 73, 74. the points *a*, *v* and O do all coincide: And in *fig.* 75, to 78, it will be proved as above, that when the point *b* is very near to *a*, it will be, *a*O : *av* :: *m* : *n*. Therefore, *&c.* *Q. E. D.*

210. *Corol.* 1. The farther is the point B from A, and *b* from *a*; [e] 56. I. [e] the greater will be O*s* and O*v*.

Let *s* and *v* be points belonging to pencils, that have the least obliquity to the axis of the lens. Then will,

211. *Corol.* 2. A*s* = $\frac{n}{m}$AO; and *av* = $\frac{n}{m}$*a*O. Let the thickness A*a* of any lens be called *t*: Then in a plano-convex and plano-concave lens, A*s* = $\frac{n}{m}t$. And in a double convex and double concave, whose sides are segments of the same sphere, A*s* = *av* = $\frac{n}{2m}t$.

212. *Corol.* 3. Hence, in a plano-convex and plano-concave glass A*s* = $\frac{3}{8}t$. And in a double convex glass of equal convexities, and in a double concave glass of equal concavities, A*s* = *av* = $\frac{1}{3}t$.

213.

213. *Schol.* 1. The places of the points s, v are affected only by the different inclinations of the axis of the pencils, refracted by any lens, and not at all by the different distances from the lens, of the focus's of the said pencils. The point v will be always without a concavo-convex lens; but the point s may be either without or within that lens, according as the ratio of AO to Aa, is greater or less than the ratio of m to n.

214. *Schol.* 2. As the angles subtended by objects and their images, are properly those which are contained between the axis's of the pencils belonging to their extreme parts; so these angles, strictly speaking, are not formed at any common point in or out of a lens, but at the two distinct points s, v according to the respective side of the lens, the object and image are of.

215. *Schol.* 3. The focal length of any lens, is to be reckoned from the point v, if the flattest side is exposed to parallel rays; and from the point s, if the parallel rays are incident upon the most convex or concave side. The point v in all cases, being the most remote from the flattest side (A) of the lens. *Note*, the points s, v we shall sometimes call the *focal centers* of the lens. Focal centers of a lens.

[215.] *Corol.* Hence, the focal length of a convex lens is, properly, the distance between the principal focus and the next focal center. And the focal length of a concave lens, is to be reckoned from its principal focus, to the farthest focal center. See *fig.* 57, to 64. and *fig.* 73, to 84.

216. *Schol.* 3. As the axis's of the several pencils that are not very oblique, are refracted orderly from the same point s or v nearly; so the refraction of these axis's cause no sensible confusion in the image.

217. *Schol.* 4. Because $As = {}^{a}\frac{n}{m}AO$; and $av = {}^{a}\frac{n}{m}aO$. [a] 211. I.

Also because $R : r :: {}^{b}AO : aO$; it will be, [b] 207. I.

$$R \pm r : \frac{R}{r} :: t\,(AO \pm aO) : \frac{AO}{aO};\ \text{therefore,}$$

$$AO = \frac{Rt}{R \pm r};\ \text{and}\ aO = \frac{rt}{R \pm r}.\ \text{Wherefore,}$$

$As = \frac{n}{m} \times \frac{Rt}{R \pm r}$; and $av = \frac{n}{m} \times \frac{rt}{R \pm r}$; and multiplying one side of *each* equation by $m \times \overline{R \pm r}$; it will be,

$$As : av :: nRt : nrt.$$

PROP.

PROP. XXXII.

218. *If rays come both ways upon any lens, parallel to the axis; the distances of the geometrical foci on each side, from the respective points* s, v *will be equal.*

Let as before the thickness of the lens be called t, and the radii of the sides AB, ab be called R, r.

Fig. 57 *to* 60.

CASE 1. *When one side of the lens is plain.*

Because AF (*fig.* 57.) $=$ [a]EF $+ \frac{n}{m}t$ (*fig.* 59.) $=$ [b]EF $+$ Es. And EF (*fig.* 58.) $=$ [a]AF $- \frac{n}{m}t$ (*fig.* 60.) $=$ [b]AF $-$ Es. Therefore, AF (*fig.* 57.) $= s$F, *fig.* 59. And in *fig.* 58. $\left\{\begin{matrix} s\mathrm{F} \\ \mathrm{AF} \end{matrix}\right\} = \left\{\begin{matrix} \mathrm{AF} \\ s\mathrm{F} \end{matrix}\right\}$ *fig.* 60. Therefore, *&c.* *Q. E. D.*

[a] 177. I.

[b] 211. I.

Fig. 61 *to* 64.

CASE 2. *When both sides of the lens are spherical.*

Because in *fig.* $\begin{matrix} 61 \\ 62 \end{matrix}$; $mn\mathrm{R}r \mp n\mathrm{R}t : mn\mathrm{R}r \mp nrt$ :: [c]AQ : ax. By adding or subtracting the like proportionals $n\mathrm{R}t : nrt$ :: [d]As : av; it will be, $mn\mathrm{R}r : mn\mathrm{R}r$ :: AQ $\pm$ As (sQ) : $ax \pm av$ (vx). Also because in *fig.* $\begin{matrix} 63 \\ 64 \end{matrix}$; $mn\mathrm{R}r \mp n\mathrm{R}t : mn\mathrm{R}r \pm nrt$ :: [a]AQ : ax; it will also be, $mn\mathrm{R}r : mn\mathrm{R}r$ :: AQ $\pm$ As (sQ) : $ax \mp av$ (vx). Therefore in all cases sQ $=$ vx. *Q. E. D.*

[c] 189. I.

[d] 217. I.

219. *Schol.* 1. The radius of a plano-convex or plano-concave lens being the same, the focal length (AF, *fig.* 57, 58. and sF, *fig.* 59, 60.) will be the same, whatever be the thickness of the lens.

220. *Schol.* 2. All other things being the same. In a double convex lens, and in the concavo-convex, *fig.* 64. the greater is t, the greater will be sQ or vx; and in the double concave and the menisicus, *fig.* 63. the greater is t, the less will be sQ or vx. For,

[e] 185 & 217. I.

Fig. 61. $$\frac{\frac{mn\mathrm{R}r}{m-n} - n\mathrm{R}t}{m\mathrm{R} + mr - mt + nt}(\mathrm{AQ}) + \frac{n\mathrm{R}t}{m\mathrm{R} + mr} = {}^{e}\, s\mathrm{Q}.$$

Fig. 62. $$\frac{\frac{mn\mathrm{R}r}{m-n} + n\mathrm{R}t}{m\mathrm{R} + mr + mt - nt}(\mathrm{AQ}) - \frac{n\mathrm{R}t}{m\mathrm{R} + mr} = {}^{e}\, s\mathrm{Q}.$$

Fig. 63. $$\frac{\frac{mn\mathrm{R}r}{m-n} - n\mathrm{R}t}{m\mathrm{R} - mr + mt - nt}(\mathrm{AQ}) + \frac{n\mathrm{R}t}{m\mathrm{R} - mr} = {}^{e}\, s\mathrm{Q}.$$

Fig. 64. $$\frac{\frac{mn\mathrm{R}r}{m-n} + n\mathrm{R}t}{m\mathrm{R} - mr - mt + nt}(\mathrm{AQ}) - \frac{n\mathrm{R}t}{m\mathrm{R} - mr} = {}^{e}\, s\mathrm{Q}.$$

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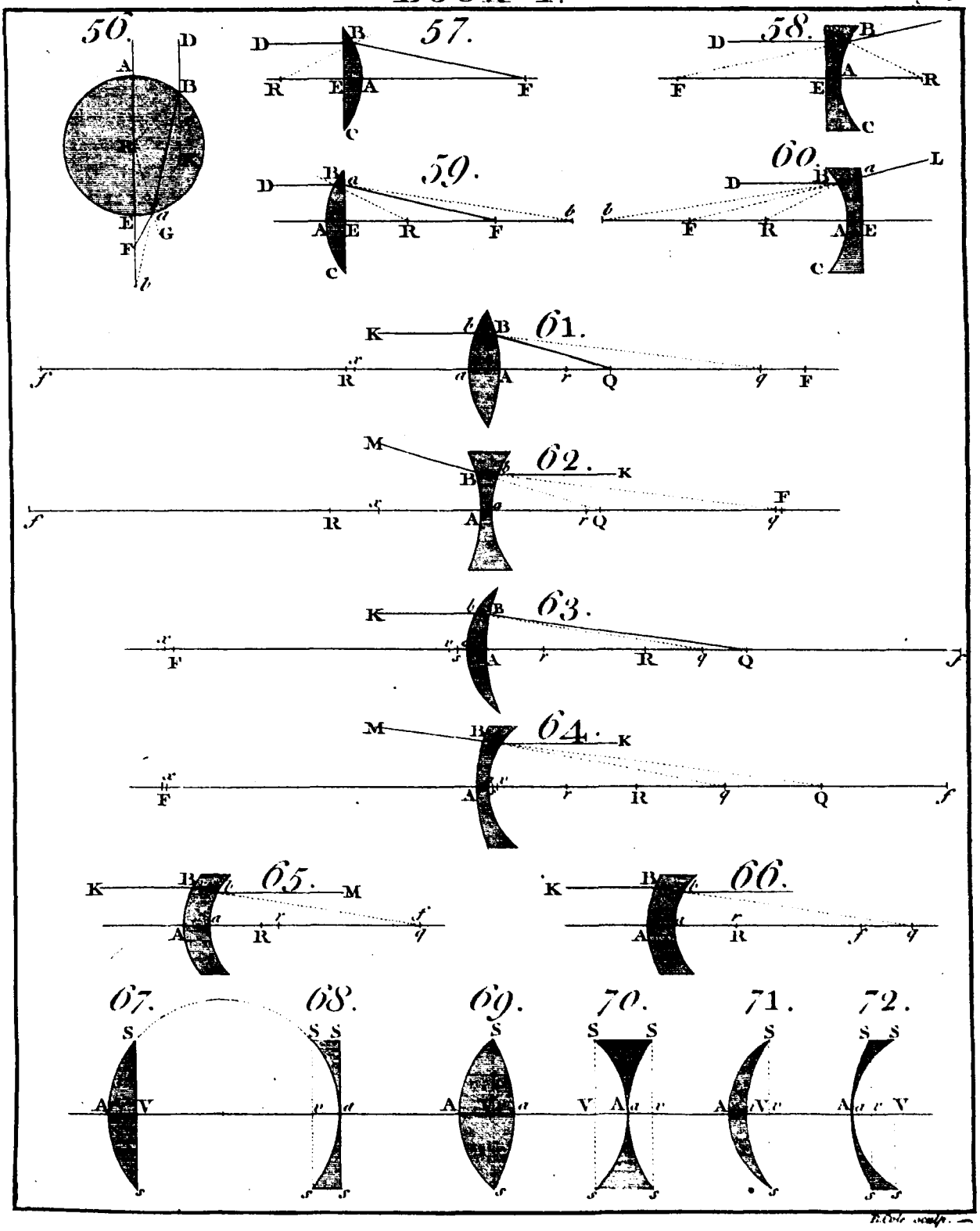
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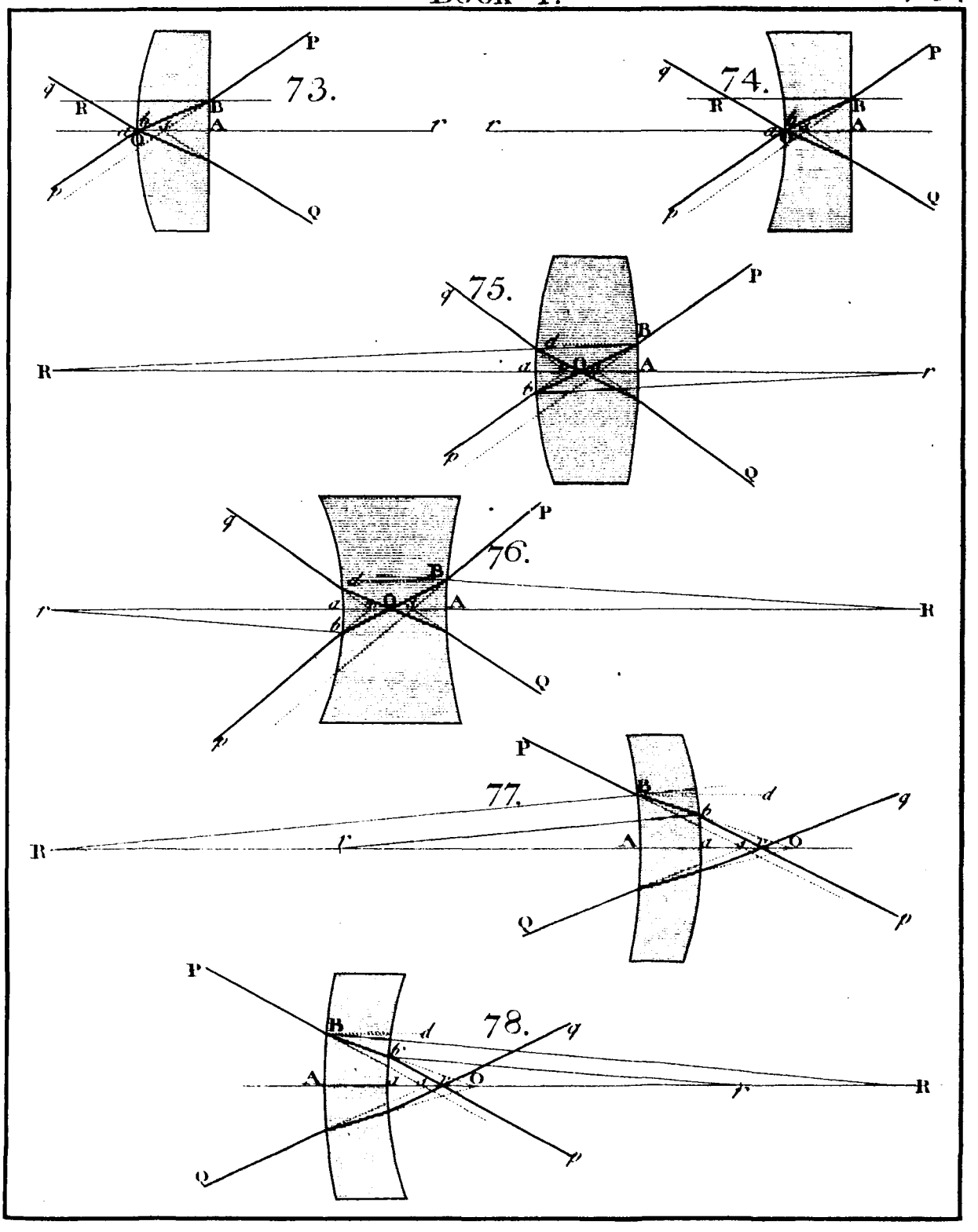
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In each case, the variable quantities $-nRt$, $+nRt$ in the numerators have contrary signs; whence it is manifest that in *fig.* 61, 64. the greater is t, the greater will be sQ; and in *fig.* 62, 63. the greater is t, the less will be sQ.

PROP. XXXIII.

221. *The foci of both direct and oblique pencils of parallel rays, are nearly at the same distance from the focal center* (s *or* v, *as the case is*) *of any lens.*

Let ArQ be the axis of the lens, r the center of the first surface ab, Q the principal focus of that surface, and F the focus of the lens. Fig. 79 *to* 84. Let pbBP be the axis of any oblique pencil; and let the emergent part BP, produced backwards cut the axis of the lens in s. Of all the rays parallel to the axis pbBP, there shall be one as Dd which will fall perpendicular upon the first surface ad; and therefore if the medium was continued, this ray (produced backwards from a concave lens, but continued through a convex lens) would pass through the center r, and the geometrical focus of rays parallel to it would be at q, so that $rq = rQ$. This is manifest, because drq is perpendicular to the surface a, as well as arQ.

Again, of all the rays parallel to the axis pb, there shall be one as Ee (if the lens is broad enough) which shall be refracted into eG perpendicular to the second surface AB; and therefore the focus of this ray shall be in sP produced, and in eG produced; that is, in the point of intersection f. Neglecting the aberration of the first surface ab; eG produced would unite with the axis drq in q. We are now to shew that $sf = s$F nearly.

CASE 1. *When one side of the lens is plain.* Fig. 79, 80.

Because sr, fq; and sf, rq [a] are parallel; $sf = {}^{b}rq = {}^{c}rQ = {}^{d}s$F.

[a] Hypoth.
[b] 34. I. *Euc.*
[c] Construct.
[d] 218. I.

CASE 2. *When both sides of the lens are spherical.*

Let R be the center of the second surface AB, and because eG is perpendicular to this surface, it will pass through the center R. And because rq is [a] parallel to sf, it will be $Rr : Rs$ [e] $:: rq : sf$. Now the two first terms (Rr, Rs) are invariable quantities, and admitting rq to remain invariable, sf will be also invariable; that is, sF will describe the sector sFf, and sf will be $= s$F. Fig. 81 *to* 84. [e] 4. VI. *Euc.*

But because of the aberration by the first surface de, no ray, as Ee parallel to and remote from the axis Drq, can be refracted to the focus q; [f] but it will cut the axis rq, in some point y, between q and r; and therefore a ray parallel to pb, that shall be refracted perpendicularly to the second surface AB, must be some ray hk, between Ee and the axis pb, if the centers R, r are on different sides, *fig.* 81, 82; otherwise hk Fig. 79 *to* 84. [f] 134. I.

will be farther from the axis than E*e*, *fig.* 83, 84; and consequently this ray, will cut the axis *sf* in some point *x*, between *f* and *s*. But it has been shewed above, that if the lens had no aberration, *f* would be the focus of all the rays parallel to the axis *pb*B*f*; and as the aberrations at the different surfaces, are contrarily, it is sufficiently manifest that the focus of those rays that are nearest to the axis *pb*B*f*, will not be remote from *f*. *Q. E. D.*

222. *Schol.* In every lens whatsoever, the true focus *g* of an oblique pencil of parallel rays will be between *f* and *s*; that is, *sg* will be less than *sf* or SF.

Because if there was no aberration at either surface of the lens, *sg* would be [e]equal to *s*F or *sf*; it follows that if the aberrations at each surface were equal, they would destroy each other. Let rays of light be within the lens, on each side of the axis *b*B, and let their inclination to that axis be such, as that they would emerge at the side *b* parallel to *bp*. Now it is manifest, that a ray parallel to the oblique axis *pb*BP, and between the said axis and the perpendicular *dr*, will be more oblique to the emergent side A, than to the incident side *a*; and therefore in this case, the aberration at the side A is the greatest, and this excess of the aberration, contracts the focus nearer to *s* than *f*, as to *g*. In like manner, if the parallel ray be on the other side of the oblique axis, the greatest aberration will be on the incident side *a*; and in the present case likewise, this excess of aberration, contracts the focus nearer to the lens, as to *g*; and it is manifest, that the more oblique is the pencil, the greater will be *fg*. Therefore, *&c.* *Q. E. D.*

[e] 221. I.
Fig. 81, 82.

§. 3. *Of diverging or converging rays, and of images.*

PROP. XXXIV.

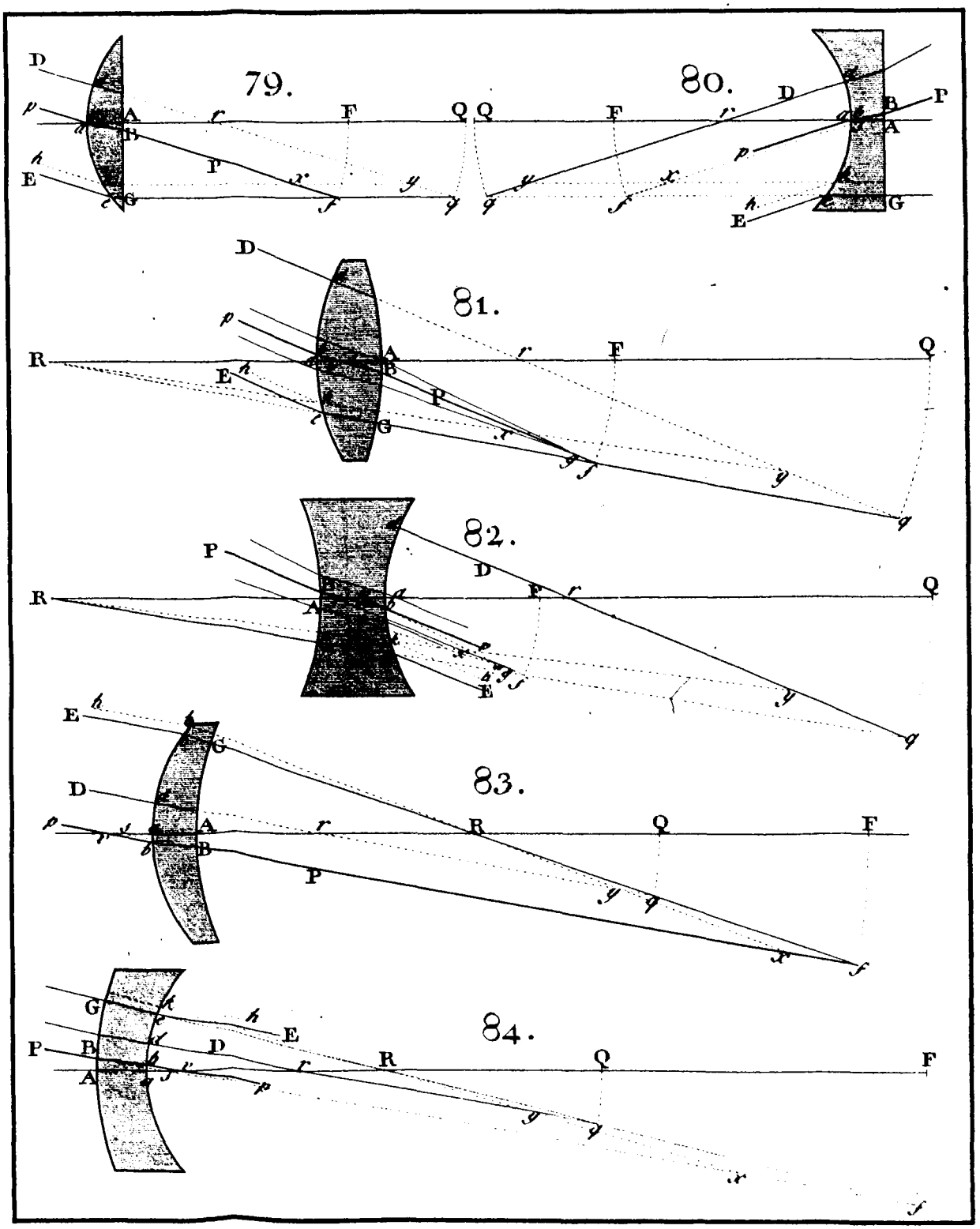
223. *The focus of incident rays, either diverging or converging upon any lens, being given; to find the focus of the emergent rays.*

Let F*f* be the axis of any lens B*b*, whose focal centers are *s*, *v*; and principal foci are F, *f*. Let any point Q in the axis of the lens be the focus of incident rays, either diverging as QB, or converging as MB; and let *f* be the geometrical focus of those rays parallel to the axis of the lens, whose incidence is on the same side with the ray QB, or MB. To find the focus of the emergent rays, say QF : F*s* :: *vf* (F*s*) : *fq*; and placing *q* the same way from *f*, that F is from Q; the point *q* thus found will be the focus required.

Fig. 85 *to* 90.

Construction. Let the point of incidence B, be not remote from the vertex of the lens: from the center *v*, describe the arch *fd*, and draw *vd* parallel to the incident ray QB or MB; [a]then will *d* be the point where the emergent ray intersects the axis *vd*, and therefore the point *q*, where

[a] 221. I.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where the emergent ray bd (or Nb) produced, interſects the axis of the lens, is the focus required.

Again, draw sD parallel to qB; then will the △s QDs, vdq be [b]equiangular, and therefore QD : Ds :: [c]vd : dq. But D is a point where the incident ray qb, or Nb, after refraction by the lens, will interſect the axis sD, ſo that sD = [a]sF nearly; and when the △s QDs, vdq are vaniſhing, the point D will coincide with F, and d with f; wherefore QF : Fs :: vf : fq. *Q. E. D.*

[b] 29, & 32. I. *Euc.*
[c] 4. VI. *Eu.*

224. *Corol.* 1. Becauſe Fs = [c]vf, Fs or vf is a mean proportional between QF and fq; that is, QF : Fs :: Fs : fq.

[c] 218. I.

225. *Corol.* 2. QF : Fs :: (vf : fq ::[d]) Qs (QF ∓ Fs) : vq (vf ∓ fq).

226. *Corol.* 3. QF : Qs :: (Fs : vq ::[d]) Qs (QF ∓ Fs) : Qs ∓ vq.

[d] Vide 12. V. *Eucl.*

227. *Schol.* 1. When the lens is not very thick, in proportion to its focal length, the focal diſtances sF, vf, may without ſenſible error be reckon'd from the point O, which biſects vs; and then the preceding rules, will be convertible into the following, *viz.*

1. QF : FO :: FO (= Of) : fq.
2. QF : FO :: QO : Oq.
3. QF : QO :: QO : Qq.

228. *Corol.* 4. If q be the focus of incident rays, Q will be the focus of the emergent rays.

229. *Corol.* 5. The focus's Q, q, move both the ſame way; and always lie the contrary way from F and f. And the diſtances FQ, fq, vary reciprocally; that is, as one increaſes, the other decreaſes, but with a different velocity, according as they are differently ſituated. When Q coincides with F, q will be at an infinite diſtance; that is, the refracted rays will emerge parallel to the axis. And, in *fig.* 85, 86, when QO = 2 OF, Oq will be = OQ; whence the difference of the focal diſtances of the refracted rays when QO = 2 OF, and when OQ is infinite, is only equal to Of.

230. *Corol.* 6. When the focus Q, of rays diverging upon a convex lens, *fig.* 87. lies between F and the lens; the refracted rays (bN inſtead of croſſing the axis) will diverge from the focus q. And if incident rays upon a concave lens, converge towards a point that is nearer to it than F, as in *fig.* 88. the lens in this caſe will have a real focus; that is, the refracted rays will croſs the axis in q. And univerſally,

231. *Corol.* 7. When the focus's Q, q lie both on the ſame ſide of the lens; if the incident rays diverge from Q, (*fig.* 87, 90.) the emergent rays

Fig. 87 to 99.

rays will diverge from *q*. And if the incident rays converge towards Q, (*fig.* 88, 89.) the emergent rays will converge towards *q*. And in all these cases, the nearer Q is to O, the nearer also will *q* be to O; and if one of these focus's be in O, the other will also coincide with it.

Fig 85,86. And the contrary happens, when Q and *q* are on different sides of the lens; that is, rays diverging from Q (*fig.* 85.) will converge towards *q*. And rays converging towards Q (*fig.* 86.) will diverge from *q*.

232. *Schol.* 2. Convex and concave lens's, may be compared to convex and concave surfaces; and what has been shewed with respect to the motions of the focus's Q,*q* by the refraction of the one, is also applicable to the other. Thus, in *fig.* 85, 86. the focus's Q,*q* are on opposite sides of the lens; and neither can approach nearer to the lens, than the focus's F,*f*. If one of the focus's is between F and the lens, *fig.* 87 to 90, the other will be also on that same side of the lens. In *fig.* 87, 88. the whole motion of Q is from F to the lens; but the motion of *q* is not limited to any distance from the lens to infinity. In *fig.* 89, 90. the place of *q* is limited from *f* to the lens; and the motion of Q, is like to that of *q* in the preceding case. The three cases of convex lens's pass from one into the other, as do also those of concave lens's.

233. *Schol.* 3. All sorts of convex lens's, that have equal focal lengths, will have the same effect if put into each other's places; because the rules above, regard only their focal lengths, and not their figures. The same is also true of concave lens's. So that the six figures referred to, contain all the variety that can happen with respect to the focus's Q, *q*.

PROP. XXXV.

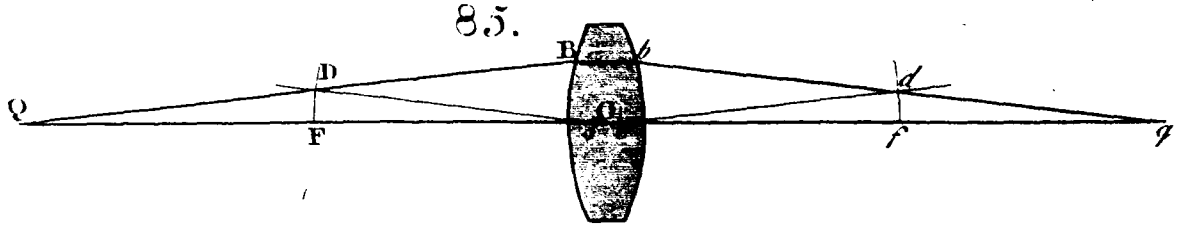
Of Images, formed by the refraction of Lens's.

Fig. 91 *to* 94. 234. *If an arch of a circle* PQR, *described from the focal center* s *of any lens, be considered as an object; its image* pqr *may, in most cases, be also considered as a similar arch, whose center is the other focal center* v. *The length of the object will be to the length of its image, as* sQ *to* vq: *And the image with respect to the object, will be erect or inverted, according as they lie on the same side, or contrary sides of the lens.*

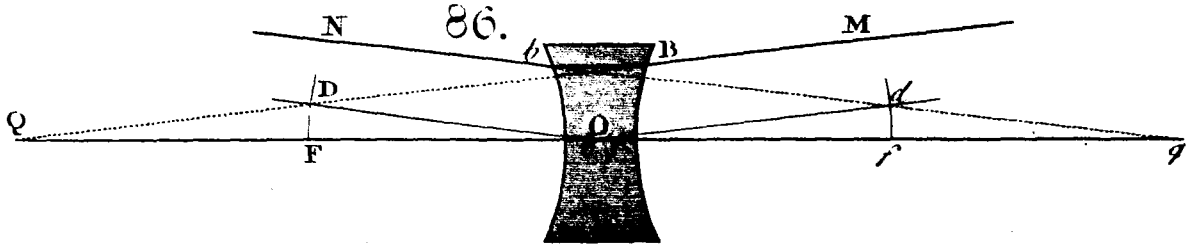
Fig. 93. This proposition, with respect to a sphere is self-evident; because the object being concentric thereto, (the points *s*,*v* in this case coinciding with the center O,) every part of the object is alike exposed to the surface of the sphere. Therefore, *&c.*

And

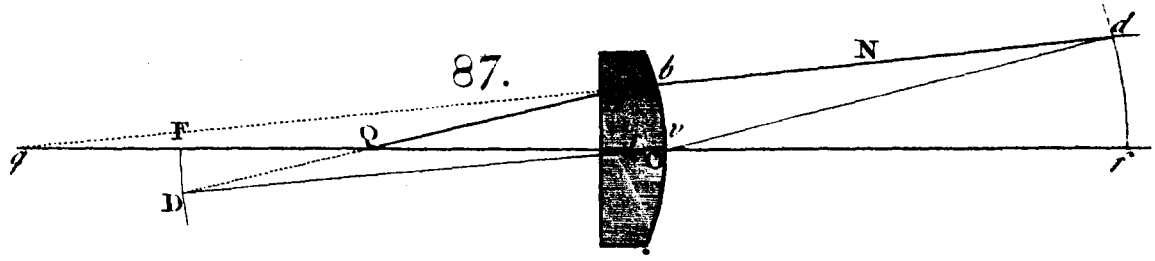
85.



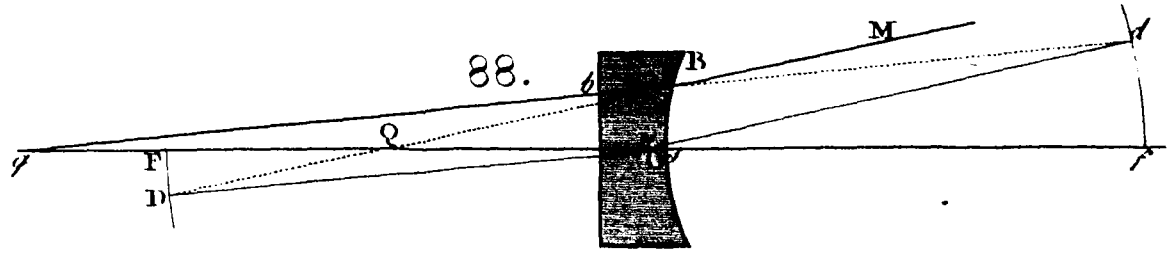
86.



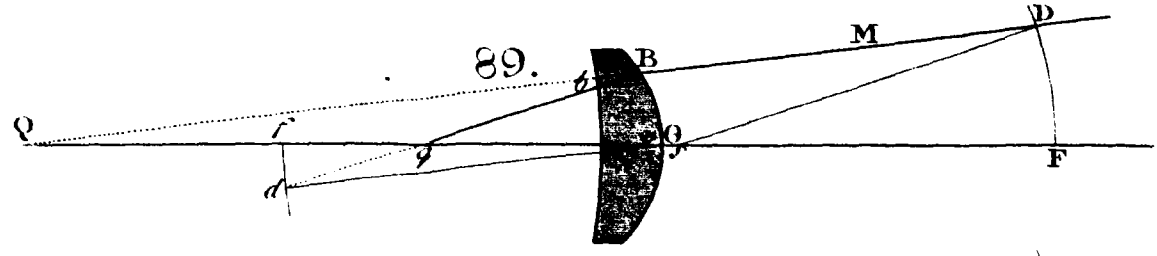
87.



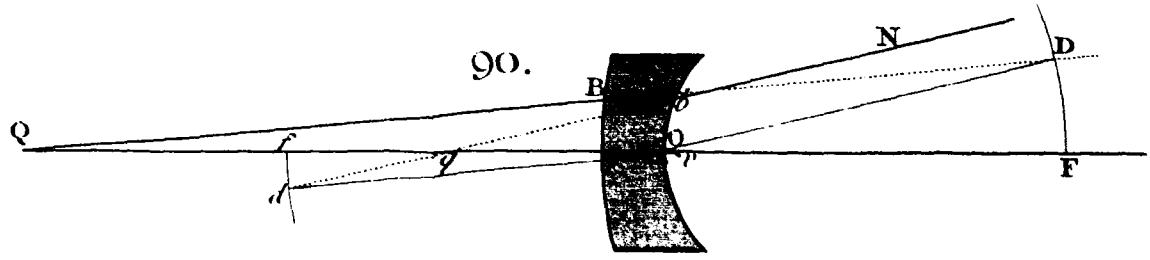
88.



89.



90.



And in a lens, becauſe the geometrical foci of parallel rays, E, F, G; *e*, *f*, *g* [a]are nearly in arches, whoſe centers are *s*, *v* reſpectively; neglecting the aberrations of the lens, (or ſuppoſing the object, and the thickneſs of the lens, to be ſo ſmall that the refractions of the two ſides are equal,) it will be PE : E*s* :: [b]*ve* : *ep*; QF : F*s* :: [b]*vf* : *fq*; and RG : G*s* :: [b]*vg* : *gr*. But in theſe three couplets of proportionals, the three firſt terms [a]are nearly equal each to each reſpectively; and conſequently the fourth terms are equal alſo nearly, and therefore *pqr* may without ſenſible error, be reckoned a circular arch, whoſe center is *v*. *Q. E. D.* Fig. 91, 92, 94.

[a] 221. I.

[b] 223. I.

235. Again, becauſe the axes P*s*, *vp*; R*s*, *vr* of the oblique pencils, [c]are parallel, the angles P*s*R, *pvr* are equal; and therefore the object PQR is to its image, as their diſtances *s*Q, *vq*: And according as their correſponding extremes P, *p*; R, *r* lie on the ſame, or on contrary ſides of *s*, *v*; they lie alſo on the ſame or on contrary ſides of their middle points Q, *q*; that is, the image with reſpect to the object, is erect or inverted. *Q. E. D.*

[c] 205. I.

236. *Schol.* 1. *In common caſes, or when the lens is not very thick; [d]the axes of the oblique pencils may, without ſenſible error, be conſidered as interſecting the axis of the lens in the point O, which biſects the diſtance *s v*.

[d] 227. I.

237. *Schol.* 2. If the object is a ſtrait line ſubtending a ſmall angle at the lens, its image will be ſo too, nearly; and the ſmaller the object and image are in proportion to their diſtances from the lens; that is, the ſmaller are the angles which they ſubtend, the more exactly will they correſpond. And univerſally, the images of all ſurfaces ſubtending very ſmall angles at the focal centers of any lens, will be, accurately enough, alike to their originals.

238. *Schol.* 3. Becauſe the focal diſtances OE, O*e*, of an oblique pencil, [e]are leſs than the focal diſtances OF, O*f*; a circular object whoſe center is O, will not have a ſimilar arch for its image. If the object and its image are on different ſides of the lens, as in *fig.* 91. or if the lens is concave; the image will have a greater curvity, than the object; that is, the ratio of O*q* to OQ, will be greater than the ratio of O*p* to OP: But if the object and image, are both on the ſame ſide of a convex lens, *fig.* 92. the ratio of O*p* to OP, will be greater than the ratio of O*q* to OQ. As will be manifeſt, by comparing together the rules QF : FO :: QO : O*q*; PE : EO :: PO : O*p*.

[e] 222. I.

* In the four laſt plates, the lens's are made thicker than ordinary, for the ſake of repreſenting the places of the points *s*, *v*, and the rays belonging to them diſtinctly.

Fig. 91 to 94.

239. *Schol.* 4. Neglecting the aberrations of the lens's, a ſtrait object (PQR) at right angles to the axis of any lens, will have a curvilinear image: And if the object and image are on different ſides of the lens, as in *fig.* 91. the image will be concave towards the object; and the image will be convex towards the object, if they are both on the ſame ſide, as in *fig.* 92, 93, 94. For it being QF : OF :: [f] O*f* : *fq*; and PE : OE :: [f] O*e* : *ep*; it will alſo be,

[f] 234, & 236. I.

QO (QF ± OF) : OF :: O*q* (O*f* ± *fq*) : *fq*; and
OF (=OE) : PO :: *ep* : O*p*; and by multiplying theſe two rows of proportionals together, we ſhall have,

QO : O*q* × *ep* :: PO : O*p* × *fq*. But in *fig.* 91, 94. (becauſe QF is leſs than PE, and the middle terms OF, O*f*, OE, O*e* are equal,) it is manifeſt that *ep* is leſs than *fq*; and therefore the ratio of O*q* to OQ, is greater than the ratio of O*p* to OP. And in *fig.* 92, 93. *ep* is greater than *fq*; whence there the ratio of O*q* to OQ, is leſs than the ratio of O*p* to OP. Whence in all caſes, the image is curvilineal; and in *fig.* 91. the image is concave, and in *fig.* 92, 93, 94. it is convex, towards the object. *Q. E. D.*

If the aberrations of the oblique pencils be conſidered, becauſe then the focal diſtances OE, O*e* are leſs than OF, O*f*; it is manifeſt that the curvity of the image will be ſtill greater, and the ſame way as before. And the greater is the angle QOP, the leſs in *fig.* 91, 94. and the greater in *fig.* 92, 93. will be the focal diſtance O*p*. So that *qp* is not a circular arch, but approaches towards one or other of the conic ſections, having *q* in its vertex.

And it may be farther obſerved, that the images of ſtrait objects, by convex or concave lens's, correſpond to thoſe of like objects, by convex or concave ſurfaces. *Vide Art.* [153.] I.

240. *Schol.* 5. When an object, as PQR, is farther from a convex lens (*fig.* 91.) than its principal focus F, there will be formed a real image *pqr*, whoſe poſition with reſpect to the object, is inverted.

But an object placed nearer a convex lens, (*fig.* 92, 93.) than its principal focus F, or any where before a concave lens, (*fig.* 94.) will have no real image; the rays *b*,*b*; *a*,*a*; *c*,*c* in each pencil, after their emerſion out of the lens, diverging from the foci *p*, *q*, *r*, reſpectively. In both theſe laſt caſes, the image *pqr* is erect; and in the firſt caſe (*fig.* 92, 93.) the image is always greater, and in the ſecond caſe, (*fig.* 94.) the image is always leſs than the object.

Fig. 95.

241. *Schol.* 6. The theory of real images, may be thus illuſtrated by an experiment. Upon a long table draw the line BD, and over ſome point

point there in A, place the convex lens O, whose principal focus's are F, f, so that the axis Ff of the lens, be parallel to BD. In the line BD, take AF, Af each equal to OF or Of in the axis Qq. On one side of A divide the line AB into the parts 1, 2, 3, 4, &c. each equal to AF; and on the other side take f1 equal to Af or AF, and divide it into $f\frac{1}{2}$, $f\frac{1}{3}$, $f\frac{1}{4}$, &c. so that these parts be respectively equal to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c. of f1 or Af. Now if the room be darkened, and a lighted candle be placed over any one of the divisions in the line AB, as at 2; the image of the candle, will be seen distinct but inverted, upon a paper held over the corresponding fraction on the other side, as at $\frac{1}{2}$. If the candle be placed at the point 3 or 4, &c. the paper for receiving the image must be held over $\frac{1}{3}$ or $\frac{1}{4}$, &c. So that if the candle be moved from 2 to an infinite distance, the whole motion of the image will be from $\frac{1}{2}$ to f. If the candle be placed at 1, the image will be at 1, at the same distance from the lens on the other side. If the candle be brought nearer to F, the motions of the image and candle will now be reciprocal, to what they were before. But if the candle be placed any where, between F and the lens, there will be no image formed.

242. *Schol.* 7. Since an object placed at any distance, and its image, subtend [g] equal angles at the focal centers s, v of any lens; the angle under which any remote object appears, being given, it will be easy to find the diameter of its image. [g] 235. I.

Let q be the principal focus of the convex lens O, and let the angle PsR$=p\,v\,r$ be that under which the sun (or any other very remote object) appears to the naked eye: Bisect the $\angle$ $p\,v\,r$ with the line vq; then in the right-angled $\triangle$ vqp or vqr, we shall have the side vq and all the $\angle$s, to find qp or qr, the semi-diameter of the image. Fig. 96.

PROP. XXXVI.

Of the Camera Obscura.

243. *By a* Camera Obscura *in optics, is meant any darkned room or box, out of which all light is excluded, but what comes through a lens upon a white sheet properly placed, so that the pictures of objects that are without, are seen thereon. This phænomenon is very curious, and being altogether explicable upon the principles already delivered; its * theory may not improperly be explained here.*

Let OB be a darkened room or box, into which no light is admitted, but what comes through the lens O, whose principal focus is F, and axis is Qq: And let q be the focus, after refraction, of rays which come from Q. Fig. 97.

244.

* The mechanism of different sorts of *Camera Obscura's*, will be exhibited in Book III.

244. *Phænomenon.* Now if a ſheet, of * white paper or canvas, be placed at *q* perpendicular to the axis O*q*; the pictures of objects that are before the lens, at the diſtance OQ, will be fairly and orderly repreſented upon the paper, in their natural colours and proportions, but inverted.

In the figure, to prevent confuſion from too many lines, there are drawn only the three pencils that come from the extremes P, R, and middle Q, of the object PQR; and in theſe pencils, there are alſo drawn only the two extreme rays, beſides the axes. But the rays flowing from any point P, for inſtance, upon the lens, are innumerable, the whole conical ſpace *b*P*d*, being filled with them; and theſe being all collected and united at the focus *p*, and there received upon the white paper, whoſe ſurface is uneven, are reflected by it all manner of ways; after the ſame manner, as objects themſelves reflect the light falling upon them. So that *p* to a ſpectator in the room, is now, as it were, a real object, exactly ſimilar to the phyſical point P, and in proportion to it, as O*p* to OP. And *p* is of the ſame colour with P, becauſe the rays flowing upon the lens from P, are united at *p*, diſtinct and ſeparate from the rays coming from other parts of the object.

In like manner, every other phyſical point of the object; ſends forth its cone of rays, which by the lens are brought to unite orderly at *pqr*; and being there reflected by the paper, the image of the whole object is diſtinct and viſible, like a picture drawn upon canvas; but much more lively and perfect, than the beſt finiſhed drawings of the greateſt artiſt.

If the objects are very remote, in proportion to the focal length of the lens, we ſhall have the pictures of thoſe that are in the ſame neighbourhood, pretty diſtinct at the ſame time, though they are at different diſtances from the lens; becauſe in that caſe the focal diſtances of the refracted rays, differ inſenſibly.

245. *Schol.* 1. If the paper be moved nearer towards the lens, as to *x*, or farther from it, as to *y*; the picture will begin to be confuſed; becauſe the rays proceeding from the next adjacent points of the objects begin to interfere and mix together, as the rays from the point *a*, will be mixed with thoſe from P, *&c.* The diſtinctneſs of the picture, is entirely owing to the ſeparation of the rays belonging to every point of the object, upon their reception by the paper. And if the rays belonging to any phyſical point of an object could be ſeparated from all others, the image of that point would be diſtinct upon the paper placed at any diſtance either before or behind the focus; only it would be leſs bright, becauſe

* The picture appears beſt upon white, becauſe white bodies reflect light more uniformly and copiouſly, than bodies of other colours; and paper reflects light more uniformly than canvas.

because it would contain fewer rays, according to the greater distance of the paper from the focus.

If the paper be moved farther and farther from the focus, the picture will become more and more indistinct, and at length totally vanish, no one part being distinguishable from the rest; because now the rays, even from remote parts, are mixed together. It is for a like cause, that the room must be dark; otherwise the foreign light mixing with the pencils that form the picture upon the paper, weaken that picture, till at last as the light grows stronger it becomes insensible. A picture will be perfectly formed, upon a paper placed in the focus of a lens, in the open field; but the paper being at least equally enlightened from all other parts, and its uneven surface reflecting the light it receives all manner of ways; the picture formed by the lens is quite obliterated, and nothing is seen but the paper itself.

246. *Schol.* 2. Why the image is inverted, is evident by a bare inspection of the figure; and it is manifest also, that this inversion is not owing to the lens: For if the lens be removed, and the light be admitted into the chamber, through a moderate hole of about an inch or less in diameter, we shall have on the opposite wainscot or wall, an inverted (tho' a very imperfect) picture of the objects without, after the same manner as when the light came through the lens upon the paper screen; the several pencils in both cases necessarily crossing each other, as well in the open hole, as at the lens. But without the lens, the picture will be very faint, and also confused: It is faint for want of sufficient light; there being now but very few rays for forming each point of the image, and these being reflected by the wall or paper all manner of ways, the several pencils that enter the eye are so rare, or contain so few rays, as to be scarce sensible: And the picture is confused, because the rays coming from the adjacent objects, interfere and mingle together; and the larger the hole, the greater will be this confusion.

§. 4. *Of the aberrations of rays, from the geometrical focus's of lens's.*

PROP. XXXVII.

Having the angle of incidence upon any given lens, of a ray parallel to the axis; to find the proper focus of the emergent ray.

It will be sufficient to trace the progress of the ray, through the sphere, and the three convex lens's; the method of tracing the ray through the concave lens's, being much the same. This admits of four

Fig. 98 *to* 102. cases: in each of which A*a* is the axis of the lens; DB is the incident ray parallel to the axis; RA = RB the radius of the side AB; and in *fig.* 101, 102. *ra* = *rb* is the radius of the side *ab*, and *rbk* is a strait line. Q is the geometrical focus of the first surface AB, *q* the proper focus of the ray after its first refraction by that surface; F the geometrical focus of the lens, and *f* the focus required. In each of these cases, the angle of incidence DB*d* (or its supplement) = ARB, is given.

Fig. 98. 247. Case 1. *Of a sphere.*

1. Say *m* : *n* :: Sine ∠ DB*d* (= ∠ RB*b*) : Sine ∠RB*b*; then draw B*b*[a], which is the refracted ray within the sphere, and produce it to *q*; through the point *b*, from the center R, draw the line R*bk*. Now the ∠ *kbq* = [b]R*b*B = [c]RB*b*. 2. Say *n* : *m* :: [a]Sine ∠*kbq* : Sine ∠ *kbf*; and draw *bf*. 3. To find the point *f*; in the △ R*bf* will be given, the radius R*b*, the ∠ *b*R*f* (= 180° — $\overline{\text{BR}b+\text{ARB}}$), and the ∠ R*bf* (= 180° — *kbf*), and consequently the ∠*f*; whence R*f*[d] is easily found. *Q. E. I.*

[a] Laws of refraction.
[b] 15. I. *Euc.*
[c] 5. I. *Eucl.*
[d] Trigonom.

Fig. 99. 248. Case 2. *A plano-convex lens, with its plane side towards the incident ray.*

In this case, the incident ray being perpendicular to the plane side of the lens, [a]it will pass on directly to B unrefracted; and therefore the ∠ RBD = [b]*d*B*k* is properly the angle of incidence. Wherefore, 1. Say *n* : *m* :: [a]Sine ∠ *d*B*k* : Sine ∠ *d*B*f*, and draw B*f*. Then 2. In the △ RB*f* will be given, the side RB, and all the ∠s, [c]to find R*f*. *Q. E. I.*

Fig. 100. 249. Case 3. *A plano-convex lens, with its convex side towards the incident ray.*

1. Say *m* : *n* :: [a]Sine ∠ DB*d* (= [b]RB*b*) : Sine ∠ RB*q*. Through the point *b* (where the ray B*q* cuts the side *ab*) draw *bk* parallel to AR; and produce DB to *h*. Then,

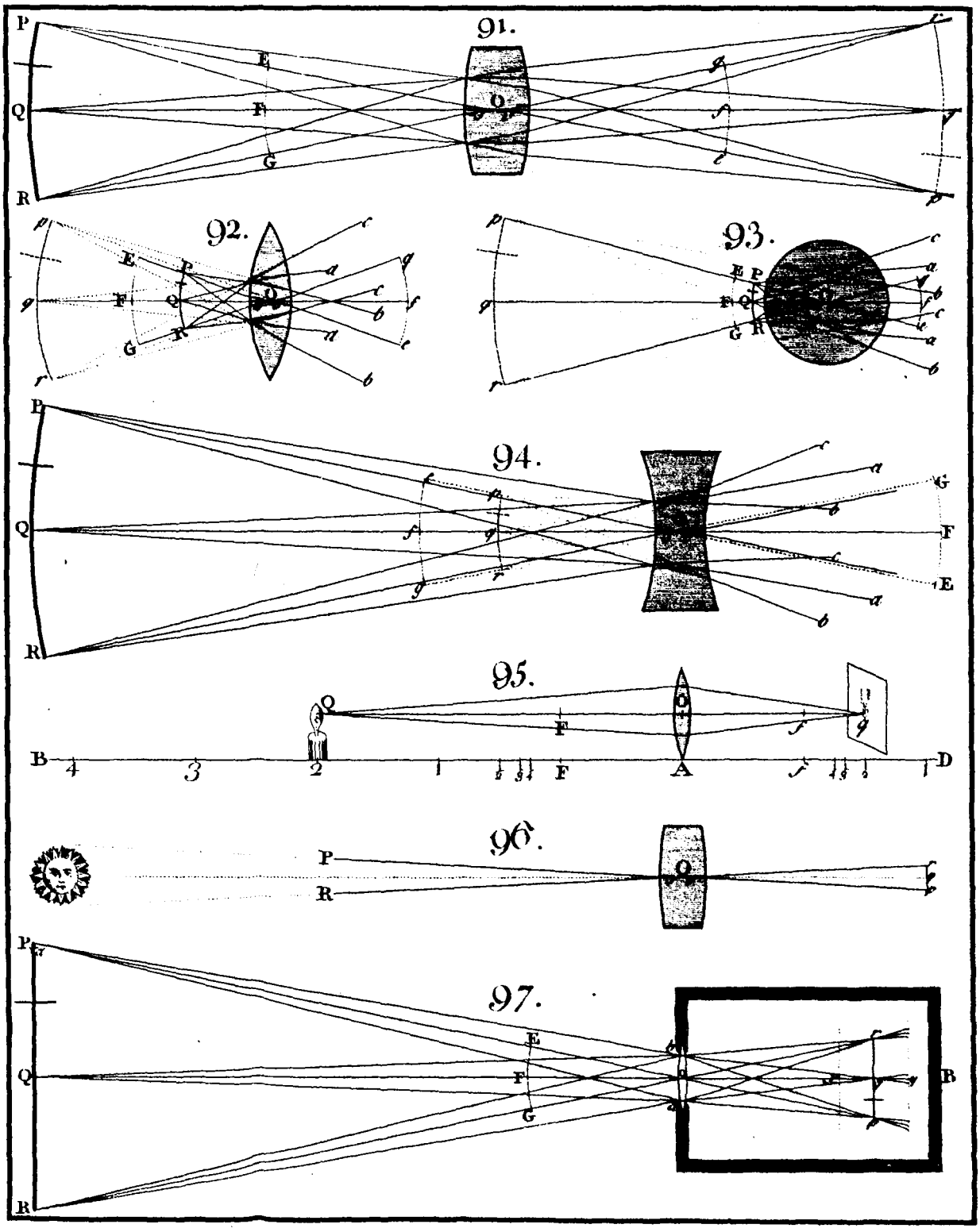
2. Say *n* : *m* :: [a]Sine ∠ *kbq* (= RB*h* — RB*q*) : Sine ∠ *kbf*; and draw *bf*. To find the point *f*. 3. In the right-angled △ B*hb* are given the side B*h* (= A*a* — A*v*) and the ∠ *b*B*h* (= RB*h* — RB*q*); [c]to find *hb*. 4. In the right-angled △ *baf*, will be given the side *ba* (= B*v* — *bh*), and the ∠ *abf* (= 90° — *kbf*); [d]to find *af*. *Q. E. I.*

Fig. 101, 102. 250. Case 4. *When both sides of the lens are spherical.*

1. Make as before, *m* : *n* :: [a]Sine ∠ DB*d* (= RB*h*) : Sine ∠ RB*b*, or RB*q*.

[a] 23. I. *Euc.* 2. In the △ *r*RB, are given the sides *r*R, RB, and the ∠ R (= [c]DB*d* or its supplement); to find the ∠s R*r*B, RB*r*, and the side *r*B.

3.


91.
92.
93.
94.
95.
96.
97.

3. In the $\triangle rBb$, will be given the sides rB, rb, and the $\angle rBb$ ($= RBq \pm RBr$); [d] to find the $\angle Brb$, and also the $\angle rbB = {}^{b}kbq$.

4. Say $n : m$ :: [a] Sine $\angle \frac{kbq}{rbq}$: Sine $\angle \frac{kbf}{rbf}$; and draw bf.

5. In the $\triangle rbf$, will be given the side rb, the $\angle brf$ ($= BrR \mp BrB$, or their supplement) and the $\angle rbf$; [d] to find rf. *Q. E. I.*

If a ray HG parallel to the axis, enters the concave side of a menis- Fig. 102.
cus lens; the point L where the emergent ray cuts the axis, and consequently the aberration LM, from the geometrical focus M, may, as above, be found thus:

1. Let it be $m : n$ :: Sine $\angle rGH$: Sine $\angle rGK$; and produce KG through Z to g; then will GZ be the refracted ray, within the lens.

2. In the $\triangle RrG$, are given the sides Rr, rG, and the $\angle r$; to find the $\angle$s rRG, rGR, and the side RG.

3. In the $\triangle RGZ$, will be given the sides RG, RZ, and the $\angle RGZ$ ($= 180° - \overline{KGr + rGR}$); to find the $\angle ZRG$, and also the $\angle RZG = eZg$.

4. Say $n : m$ :: Sine $\angle eZg$: Sine $\angle eZL$; and draw ZL.

5. In the $\triangle RZL$, are given the side RZ, and all the $\angle$s (*viz.* the $\angle R = 180° - \overline{ZRG + GRr}$, and the $\angle RZL = 180° - eZL$); to find the side RL. *Q. E. I.*

251. *Schol.* 1. Having the foci F, f; the longitudinal aberration Ff, Fig. 98 to 101.
of a given ray DB, is therefore given: And to find the lateral aberration FG; in the right-angled $\triangle fFG$, are given the side fF, and the $\angle FfG = Rfb$.

252. *Schol.* 2. The point of incidence B, was taken at some distance from the edge of the lens; because a lens let in a frame or cell for use, has a part, towards the edge, always covered. This makes no alteration as to the two first cases; because a whole sphere cannot be used, as a lens: And in the second case, the incident ray undergoes no refraction by the first side. But in the two last cases (*fig.* 100 to 102.) if the point B be taken at the edge, that is, if $ABvZ$ be supposed the lens, the calculation will be shorter. Thus,

CASE 3. 1. Say $m : n$ :: Sine $\angle DBd$: Sine RBq. Fig. 100.

2. Make $n : m$ :: Sine $\angle hBq$: Sine $\angle hBf$; and draw Bf.

3. In the right-angled $\triangle Bvf$, are given the side vB, and the $\angle f$ ($= hBf$); to find vf.

Fig. 101, 102. Case 4. 1. Say $m : n$ :: Sine $\angle$ DBd (=RBh) : Sine $\angle$RBq.

2. In the $\triangle$ rBR, are given either the three sides, or two sides and the $\angle$ R; to find the $\angle$ RBr (whose supplement, *fig.* 101. is lBR) and the $\angle$ RrB.

3. Say $n : m$ :: Sine $\angle \begin{matrix} l\mathrm{B}q = l\mathrm{BR} - \mathrm{RB}q \\ r\mathrm{B}q = \mathrm{RB}q - \mathrm{RB}r \end{matrix}$: Sine $\angle \begin{matrix} l\mathrm{B}f \\ r\mathrm{B}f \end{matrix}$.

4. In the $\triangle$ rBf, will be given the side rB, and all the $\angle$s (*viz.* the $\angle r$, and the $\angle$ rBf (=180—lBf, *fig.* 101.) to find $r f$. *Q. E. I.*

253. *Schol.* 3. Supposing the sines of incidence and refraction from glass to air, to be as 3 to 2; the following table (if no mistake crept into the calculation, which 'twas difficult to avoid) gives the aberrations of the lens's therein specified, in such parts that the focal length of each lens is 100,000 nearly; and the several apertures that correspond, are also the same nearly; that is, making the focal length of each lens Radius, the corresponding angles of incidence will be nearly the same; or AB in each lens, and ab in the sphere, will be nearly of the same breadth.

Fig. 98.		*Fig.* 99.			*Fig.* 100.		*Fig.* 101.	
RF=100,000		AF=100,000			RA=50,000		RA=rv=100,000	
$\angle a$Rb	Ff	$\angle$ ARB	Ff	FG	$\angle$DBd	Ff	$\angle$DBd	Ff
° ′		° ′			° ′		° ′	
3 : 0 +	697.6	4 : 0	548.63	19.217	4 : 0	142.61	2 : 0	203.41
1 : 30+	173.93	2 : 0	135.2	2.36	2 : 0	35.867	1 : 0	50.33
0 : 45+	44.33	1 : 0	34.1	0.297	1 : 0	8.888	0 : 30	11.9
0 : 22½	11.31	0 : 30	8.1	0.035	0 : 30	2.295	0 : 15	3.07

Note, In computing the aberrations of the two last lens's, I neglected their thicknesses; that is, I supposed each ray to enter at the edge of the lens.

254. *Schol.* 4. By the method shewed in the preceding proposition, may be found the exact quantity of the aberration of parallel rays, from the geometrical focus of any given lens, having a given aperture. Equations have been also formed for this purpose; but as the method of raising them is unavoidably tedious, and their application very laborious, on account of their involving a great many terms; and aberrations from the figure, are but of small moment, in comparison of the aberrations on account of the different refrangibility of light; as will be seen in the next section. I shall therefore omit inserting these equations,

tions here; but shall take the liberty of borrowing from * M. *Huygens*, (who, I think, was the first that took the trouble of making the computation,) the proportions which the longitudinal aberrations of the extreme rays parallel to the axis, nearly bear to the thicknesses of the following glass lens's; the point where they enter, being supposed not very remote from the vertex, and also in the edge of the lens; as the point B in the lens ABvZ, *fig.* 99, to 102.

255. When the incident ray falls upon the plane side of a plano-convex glass, $Ff = \frac{2}{3} Av$, nearly. This has been already demonstrated in *Art.* 138.

256. When the incident ray falls upon the convex-side of a plano-convex glass, $Ff = +\frac{7}{6} Av$, nearly. And therefore the longitudinal aberration of a plano-convex glass, is near four times less in this case than in the former; being as $\frac{14}{12}$ to $\frac{56}{12}$, or as 7 to 27.

257. When the glass is a double convex, of equal convexities, $Ff = \frac{5}{3} Av$ nearly ‖. And therefore the aberration of this glass to that of a plano-convex in its best position, is as $\frac{5}{3}$ to $\frac{7}{6}$, or as 10 to 7.

258. A double convex glass having the radius of the first surface upon which the ray falls, to the radius of the second surface, as 2 to 5; is just as good as the plano-convex in its best position, it being in both $Ff = \frac{7}{6} Av$.

259. A double convex, having the radius of its first surface, to that of the second, as 1 to 6, is the best glass of all; and its aberration $Ff = \frac{15}{14} Av$. But if the other side of this glass be exposed to the parallel rays, Ff will be $= \frac{145}{42} Av$.

260. A plano-convex glass, with its $\left\{\begin{matrix}\text{convex}\\ \text{plane}\end{matrix}\right\}$ side, towards the incident parallel rays, has less aberration than any meniscus, with its $\left\{\begin{matrix}\text{convex}\\ \text{concave}\end{matrix}\right\}$ side exposed to parallel rays. Whence it necessarily follows, that that meniscus is best, which approaches nearest in shape to a plano-convex lens.

261. The above proportions of Ff to Av, will be also the same in concave glasses under like circumstances; Av being now taken at the circumference of the lens. See *Art.* 137 & 199. I.

* *Huygens*'s Dioptrics, Prop. XXVII. See also Dr. *Smith*'s Optics, Book II. Chap. 13.

† This may be proved from *Art.* 138, 61. For by *Art.* 138. the aberration Qq of the first surface is $= \frac{4}{3} Av$; this then will cause an aberration from $F = \frac{2}{3}$ of $\frac{4}{3} Av$ nearly $= \frac{8}{9} Av$. And by *Art.* 61. the aberration Ff of the plane surface, if there had been no aberration caused by the first surface, would have been (by *Art.* 58.) $= \frac{5}{18} Av$ nearly (because QA is thrice RA); by adding these together we have ($\frac{11}{18} Av =$) $\frac{7}{6} Av$ for the aberration Ff.

‖ So far reaches the Table, *Art.* 253, with which the above numbers agree nearly.

262. *Schol.* 5. Which ever ſide of a lens is beſt when expoſed to parallel rays, that ſide will be the worſt when expoſed to rays diverging from a focus, near the principal focus of the lens: And *vice verſa*. This is ſufficiently manifeſt of it ſelf.

PROP. XXXVIII.

263. *Having the focus of a ray, either diverging or converging upon a given point, of any given lens; to find the focus of the emergent ray.*

It is needleſs to have figures on purpoſe to illuſtrate this propoſition; but in thoſe laſt referred to, in the axis produced, we may ſuppoſe ſome point Y, to be the focus of the incident ray. The propoſition may be reduced into two caſes.

Fig. 98, 100, 101, 102. Case 1. *When the incidence is upon a ſpherical ſide.*

Let B be the point of incidence, and in the △ YRB will be given the ſides YR, RB, and the ∠R, to find the angle of incidence Y; then the reſt of the proceſs will be as in the preceding propoſition.

Fig. 99. Case 2. *When the incidence is upon the plane ſide of a plano-convex lens.*

Let the point of incidence be *h*; and in the right-angled △ Y*ah*, will be given the two ſides Y*a*, *ah*; to find the ∠ of incidence Y. 2. Find the refracted ∠ at *h*, and ſuppoſe for inſtance the point O to be the focus of the ray refracted at *h*; to find the point O, in the right-angled △ *ha*O will be given the ſide *ah*, and the ∠ *h*; to find *a*O. Let B be the point where the ray emerges out of the lens: In the △ RBO will be given the ſides RB, RO, and the ∠ O; to find the ∠ RBO, whoſe ſupplement is the ∠ *d*BO; then ſaying *n* : *m* :: Sine ∠ *d*BO : Sine ∠ *d*B*f*, we may draw the ray B*f*. Laſtly, in the △ RB*f* will be given, the ſide RB, and all the ∠s, to find R*f*. *Q. E. I.*

PROP. XXXIX.

Fig. 99 *to* 102. 264. *In every lens having different apertures, the longitudinal aberrations* (Ff) *of the extreme rays parallel to the axis, are as the ſquares, and their lateral aberrations* (FG) *as the cubes, of the diameters of thoſe apertures, nearly. Or, the longitudinal and lateral aberrations of parallel rays, incident at different diſtances from the axis of any lens, are reſpectively as the ſquares and cubes of theſe diſtances, nearly.*

[a] 140. I. This has been already demonſtrated of [a] ſingle ſurfaces, and conſequently alſo of plano-convex and concave lens's, with their plane ſides expoſed to the incident rays. And from what has been ſhewed in *Art.* 60, 136, it may be gathered, that in all lens's, the longitudinal aberrations *

tions * are in ſome conſtant ratio to Av the thickneſs of the lens, at leaſt nearly; and Av [b] is as the ſquare of the breadth BZ nearly. [b] 36. I. Therefore, *&c.*

Again, whilſt the aperture BZ remains the ſame, FG will be in the ſimple ratio of Ff, that is as BZ^2; but as the aperture BZ is increaſed or diminiſhed, FG at the ſame time increaſes or diminiſhes, not only as Ff, but moreover in the ſimple ratio of the ſine of AB; that is in the whole, as the cube of the ſine of AB, or as the cube of the chord of BZ, nearly. *Q. E. D.*

265. *Schol.* 1. In different lens's, that have the ſame apertures and focal lengths; the lateral aberrations of parallel rays in each, are nearly as the longitudinal ones: But in that lens which has the greateſt aberration, the ratio of the lateral aberration to the longitudinal one, is the greateſt. Let OF, *fig.* 99. be = fF, *fig.* 100. then will Fp in the former be = FG in the latter; vB and vF in each being ſuppoſed reſpectively equal. Now, *fig.* 99. the ratio of GF to Ff, is greater than the ratio of pF to FO, becauſe the ∠ O is leſs than the ∠ f, and the ∠ F in both △s is common. Therefore, *&c.* *Q. E. D.*

266. *Schol.* 2. Now it is manifeſt that no lens can gather all the rays into one point; and that the focus's of all the rays, as they are more and more remote from the axis, will lie orderly nearer and nearer to the lens, than F. Nevertheleſs it is manifeſt alſo, that F or a point very near to F, is the burning focus of the lens, or that place where the rays are denſeſt of all.

Let the arches Aa, ab, bc, cB, cC, on each ſide of the axis AF, intercepted between the parallel rays 0, 1, 2, 3, 4 at their emerſion out of the plano-convex lens BAC, be all equal; and let F be the geometrical focus of the lens, and f the focus of the extreme rays 4, 4. At the point F, upon the axis AF, raiſe on each ſide the perpendiculars FG, FH. Divide Ff into as many equal parts, as the ſquare of the number of equidiſtant rays on each ſide the axis, contains units; that is, in this example divide Ff into 16 equal parts; and from the point F towards f, lay off the ſquares of the numbers 1, 2, 3, *&c.* reſpectively; that is, let F1 be = 1, F2 = 4, F3 = 9, and F4 or Ff = 16; and thro' the points 1, 2, 3, f draw the rays a1, a1; b2, b2, c3, c3; Bf, Cf, and produce them till they meet the line GH; then will the lines FG, FH be divided at the points 1, 2, 3, 4, as the cubes of theſe numbers, nearly; that is F1, F2, F3, F4, will be as the numbers 1, 8, 27, 64, nearly. Or if GH be firſt divided after this manner, rays drawn to the reſpective points therein will divide Ff as above, nearly. Fig. 103.

267.

* See the Table, *Art.* 253.

267. *Schol.* 3. The rays refracted by the lens, will be ſome of them ſpread all over a circle, whoſe diameter is GH; and as they increaſe in denſity from G and H towards F, *ad infinitum*; and from F decreaſe in the ſame manner, ſo as at G and H the denſity becomes nothing; at the center F, they will be infinitely denſe; and at the extremities G, H, they will be infinitely rare, and therefore cannot there be ſenſible. Let
Fig. 104. the line GH, *fig.* 104. be divided in the ſame manner with GH, *fig.* 103. And from the center F, deſcribe the circles 1F1, 2F2, 3F3, 4F4. Now the light contained in the ſmalleſt circle, is equal to that which is contained in the ſpace between the peripheries of any two adjacent circles; and the light contained in the circle whoſe diameter is 2F2, is equal to all the reſt: Whence it ſeems probable, that in a lens having a large aperture, the diameter of the greateſt circle of aberration that is ſenſible cannot much exceed 2F2; at leaſt I think we ſhall be much within compaſs if we aſſume this diameter not leſs than $\frac{1}{4}$ GH.

Fig. 103. 268. *Schol.* 4. The leaſt circle, through which all the rays refracted by any lens can paſs, has for its diameter a certain line *gh*, drawn between the remoteſt points *g*, *h* from *f*, where the extreme rays cut any of the other rays on contrary ſides of the axis. If the refraction be by a ſingle ſurface, or by a plano-convex lens with its plane ſide expoſed to parallel rays, the line *gh* will be = * $\frac{1}{4}$ GH. Alſo the diameter of the leaſt circle, through which all the parallel rays, reflected
Fig. 24. by a concave ſpherical ſpeculum, can paſs, will be = † $\frac{1}{2}$ FG.

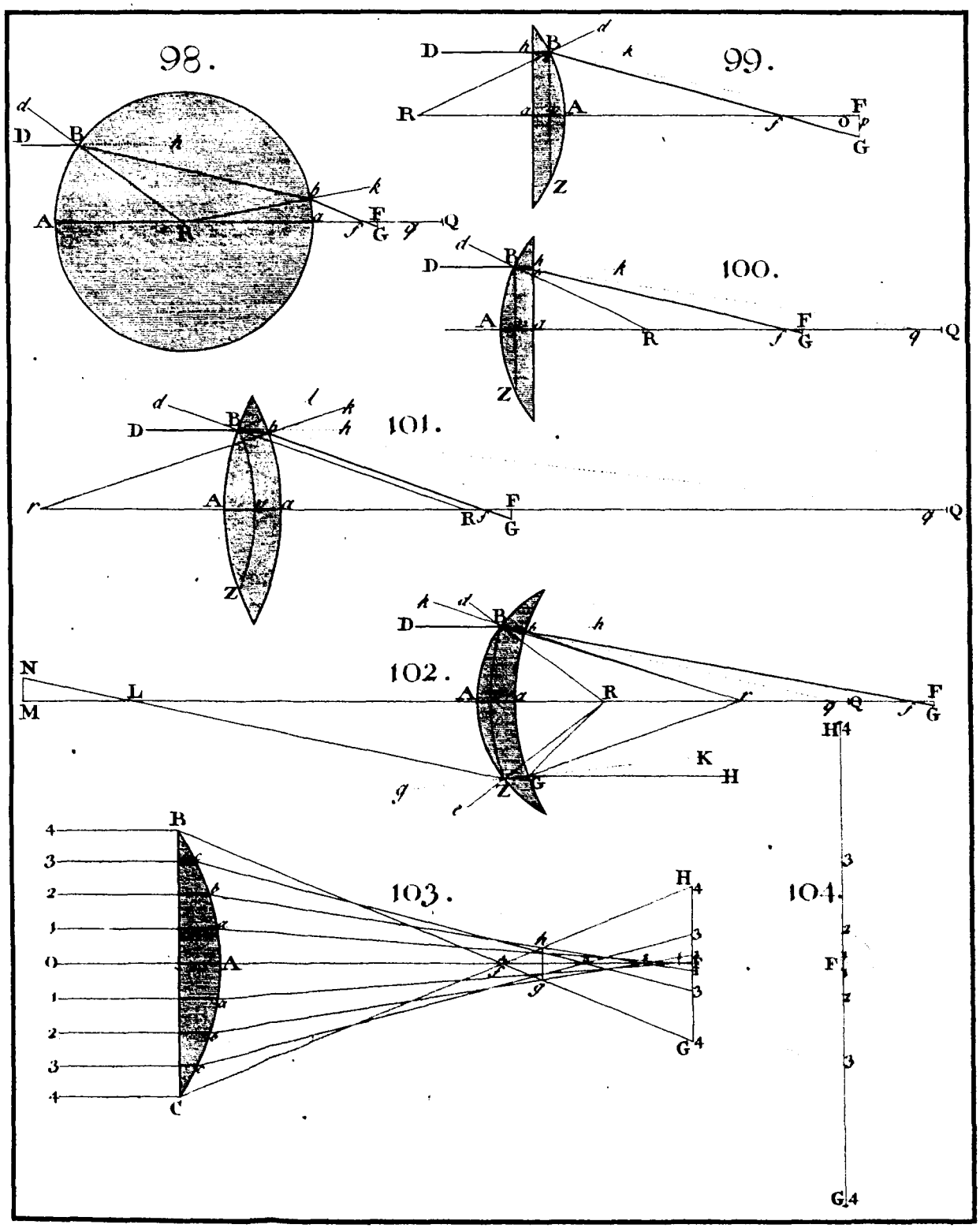
PROP. XL.

269. *If an incident ray that is not very oblique to the axis of any lens, be conſidered as having its focus in the point where it interſects that axis; or as belonging to the oblique pencil, whoſe axis is parallel to the ſaid ray; the lateral aberrations of the emergent ray in both caſes, will be as the diſtances of its focus's from the lens, nearly.*

Let F*f* produced be the axis of the lens, Q the focus of an incident
Fig. 105, 106. ray, either diverging as QB, or converging as MB, to the point B. Parallel to QB draw the axis O*b*. Let *q* be the geometrical focus of emergent rays, whoſe focus of incidence is Q; *b* the geometrical focus of rays parallel to O*b*; and let B*y* produced be the emergent ray. Upon the axis's O*q*, O*b* raiſe the perpendiculars *qt*, *bs*, till they meet the emergent ray. I ſay the lateral aberrations *qt*, *bs* are as the diſtances from the lens B*q*, B*b*, or O*q*, *of*, nearly; that is, *qt* : *bs* :: O*q* : *of*; nearly.

In the △s B*bs*, B*qt*; had the ∠s *b* and *q*, or *s* and *t* been equal, it would have been exactly *qt* : *bs* :: B*q* : B*b*. But in theſe △s, the ∠ B being very

* *Newton*'s Optical Lectures, page 184, 185. † Dr. *Smith*, ibid. Alſo Dr. *Smith*'s Optics, *Art.* 339.


98.
99.
100.
101.
102.
103.
104.

very acute, and the ∠s q,b being not very oblique; the inequality of these ∠s can create but a very small and imperceptible error in the above proportion of Bq : Bb :: qt : bs. And the nearer B is to O, the less will that error be; also in that case, the nearer will Bq, Oq, and Bb, Of be to a ratio of equality. Therefore, &c. *Q. E. D.*

270. *Schol.* 1. From what has been before shewed about the aberrations of single surfaces, it is manifest that this proposition is universal for them also; and likewise for concave, as well as convex lens's. See *fig.* 43 to 48.

271. *Corol.* When Oq is greater than Ob or Of; the ratio of qt to bs, is less than the ratio of Bq to Bb. And when Oq is less than Of, the ratio of qt to bs, is greater than the ratio of Bq to Bb.

272. *Schol.* 2. In a lens having a moderate aperture, the aberration bs, is nearly equal to the aberration fv, of a ray NB parallel to the axis. For in the △s Bbs, Bfv, the ∠s bBs, fBv are nearly equal; and so are also the other correspondent ∠s, and the sides Bb, Bf are also nearly equal. Therefore, &c.

273. In order to avoid the aberrations, arising from the spherical figures of lens's, several of the * writers of dioptrics, before Sir *Isaac Newton* published his optical discoveries, had bestowed much pains in investigating the figures of lens's, that would refract a direct pencil of (homogeneal) rays, having a given focus, into a point, without any aberration. But these authors, not regarding the oblique pencils, their † *theoretical lens's*, if they really could have been made, would not have answered all purposes better than those we now have; and upon the whole they would not have been so good. For if an object of some breadth be placed in the given focus, the extremes of the object seen through such a lens, would be far less distinct than the middle part; and this indistinctness would increase very fast, as the object subtended a larger angle; and in common lens's, we do not so much want to correct the aberrations of the direct pencils, as we do to correct those of the oblique pencils. Again, if the focus of incident rays be changed, that

* Monf. *Huygens* in particular, seems to have bestowed more pains upon Dioptrics, and to have treated that science in a more elegant manner, than any author before him. But this truly great man no sooner saw Sir *Isaac Newton*'s dissertations about the different refrangibility of light, than he ingenuously embraced this new theory, and laid aside all farther thoughts of improving refracting telescopes, which was his principal aim, by correcting the figures of their glasses.

† Sir *Isaac Newton* hath shewed (*Newton*'s Optics, 8vo *English* edit. page 90.) how to make a better lens for all purposes, than can be made of one piece of glass, by cementing two like concavo-convex glasses, with water between them, together; the radii of the sides of each glass having a certain ratio to each other. A contrivance somewhat like this, had been also thought of by Dr. *Hook*, before Sir *Isaac Newton*. But as the aberrations caused by the figures of common lens's, are altogether inconsiderable when compared to the aberrations from the different refrangibility of light; this contrivance will not answer the pains, as Sir *Isaac* himself hath shewed.

that is, if an object be placed nigher to, or farther from the lens, than the focal distance for which it was formed, there would be then none of the pencils of light, that came from the object, free from aberrations after refraction by the lens; and a spherical lens in this case also, would do better than the other. So that upon the whole, spherical surfaces, or common lens's, as well for the reasons above assigned, and also because they are the easiest formed, may be reckoned preferable to any other.

SECTION V.

Of the aberrations caused by the different refrangibility of the rays of light.

Rays of light differently refrangible; discovered by Sir *Isaac Newton.*

274. FROM the common *phænomenon of a distorted coloured image produced by a glass triangular prism, and from several experiments devised by himself; Sir *Isaac Newton* hath clearly demonstrated, that the rays of light are not all equally refrangible: that is, rays of light impinging any refracting surface, at the same degree of obliquity from the perpendiculars to that surface at the points of incidence, will not be all equally bent or refracted towards or from the said perpendiculars; but some will be more, and others less refracted, and that in all possible degrees between the limits of the least and most refrangible rays.

Cause of colours, discovered also by the same author.

275. The same great author also discovered, that the cause why bodies appear of different colours, is their reflecting some of the differently refrangible rays, more copiously than others; the most refrangible rays producing *violet*, and the least refrangible rays *red* colours. And so others through all the intermediate degrees of refrangibility, producing all the varieties of different colours. And hence, the peculiar colours of particular bodies, are owing to some peculiar textures or properties of these bodies, that dispose them to reflect such rays of light, as are suited to produce the idea of such and such a colour.

276.

* The prism was in many hands, and the phænomenon above well known, but not the cause of it, before Sir *Isaac Newton*'s time. There is no meddling with the subject of this section, without borrowing from Sir *Isaac Newton*'s Optics; but I shall be very brief in my extracts from it, and refer the reader to the book itself, in which are many curious and surprizing discoveries about light, which do not immediately concern us here. And of all Sir *Isaac*'s great performances, his Optics is justly esteemed as one of the chief. That Edition I refer to, is the *English* Octavo.

276. The different refrangibility of light, is a great impediment to the perfection of optical instruments. But this disadvantage is infinitely outweighed, by the inestimable benefits accruing from the diversities of colours; these, by their variety, not only contribute greatly towards the beauty of the material world, but also help us to distinguish bodies and things one from another. And it is almost demonstrable, that if all the rays of light had been perfectly alike in their properties, all the bodies in the universe would have appeared of one colour.

277. After what manner the ideas of different colours are produced in the mind, is a proposition as far above our reach, as how we have the ideas of different smells, tastes, *&c.* But it cannot be doubted, that the impulses of light upon the optic nerve, which excite the ideas of different colours, must be themselves different; and this difference must arise from a difference in the constitution of light itself. And as those rays that are the least refrangible, undergo the least change by the refraction of bodies; it seems probable that their impetus should be greater than that of rays which are more refrangible: And this greater impetus of the least refrangible rays may proceed, 1. from the particles of these rays being larger than those of the other rays; or, 2. these particles having a greater density, their magnitudes being the same with those of the rest; or, 3. from their having a greater velocity; or lastly, from these several causes combined together.

278. *Defin.* Rays of light that are alike refrangible, are called *homogeneal rays*; also *simple*, and *similar rays*: And those rays which differ in their refrangibility, are called *heterogeneal*, *compound*, or *dissimilar rays*.

279. *Phenom.* If the sun's rays admitted into a dark chamber, through a small round hole O in the window-shutter, be intercepted by a triangular glass prism BAC, and the prism be turned * slowly round its axis until the sun's image be thrown on the opposite side of the room; instead of a round image, there will be formed an oblong one PT, terminated with two rectilinear and parallel sides, and two semi-circular ends. The sides will be pretty well defined, but not the ends, the light there decaying and vanishing by degrees. The breadth of the oblong image, answers to that of the sun's image at a like distance from the hole, when its rays are not refracted; and its length being much greater, shews that the rays which go to T, are not so much refracted as those which go to P; and so all the rays that fall from T towards P, have greater degrees of refrangibility, as they are more remote from T. So the oblong Fig. 107.

L

* See this experiment, with all the minute circumstances that are necessary in order to make it succeed best, described at length in Sir *Isaac Newton*'s Optics, p. 21, *&c.* The short account above is sufficient here, being only intended as an introduction to what follows.

Fig. 108. long image PT is compoſed, as it were, of innumerable circles (*p*, *q*, *r*, *s*, *t*, &c.) each anſwering to the ſun's image, and whoſe centers lie all in a ſtrait line betwixt T and P. And hence it is, that the ſides of PT are rectilinear, and well defined, and the ends ſemi-circular and faint.

Fig. 107. 280. The ſpectrum PT conſiſts, as the phraſe is, of all the colours of the rainbow, all orderly ranged, as expreſſed in the figure. The dotted lines ſhew the confines of the colours, all the ſpace between them, oppoſite to the name, being allotted to ſuch a colour. But the colours are not totally ſeparated in any one part, but contiguous, and running into one another. For example, the ſpace allotted to red, contains innumerable degrees of red; the laſt of the red being not diſtinguiſhable from the firſt of the orange, *&c.*

281. The number of primary colours ſeems to be altogether arbitrary: Sir *Iſaac Newton* was pleaſed to diſtinguiſh them into ſeven, *viz. violet*, *indigo*, *blue*, *green*, *yellow*, *orange*, *red*, as expreſſed in the figure; out of theſe *white* and *black*, vulgarly called colours, are excluded: Whiteneſs ariſing from a mixture of light of all colours in equal proportion; and blackneſs from a reflection of a very ſmall quantity of a mixture of rays of all ſorts. The ſpace PT is divided (by the dotted lines ſeparating the colours) as a muſical chord, agreeable to Sir *Iſaac Newton*'s method: See his Optics, page 110. The circular ſpaces at the ends repreſent a little faint penumbra.

282. † Rays of the ſame colour, have always the ſame degree of refrangibility. And the colour exhibited by any ray ſeparated from the reſt, always continues invariably the ſame; however that ray may be refracted or reflected, ſo long as it is not blended with other coloured rays: For inſtance, a red ray always exhibits redneſs, and never any other colour; and a green ray always retains its greenneſs, *&c.* In like manner, all ſorts of rays mingled together, always exhibit whiteneſs, whether theſe rays are parallel, condenſed in a focus, diverging or converging, ſo long as they are not ſeparated aſunder. But if any of the coloured rays are wanting in the mixture, whiteneſs will not be produced; and the colour will be different, according to which rays are moſt predominant. And hence, all the varieties of colours of natural bodies, proceed from the different proportions in which they reflect ſome, and abſorb others of the different coloured rays of light. And when moſt of the rays are abſorbed, and but few reflected, ‖ blackneſs is

† See *Newton*'s Optics, Book I. Part II. Alſo Book II. and III.

‖ There ſeems to be ſomething in the conſtitution of black, beſide a mere deficiency of light; for bodies will retain their colours, in very different degrees of light. However black is not one of the priſmatic colours; and perhaps it may proceed from ſome kind of mixture of ſome peculiarly coloured rays, when others are totally abſorbed, or reflected a different way.

is exhibited. So that colours are not in bodies, for they will appear of any colour, if they be enlightened only with rays of ſuch a peculiar colour; but they are ideas excited by ſome difference in the impulſes upon our optics, of the differently refrangible rays *.

283. If the round hole in the window-ſhutter enlightened only by the clouds, be ſeen by rays refracted by a priſm, its image inſtead of appearing round will be oblong, and coloured like the ſun's image, as before deſcribed. And univerſally, the light reflected from all natural bodies, when refracted in the ſame manner, will have like effects. Alſo priſms made of water (by confining it between three glaſs plains cemented together,) or any other tranſparent medium hitherto known, will have like effects; only the length of the coloured image will differ, according to the different degrees of the refrangibility of the priſm.

284. Let RS be any refracting ſurface, ſuppoſe of glaſs, PO a perpendicular thereto at the point B, and let the line DB make an oblique angle with PB. If the ſolar rays be conceived to flow along DB, continually ſucceſſive to one another, ſo that they may one after another impinge on the point B, and there be refracted into † a denſe medium; or, if DB be ſuppoſed an indefinitely ſlender pencil of parallel rays, falling on the refracting ſurface at the points neareſt to B; theſe rays after being refracted, inſtead of keeping cloſe, as it were, in one line, as before, will diverge from one another into different lines B*p*, B*q*, B*r*, B*s*, B*t*, &c. and into innumerable others through the intermediate ſpaces, according as each ray is ſuited to ſuffer a greater or leſs refraction. The rays B*p* that are refracted the moſt produce purple colours, and thoſe B*t* refracted the leaſt produce red colours, and all the intermediate rays produce different colours ‖. Fig. 109.

285. *Schol.* If RS be a plane ſurface, and the medium be terminated by another plane ſurface TV parallel to the former; the rays after both refractions will emerge in lines [a] parallel to the incident ray DB. [a] 67. I.

286. In like manner, if DB be ſuppoſed a very ſlender pencil of parallel rays, incident upon any refracting ſurface or lens BA*b*, at points infinitely near to B; theſe rays after refraction will interſect the axis AF of the lens, at different points F, *f*; that is, the moſt refrangible rays will have their focus at *f*, nearer to the lens; and the leaſt refrangible rays, will have their Fig. 110, to 114.

 focus

* See 277. I.

† The caſe is the ſame when the incidence is from a denſe into a rare medium, only the moſt refrangible rays will be farthest from the perpendicular.

‖ If the different ſorts of coloured rays follow one another in the ſame line, their velocity is ſo exceeding great, that no interruption between the different ſorts can be perceived. And if they come together like threads, they are ſo cloſely united and prodigious fine, that the ſmalleſt perceivable thread or pencil, or what we would call a ray of light, conſiſts of an indefinite number of different ſorts of coloured rays.

focus at a point F, more remote from the lens: and innumerable other rays will interſect the axis in different points, all lying orderly between F and *f*, according as thoſe rays differ in degrees of refrangibility.

287. The ſines of incidence and refraction of all ſorts of rays *, are in a conſtant ratio; that is, whatever is the ratio betwixt theſe ſines, at any one degree of incidence of any particular ray, there will be the ſame ratio between them, at all other degrees of incidence of the ſame ray; at leaſt, as exactly as can be diſtinguiſhed by experiments.

288. If the common ſine of incidence from glaſs to air is 50 †, the ſines of refraction of the leaſt and moſt refrangible rays will be 77 and 78 reſpectively. Whence in ſmall refractions (that is, ſuch that the angles and their ſines are nearly proportional) the refraction of the leaſt refrangible rays, is to the refraction of the moſt refrangible rays, as 27 to 28 nearly; or [a] $\angle EBF : \angle EBf :: 27 : 28$, nearly. And the difference of the refractions of the leaſt refrangible and moſt refrangible rays, is about $\frac{1}{27\frac{1}{2}}$ or $\frac{2}{55}$ of the whole refraction of the mean refrangible rays.

[a] Fig. 110.

289. By dividing the difference between 77, 78, the ſines of the leaſt and moſt refrangible rays, in the ſame manner as the *ſpectrum* PT, *fig.* 107. is divided by the dotted lines; Sir *Iſaac Newton* ‖ computes the ſines of refraction from glaſs to Air, of the ſeveral ſorts of rays contiguous to theſe dotted lines, to be $77, 77\frac{1}{8}, 77\frac{1}{5}, 77\frac{1}{3}, 77\frac{1}{2}, 77\frac{2}{3}, 77\frac{7}{9}, 78$; their common ſine of incidence being 50; that is, the ſines of incidences of all *red-making rays* out of glaſs into air, are to the ſines of their refractions from 50 to 77, and 50 to $77\frac{1}{8}$. The common ſine of incidence being 50, the ſines of refractions of the *orange-making rays* are from $77\frac{1}{8}$ to $77\frac{1}{5}$; thoſe of the *yellow-making* from $77\frac{1}{5}$ to $77\frac{1}{3}$; thoſe of the *green-making* from $77\frac{1}{3}$ to $77\frac{1}{2}$; of the *blue* from $77\frac{1}{2}$ to $77\frac{2}{3}$; of the *indigo* from $77\frac{2}{3}$ to $77\frac{7}{9}$; and thoſe of the *violet* from $77\frac{7}{9}$ to 78.

290. If the above numbers be made the ſines of incidences from air to glaſs, of the ſeveral ſorts of rays reſpectively; [a] the ſine of refraction of them all will be 50.

[a] 13. I.

PROP.

* *Newton*'s Optics, p. 64 to 70. & p. 82.
† *Newton*'s Optics, p. 73.
‖ Ibid. p. 111.

PROP. XLI.

291. *If heterogeneal rays or rays of all sorts are incident upon the spherical surface of a rare medium, whether it be concave or convex*, or upon the plane side of a plano-convex lens, parallel to the axis and not † very remote from it;*

The focal distance from the refracting surface or lens of the least refrangible rays, will be

To the focal distance of the most refrangible rays;

As the difference between the sines of incidence and refraction of the most refrangible rays,

To the difference between the said sines of the least refrangible rays.

Let O be the center of the spherical refracting surface BA, OF a line perpendicular to that surface, and also to the plane surface Bvb; DB a line parallel to the axis OF, along which let the incident rays be supposed to flow; F the focus of the least refrangible rays, and f the focus of the rays that are most refrangible. Let the greatest of the sines of incidence and refraction of the most refrangible rays, be called M; and the greatest of the said sines of the least refrangible rays, be called L; and the least of the sines of incidence and refraction of both sorts of rays call I. I say AF : Af :: M—I : L—I. Fig. 110.

For L—I : I :: [a]OA : AF; and M—I : I :: [a]OA : Af. Therefore M—I : L—I :: AF : Af. *Q. E. D.* [a] 125, & 127. I.

292. *Corol.* 1. $M-L\,(\overline{M-I}-\overline{L-I})\,{}^{M}_{L}-I :: fF\,(AF-Af)\,{}^{AF}_{Af}$.

293. *Corol.* 2. If the refraction be between glass and air, it will be AF : Af :: 28 : 27.

For M—I : L—I :: [b]78—50 : 77—50 :: (28 : 27) AF : Af. Whence also the longitudinal aberration $fF = \frac{1}{27}AF = \frac{1}{27}Af$. [b] 288. I.

PROP.

* See *fig.* 41, 42.

† This limitation is necessary, to prevent the aberration from the figure affecting the truth of the proposition.

PROP. XLII.

294. *If heterogeneal rays are incident upon a spherical surface, whether convex or concave*, of a dense medium, parallel to the axis and not very remote from it; or upon the plane side of a plano-concave lens with like restrictions, it will be,*

As the distance of the focus of the least refrangible rays from the center of the spherical refracting surface,

Is to the distance of the focus of the most refrangible rays from the said center;

So is the difference between the sines of incidence and refraction of the most refrangible rays,

To the difference between the said sines of the least refrangible rays.

Fig. 111. That is, retaining the same references as in the preceding proposition, $OF : Of :: M—I : L—I$.

[a] 125 & 126. I. For $AO : OF ::$ [a] $L—I : I$; and $AO : Of ::$ [a] $M—I : I$; Therefore $OF : Of :: M—I : L—I$. *Q. E. D.*

295. *Corol.* 1. The refracting mediums and the radii AO, *fig.* 110, 111. being the same, the longitudinal aberrations fF of heterogeneal rays in both cases will be the same.

[b] 126 & 127. I. For AF and Af, *fig.* 110. are [b] equal respectively to OF and Of, *fig.* 111.

Fig. 111. 296. *Corol.* 2. $OF : Of :: \frac{M}{L} AF : Af$. For,

[c] 126. I. $M—I : M ::$ [c] $AO : Af$;

And $AO : AF ::$ [c] $L—I : L$. Wherefore

$M—I : L—I :: M \times AF : L \times Af$. Therefore, *&c.*

297. *Corol.* 3. Because the ratio of AF to Af, is less than the ratio of OF to Of; the ratio of fF, to AF or Af, *fig.* 110. is greater than the ratio of fF to AF or Af, *fig.* 111. And if the medium, *fig.* 111. is glass, it will be $28 : 27 :: 78\,AF : 77\,Af :: OF : Of$; or $AF : Af ::$ $(28 \times 77 : 27 \times 78 ::)$ $2156 : 2106$; whence fF is about $= \frac{1}{44}$ AF.

PROP.

* See *fig.* 39, 40.

PROP. XLIII.

298. *If heterogeneal rays, are incident upon any lens whatsoever, whose thickness is very small, parallel to the axis, and not very remote from it;*

The focal distance from the lens of the least refrangible rays,

Will be to the focal distance of the most refrangible rays;

As the difference between the sines of incidence and refraction of the most refrangible rays,

To the difference between the said sines of the least refrangible rays.

Let the focal distance after refraction, of the parallel rays that are the least refrangible, be called F; and of those that are most refrangible, be called f; and let L, M, I signify the same as in the two preceding propositions.

I say $F : f :: M - I : L - I$, very nearly.

CASE 1. Let P and p be the foci after refraction, of the least and most refrangible rays, whose incidences were on the plane side of a plano-convex lens; then it will be accurately, Fig. 112.

$M - I : L - I :: {}^{a}AP : Ap$. *Q. E. D.* [a] 291. I.

CASE 2. When the incidences of the parallel rays are upon the convex side of a plano-convex lens. Let F and f be the foci after refraction, of the least and most refrangible rays respectively; and let s, be one of the focal centers of the lens; then will $sF = {}^{b}AP$, and $sf = {}^{b}$ [b] 218. I. Ap. Wherefore, as above,

$M - I : L - I :: sF : sf$. *Q. E. D.*

CASE 3. When both sides of the lens are spherical. Retaining the same references as above; let moreover the radius of the flattest side (A) be called R; and the radius of the other side (a) call r; also let the thickness Aa of the lens be called t; and let s be that focal center of the lens which belongs to the refracted rays. Fig. 113, 114.

$$\text{Now } \frac{\frac{LIR\,r}{L-I} - IR\,t}{LR \pm Lr \mp Lt \pm It} + \frac{IR\,t}{LR \pm Lr} : sF ::^{c}$$ [c] 220. I.

$$\frac{\frac{MIR\,r}{M-I} - IR\,t}{MR \pm Mr \mp Mt \pm It} + \frac{IR\,t}{MR \pm Mr} : sf.$$

If t be considered as infinitely small, the quantities multiplied by it will be so also, and therefore may be neglected; then multiplying the antecedents

antecedents by $R \pm r$, and by L and M, and dividing by IR, we shall have $\left(\frac{MLr}{L-I} : \frac{MLr}{M-I} ::\right) M-I : L-I :: sF : sf$. *Q. E. D.*

299. *Corol.* In every lens, whose thickness is very small, it will be
[d] 292. I. $M-L : \frac{M}{L} - I :: {}^{d}fF : \frac{sF}{sf}$; nearly. And therefore in a glass lens, the longitudinal aberration $fF = \frac{1}{27} sF = \frac{1}{27} sf$, nearly.

Fig. 112. 300. *Schol.* 1. A plano-convex lens with its convex side exposed to parallel heterogeneal rays; the thicker is the lens, the less will be the aberration fF: but the difference on account of the thickness is very inconsiderable. Let s be the focal center of the lens for the least refrangible rays, and t the focal center for the most refrangible rays. Now as the thickness of the lens increases, the points s, t separate; vt being
[e] 209. I. always less than vs; for it will be $\frac{L}{M} : I :: {}^{e}vA : \frac{vs}{vt}$. But sF, tf are[f]
[f] 175, & 218. I. invariable quantities; and therefore as the point t recedes from s, the focus f approaches nearer to F; that is, fF will be so much diminished, as st is increased.

The variation of fF on account of the different thicknesses of lens's having both their sides spherical, is also very inconsiderable.

301. *Schol.* 2. In computing the aberration above mentioned, the incident rays were supposed to be the nearest of all to the axis; and in that case, all lens's whatsoever whose thicknesses are very small, are equally good. But when the incident rays are remote from the axis, there will be an aberration by the figure; and that aberration will increase the aberration of heterogeneal rays. Therefore that lens is best which has least aberration of homogeneal rays.

302. *Schol.* 3. What has been above demonstrated of convex lenses, is also true of concave ones.

PROP. XLIV.

303. *In the same lens, the longitudinal aberration* F f *of parallel heterogeneal rays is invariable: And the lateral aberration* FG *increases or decreases in the simple ratio of the aperture of the lens; and is to the semi-aperture of the lens, as* Ff *to* fv. Fig. 110, to 114.

Let fall the perpendicular Bv.

1. Ff is invariable, because it has always the [a] same ratio to the invariable quantities AF or Af. [a] 291, 294, & 298. I.

2. The right-angled △s $f v$B, fFG are similar, and therefore, FG : vB :: fF : $f v$. Therefore, &c. *Q. E. D.*

304. *Corol.* 1. In any glass lens whose thickness is very small, FG = $\frac{1}{27}$ vB, nearly.

For M — L : L — I :: [b] fF : ($f s$ or) $f v$ :: FG : vB :: 1 : 27. [b] 299. I.

305. *Corol.* 2. In the same lens, the angle of aberration of parallel heterogeneal rays, is as the aperture of the lens nearly. For that angle is as FG nearly; and FG is to vB in the constant ratio of Ff to $f v$.

PROP. XLV.

306. *In different lenses that are alike figured, their greatest angles of aberrations will be equal, if their linear apertures be as their focal lengths, and the focus of incident rays be also at like distances from the lenses.*

1. Let DB, $d b$ be slender pencils of heterogeneal rays parallel to the axes AF, $a f$ of the lenses A, a; and let AB, $a b$ be the semi-apertures of the said lenses. And neglecting the aberrations from the figures of the lenses, let F, f and K, k be the focus's of the least and most refrangible rays respectively. Now if it be, AB : $a b$ :: AF : $a f$ :: AK : $a k$; the △[a] FAB, $f a b$, and KAB, $k a b$ will be [a] similar; and therefore the ∠[a] FBK, $f b k$ will be equal. *Q. E. D.* Fig. 116, 117. [a] 6. VI. *Euc.*

2. Let r, t, and s, v be the proper focus's of the least and most refrangible rays respectively. Now if the lenses are alike figured, the aberrations Fr, $f t$, and Ks, $k v$ will be in the [b] same given ratio to their thicknesses. And when the apertures of the lenses are as their focal lengths, their thicknesses are also in the same proportion; therefore AB : $a b$ :: Fr : $f t$ :: Ks : $k v$; and consequently the △[c] ArB, $a t b$, and AsB, $a v b$ being similar, the ∠[s] rBs, $t b v$ are equal. *Q. E. D.* [b] 60, 136, & 264. I.

307. *Schol.* 1. If rays of all sorts are supposed to flow along the lines FB, $f b$ or rB, $t b$; the least refrangible rays will emerge along the lines BD, $b d$ parallel to the axis; and the most refrangible rays

will emerge along the lines BE, *be*, so as to make the angles of aberrations DBE, *dbe*, equal to the angles FBK, *fbk*, or *r*BS, *tbv*, respectively; and consequently the said angles DBE, *dbe*, will be equal one to the other.

308. *Schol.* 2. In different lenses having the same apertures, if these apertures are but moderate, so that the aberrations from the figures of the lenses may be neglected; the greatest angles of aberrations of heterogeneal rays by these different lenses, will be nearly in the inverse ratio of their focal lengths.

Fig. 114, 116. For AB : *radius* :: A*f* : *co-tang.* ∠A*f*B :: AK : *co-t.* ∠AKB. And in the different figures, the co-tangents of the ∠[s] AFB will be as AF in each; and these are to each other, as A*f*, AK. But the tangents of small angles are nearly as those angles; and therefore

∠FB*f* : ∠FBK :: AK : A*f* nearly. *Q. E. D.*

309. *Schol.* 3. In different lenses having moderate apertures; the angles of aberration FBG are nearly, as the semi-apertures AB directly, and focal lengths AF inversely. For these angles may be considered as being as their chords directly, and as FB inversely; and these chords may be reckoned as FG, and FB as FA. But in the same lens, FG is always as AB. Therefore, *&c.* *Q. E. D.*

310. *Corol.* The angles of aberrations by all lenses that are alike figured, and having the focus's of the incident and emergent rays alike situated; will be the same, if the apertures of the lenses be as their focal lengths.

PROP. XLVI.

Fig. 110, to 114. 311. *The diameter of the least circle, through which all the heterogeneal rays will pass, is a line* gh, *drawn between the points of intersection of the most and least refrangible rays, proceeding from the opposite sides of the lens: And* gh : FG :: Bh : BF.

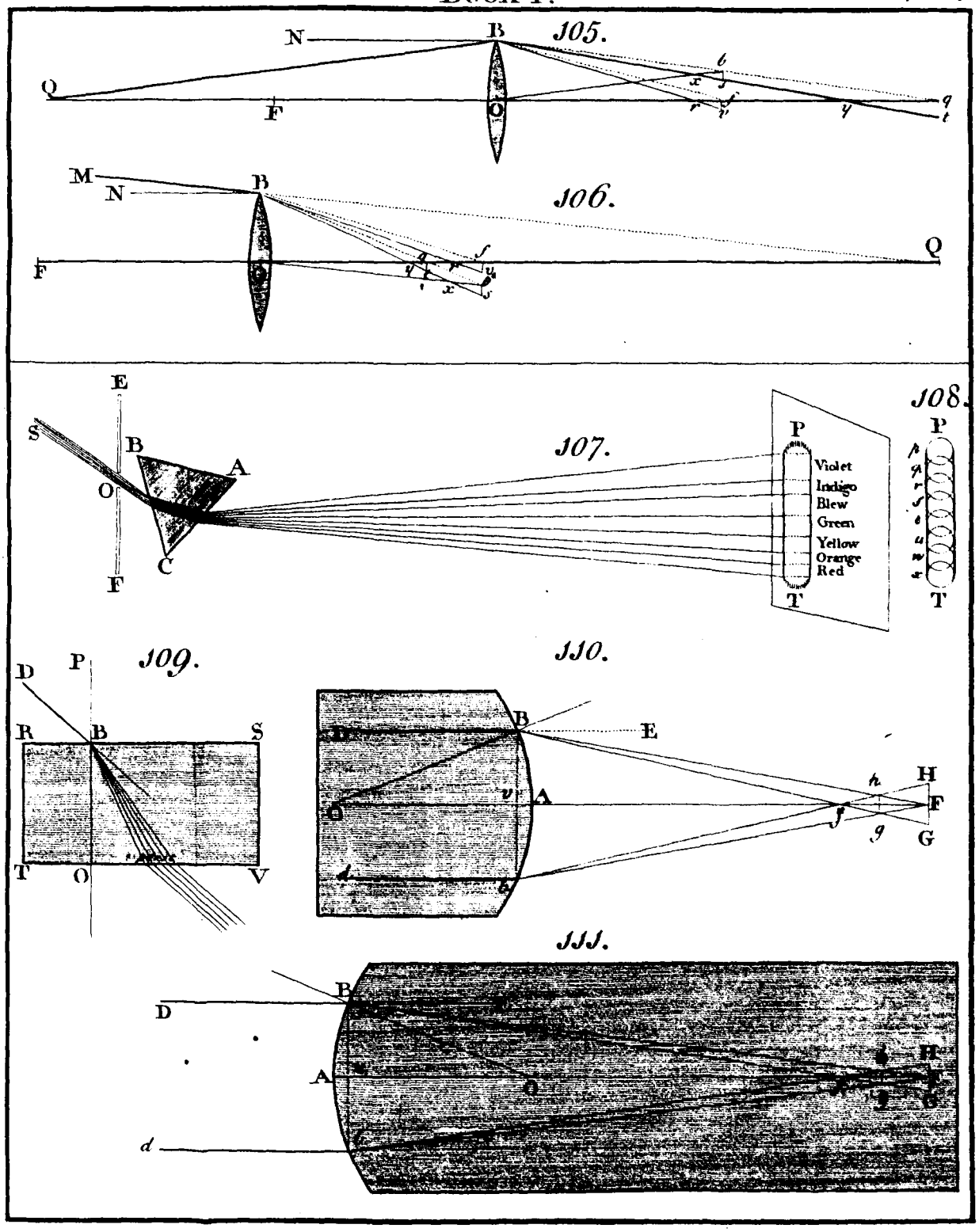
The first part of the proposition is evident by bare inspection of the figures: and it is also manifest, that *gh* is parallel to GH; whence it [a] 4. VI. *Euc.* follows, *gh* : FG :: [a] B*h* : BF. *Q. E. D.*

312. *Corol.* 1. $gh : FG :: \frac{vf + vF}{2} : vF :: vf : \frac{vf + vF}{2}$, nearly. Also $gh : Bv :: fF : \frac{vf + vF}{2}$, nearly. Hence, when the versed sine A*v* is very small, in comparison of the focal distance from a spherical refracting surface or lens, it will be nearly;

[b] 292, & 299. I. 313. *Corol.* 2. $gh : Bv :: (fF : \frac{vF + vf}{2} ::$ [b]$) M - L : \frac{M + L - 2I}{2}$. And therefore if the refracting surface or lens, is glass; it will be,

$gh : Bb :: 1 : 55 (:: 78 - 77 : 78 + 77 - 100)$.

314.


105.
N
B
Q
F
O
106.
M
N
B
F
Q
107.
E
S
B
A
O
C
F
P
Violet
Indigo
Blew
Green
Yellow
Orange
Red
T
108.
P
T
109.
D
P
R
B
S
T
O
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110.
B
E
A
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111.
B
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d

314. *Schol.* The diameter of the leaſt circle that can receive all the rays of any two colours, is after the ſame manner determinable from the proportion of their ſines. Thus all the *orange* and *yellow* will be contained in a circle whoſe diameter is about the $\frac{1}{200}$ part of the aperture of a glaſs lens, when that aperture is but ſmall in proportion to the focal length; the ſines of the outermoſt orange and yellow, being to the common ſine of incidence as [c] $77\frac{1}{8}$ and $77\frac{1}{3}$ to 50; and putting [c] 289. I.
L, M and I for theſe ſines, it will be,

M—L : M+L—2I :: 1856—1851 : 3707—2400 :: 1 : 261.

PROP. XLVII.

315. *The rays contained in the circle of aberration whoſe diameter is* gh, *are not uniformly ſpread over that circle; but are denſer at the center than in any other part, and from thence towards the circumference they grow continually rarer. And the denſity of the light in any point of this circle, is directly as the diſtance of that point from the circumference, and reciprocally as its diſtance from the center, nearly.*

For, ſuppoſing the heterogeneal rays that are refracted at the points B, *b* in the periphery of the lens, to be ſpread uniformly over the phyſical line *g h*; the rays tranſmitted through every other point in that periphery, will be alſo ſpread uniformly over lines all equal to *g h*, and interſecting one another at the center *c*; ſo that the whole light tranſmitted as above, will be ſpread over a circle whoſe diameter is *g h*. And upon the above ſuppoſition, the denſity of the light in any point of this circle, will be reciprocally as the diſtance from the center; that is, the denſity in any point *a*, will be to the denſity in any point *b*, as *c b* to *c a*. Fig. 110, to 114. Fig. 115.

If the lens or medium be ſuppoſed to be divided into an indefinite number of ſmall concentric rings, all equal in breadth; the quantity of light tranſmitted through each ring, will be as the rings themſelves, that is, as their diameters; and this light will be ſpread over circles of aberrations, whoſe diameters [a] are alſo as thoſe of the reſpective rings. [a] 303. I.
Now in each circle of aberration conſidered apart from the reſt, the denſity of the light will be reciprocally as the diſtances from the center; and the whole quantity of light in each circle being as its diameter; the denſities in each circle at equal diſtances from the common center are equal. For, in the circle *h g c*;

denſity in *a* : *denſity* in *h* :: *c h* : *c a*; and in the circle *b d c*,

denſity in *b* : *denſity* in *a* :: *c a* : *c b*. Wherefore,

denſity in *b* : *denſity* in *h* :: *c h* : *c b*. Inſtead of the denſities in *b* and *h*, let us ſubſtitute their ratio's, and it will be in the circle $\frac{hgc}{bdc}$;

denſity in $a : \frac{cb}{ch} :: \frac{ch}{cb} : \frac{ca}{ca}$. Whence it is manifeſt that the denſity at a in each circle is the ſame. And therefore the whole denſity in any point of the circle of aberration, is directly as the diſtance of that point from the periphery, and reciprocally as its diſtance from the center, nearly; that is, the denſity of the light in any point a or b, is as $\frac{ha}{ca}$ or $\frac{hb}{cb}$, nearly. *Q. E. D.*

316. *Corol.* 1. Let the ſemi-diameter ch be biſected in n; and the denſity of the light in any point b: will be to its denſity in $n :: hb : bc$.

For by the preceding,

denſity in b : *denſity* in $n :: \frac{hb}{cb} : \left(\frac{hn}{cn}\right) 1 :: hb : cb$.

317. *Corol.* 2. Hence the light at the center is infinitely denſe, and at the circumference infinitely rare, in compariſon of the light at the point n.

318. *Schol.* 1. The quantity of light in the greateſt circle hgc : is to the quantity of light in any leſſer circle $bdc :: hc^2 : hc^2 - hb^2$.

Let the ring $hbdg$ be divided into any number of indefinitely ſmall concentric rings, all equal in breadth; and let the radii of theſe rings be co, cp, cq, cr, &c. Now the denſity of the light in each ring[b] will be as $\frac{ho}{co}$, $\frac{hp}{cp}$, $\frac{hq}{cq}$, $\frac{hr}{cr}$, &c. and the rings are as co, cp, cq, cr, &c. Wherefore multiplying each ring by its reſpective denſity, we ſhall have the quantity of light in each, as ho, hp, hq, hr, &c. and the ſum of all theſe is $= hb \times \frac{1}{2} hb = \frac{1}{2} hb^2$. If the whole circle be divided in like manner into theſe ſmall rings, till we come to the center; the ſum of the light in them all, will be as $hc \times \frac{1}{2} hc = \frac{1}{2} hc^2$. And therefore, the quantity of light in the circle hgc : is to the quantity in the circle $bdc :: \frac{1}{2} hc^2 : \frac{1}{2} hc^2 - \frac{1}{2} hb^2 :: hc^2 : hc^2 - hb^2$. *Q. E. D.*

[b] 315. I.

319. *Schol.* 2. Sir *Iſaac Newton* * obſerves, that the *yellow* and *orange* being the moſt luminous of the priſmatic colours, affect the ſenſe more ſtrongly than all the reſt together; and next to theſe in ſtrength are the *red* and *green*. The *blue* compared to theſe is a faint and weak colour; and the *indico* and *violet* are ſtill darker and fainter; ſo that theſe, compared to the ſtronger colours, are little to be regarded. Whence, and from the greater denſity of the rays of light near the center of a circle of aberrations, Sir *Iſaac* infers, that the ſenſible image of a lucid point in the focus of a glaſs lens having but a ſmall aperture, is not much broader than $\frac{1}{230}$ of the diameter of that aperture.

320. *Schol.* 3. The images of objects are therefore to be placed or reckoned, not in the focus of the mean refrangible rays, which are in the

* *Newton's* Optics, p. 85, 86.

the confine of green and blue; but in the focus of those rays which are in the middle of the orange and yellow. The sines of incidence and refraction of these rays between glass and air, are as 17 and 11: And from these proportions therefore, the focal lengths of glasses for optical uses, are to be computed †.

PROP. XLVIII.

321. *In any lens retaining the same aperture, and neglecting the aberration from its figure:*

The longitudinal aberration of heterogeneal rays, diverging from, or converging to, a given focus in the axis of the lens,

Is to the longitudinal aberration of parallel heterogeneal rays by the said lens;

As the distance between the focus of incidence, and the focus of the most or least refrangible rays after refraction by the lens,

Is to the distance between the focus of incidence, and the principal focus of the least or most refrangible rays.

Let Y be the focus of the incident heterogeneal rays; Q, q the focus's of the least and most refrangible rays, after refraction by the lens O; and F, f the principal focus's of these rays respectively. Fig. 118, to 123.

I say $Qq : Ff :: Yq : YF :: YQ : Yf$.

For $YF : YO ::$ [b] $YO : YQ$; and $Yf : YO ::$ [b] $YO : Yq$; wherefore [b] 227. I.
$YF : Yf :: Yq : YQ$; and therefore,

$$Qq\,(YQ \backsim Yq) : \begin{matrix} Yq \\ YQ \end{matrix} :: Ff\,(Yf \backsim YF) \begin{matrix} YF \\ Yf \end{matrix}. \quad \text{Q. E. D.}$$

322. *Corol.* 1. $YF : Yf :: Fq : fQ :: Yq : YQ$.

323. *Corol.* 2. $OQ : Oq :: Yf \times OF : YF \times Of$.

For, $YO : OQ ::$ [c] $YF : OF$; [c] 227. I.
And $Oq : YO ::$ [c] $Of : Yf$. Therefore, &c. Q. E. D.

324. *Corol.* 3. $Qq : Ff :: \frac{Yq + YQ}{2} : \frac{YF + Yf}{2}$.

Let the point which bisects Ff be called a, and that which bisects Qq be called b; and then it will be, $Qq : Ff :: Yb : Ya$. And because Ff is an invariable quantity, Qq increases or diminishes as $\frac{Yb}{Ya}$.

325. *Corol.* 4. If the lens is glass, it will be,

$Qq : \frac{1}{27} Of ::$ [d] $Yq : YF :: YQ : Yf$. [d] 299. I.

Also, $27YF : 28Yf ::$ [d] $Oq : OQ$.

PROP.

† *Newton's* Optics, p. 86.

PROP. XLIX.

326. *In any lens retaining the same aperture, and neglecting the aberration from its figure:*

The lateral aberration of heterogeneal rays, diverging from, or converging to, a given focus in the axis of that lens,

Is to the lateral aberration of parallel heterogeneal rays by the said lens;

As the rectangle under the longitudinal aberration of the rays belonging to that focus, and the distance from the lens of the principal focus of the most refrangible rays,

To the rectangle under the longitudinal aberration of parallel heterogeneal rays, and the distance from the lens of the focus of the most refrangible rays.

Fig. 118, 119. Let OB be the semi-aperture of the lens O, QS the lateral aberration for that aperture of heterogeneal rays, whose focus of incidence is Y; and FG the lateral aberration of parallel heterogeneal rays for the same aperture;

I say, $QS : FG :: Qq \times Of : Ff \times Oq$.

For the right-angled triangles fFG, fOB; and qQS, qOB being similar; it will be,

$Ff : Of :: FG : OB$; and $qO : qQ :: OB : QS$. Wherefore $QS : FG :: Qq \times Of : Ff \times Oq$. *Q. E. D.*

The same will be also true in the four succeeding cases or figures, if the respective lines BF, BfG, FG, BqS, QS be drawn, changing the places of Q, q in fig. 120, 122.

327. *Schol.* 1. In any lens and with any aperture; $QS = OB \times \frac{Qq}{Oq}$.

Fig. 118, 119. 328. *Schol.* 2. In different lenses, similarly formed, having their apertures as their focal lengths, and forming equal images Qp behind them of a given object YP; the lateral aberration QS will be as the focal lengths of the lenses. For in that case the angles of aberrations QBS in each are equal, and in each OQ is as OB or OF.

329. *Corol.* Hence, if a given object is to have its image upon a screen magnified in a given ratio; the deeper is the lens, the more distinct will be the image. For in this case, the distinctness of the image is chiefly affected by the aberration QS. And if the distance OQ of the screen is fixed; the distinctness of the image will not be sensibly different by using lenses of different focal lengths, if it be but sufficiently bright; provided always that the figures of the lenses be similar, and their apertures as their focal lengths.

PROP.

PROP. L.

330. *The circle of aberrations caused by the sphericalness of the figure of an optic lens having a moderate aperture, compared with the circle of aberrations caused by the unequal refrangibility of the rays of light, is very inconsiderable.*

For a comparison betwixt these two sorts of aberrations, let us take a plano-convex glass with its plane side exposed to parallel rays; this lens being the worst for homogeneal rays of any in the table, *Art.* 253. Let F be the principal focus of the lens, Fp the lateral aberration of homogeneal rays, for the semi-aperture Bv; and FG the lateral aberration of heterogeneal rays for the same aperture. And putting m, n for the greater and lesser of the sines of incidence and refraction, it will be, Fig. 99.

$$Fp : FG :: \frac{Bv^3}{2RA^2} \times \frac{mm}{nn} : \tfrac{2}{55} Bv.$$

For in any plano-convex lens having a small aperture with its plane side to the parallel incident rays, it will be $OF = {}^a vA \times \frac{mm}{\overline{m-n} \times n}$ nearly. And $vA = {}^b \frac{Bv^2}{2RA}$ nearly. Also $Fp : OF :: {}^c Bv : vO = AF$ nearly. But $AF = {}^d \frac{n}{m-n} RA$. Therefore $Fp = \left(\frac{Bv^2}{2RA} \times \frac{mm}{m-n \times n} \times \frac{Bv}{RA \times \frac{n}{m-n}} =\right) \frac{Bv^3}{2RA^2} \times \frac{mm}{nn}$. And $FG = {}^e \tfrac{2}{55} Bv$. Therefore, *&c.*

[a] 136. I. [b] 13. VI. *E.* [c] 4. VI. *Eu.* [d] 175. I. [e] 313. I.

The diameter of the least circle that will contain all the heterogeneal rays is $= {}^f FG$ nearly; and the diameter of that least circle which will contain all the homogeneal rays is $= {}^g \tfrac{1}{2} Fp$. If we compare these lesser circles of aberrations together, their respective diameters will be, as $\tfrac{2}{55} Bv$, to $\frac{Bv^3}{4RA^2} \times \frac{mm}{nn}$. And by these numbers, the diameters of the respective circles of aberrations for any given aperture of a plano-convex glass, may be easily found.

[f] 311. I. [g] 268. I.

If we take $\tfrac{1}{2}$FG in the table, *Art.* 253. for the diameter of the least circle of aberrations of homogeneal rays; this will be for the several apertures there specified, beginning with the largest and so downwards, the $\tfrac{1}{13}$, $\tfrac{1}{34}$, $\tfrac{1}{213}$, $\tfrac{1}{906}$ of the diameters of the least circles of the aberrations of heterogeneal rays, for these respective apertures, nearly. But in many cases, the smallest even of these apertures is much too large; and the aberration from the figure decreasing as [h] the cubes of the apertures, and that from the heterogenity of the rays only in the [i] simple ratio of the apertures; the fractions expressing the ratio's of these respective aberrations, will become much smaller as the apertures are lessened.

[h] 264. I. [i] 303. I.

If the diameter of the least circle of aberration of heterogeneal rays, that may affect the sense, be admitted to be so small as the $\frac{1}{250}$ part of the aperture of a lens; Sir *Isaac Newton* * computes that in a 100 foot telescope with a proper aperture, this aberration would be at least 1200 times the aberration caused by the sphericalness of the figure of the object-glass.

† It is farther observable, that the rules deduced from experience, for limiting the apertures and charges of telescopes, agree exactly with the theory of the different refrangibility of the rays of light, without at all regarding the figures of the object-glasses. Therefore, *&c.* *Q. E. D.*

331. *Schol.* 1. The least circle of aberration from the imperfect figure of a plano-convex glass, with its convex side exposed to parallel rays, is equal to the least circle of aberration of heterogeneal rays, if the figure of the lens was perfect, when the aperture of the lens is half its focal length, nearly. That is, $\frac{1}{2}Fp = FG$ nearly, when $vB = \frac{1}{4}AF$.

Fig. 99.

For $OF = {}^{a}\frac{7}{6}vA = \frac{7}{6}\frac{vB^2}{AF}$ nearly; and $Fp = \frac{Bv \times FO}{vO}$.

a 256. I.

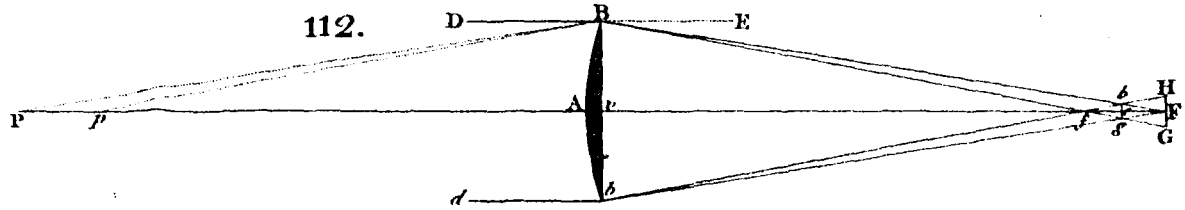
But $vO = AF$ nearly; and therefore $\frac{1}{2}Fp = \frac{7}{12} \times \frac{vB^3}{AF^2}$.

Now $FG = \frac{1}{27\frac{1}{2}}vB$, and when $vB = \frac{1}{4}AF$, $\frac{1}{2}Fp = \frac{7}{12} \times \frac{1}{16}vB = \frac{1}{27\frac{3}{7}}vB$. Therefore, *&c.* *Q. E. D.*

332. *Schol.* 2. The aberrations of coloured rays, being so very considerable in proportion to the aberrations arising from the spherical figures of lenses; there is no sensible advantage to be expected from lenses of other figures; nor yet from a medium of different refractive power from glass, unless that medium should be more homogeneal than glass. For different refrangibility being a property of light itself, the aberrations proceeding from thence must alike take place, whatever be the medium it is refracted by; unless a medium should be found that would transmit only the brightest of the prismatic colours. And all the consequence of having a medium of greater or less refractive power than glass, would be; that for the same focal lengths, lenses made of such a medium would be more or less flat, than those we have at present ||.

* *Newton*'s Optics, p. 84, 89. † Ibid. p 89. || *Note*, In the annexed plate, the aberrations Ff, Qq, are each four times too large in proportion to the focal distances Of, Oq.

112.



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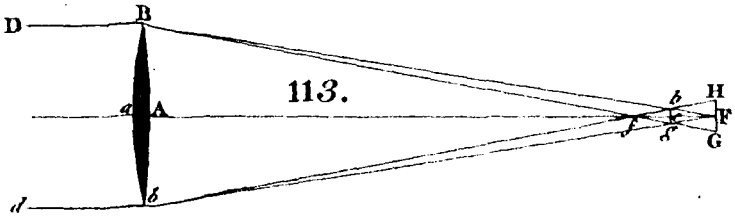
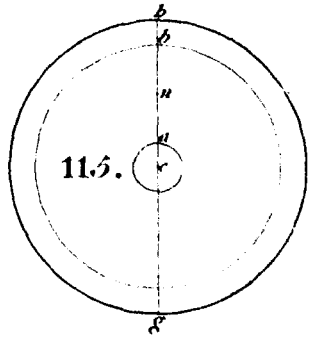


115.



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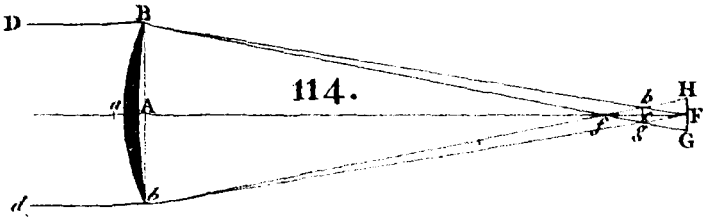

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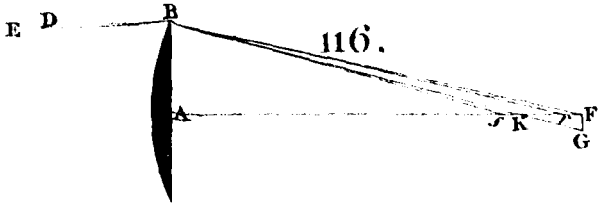
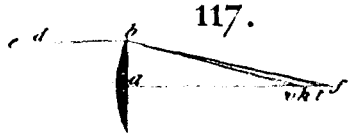


117.



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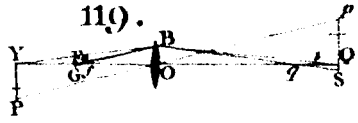
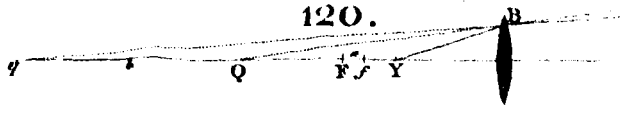
119.



120.



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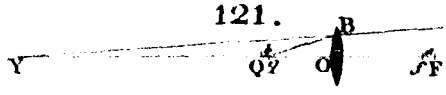

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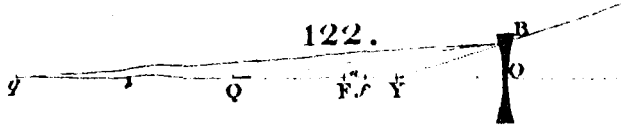

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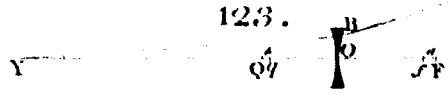

A

TREATISE

OF

OPTICS.

BOOK II.

Of VISION.

HAVING now gone through the general * theory of reflection and refraction of light, we are in the next place to enquire into the theory of vision. This enquiry in itself is not only curious and entertaining, but without it there is no accounting for several optical phænomena; and it is also very necessary that it should precede the particular theories of optical instruments, if we would rightly understand their effects in extending, as they do, so prodigiously, the natural boundaries of vision. I have therefore bestowed a distinct book upon this head, the two first sections whereof are upon vision in general; in which I have endeavoured to digest and set in the best light I could, what I have been able to gather upon the subject; the two remaining sections are for explaining the phænomena of vision by the reflection of polished speculums, and refraction of a single lens.

SECTION I.

Of the eye, and manner of vision.

§. 1. *A brief description of the eye.*

1. THE eye consists of several *coats*, or teguments, which form a *ball* perfectly globular, excepting on the fore part, which is a little more protuberant than the rest. Within this ball are included three different

* This theory has no necessary connection with vision, and may be perfectly understood by a blind man: And although a person who does not remember to have seen objects, cannot have the same ideas about colours that others have; yet he may perfectly comprehend the theory of the different refrangibility of light, and argue from thence as justly, as if he had seen the coloured spectrum formed by a prism. But there is no understanding the theory of vision, without a competent skill in that of light; otherwise it would have been perhaps more natural, to have made the description of the eye, &c. the entrance into the science of optics.

different ſorts of liquids, or ſoft tranſparent ſubſtances, called *humours*. The figure repreſents a ſection of the eye through the middle, by an horizontal plane, or a plane paſſing through both eyes; and for the ſake of repreſenting the parts more diſtinctly, the diameter of the figure is about twice the diameter of the human eye *.

Fig. 1.

Sclerotica and cornea.

2. The outermoſt coat, repreſented by the ſpace contained between the two outermoſt circles BFB, is called *ſclerotica*. Adjoining to the ſclerotica at the points B, B is a more globular coat, repreſented by the ſpace between the two circles BAB, called *cornea*. Theſe two coats make together one firm caſe of a proper form, for containing the reſt of the coats and humours. The cornea is tranſparent (like horn, whence it has its name) for the admiſſion of light into the eye.

Choroeides and Tunica Ruyſchiana.

3. The next coat under the ſclerotica, repreſented by the two innermoſt circles BFB, is a membrane of leſs firmneſs, called *choroeides*. This membrane is by M. *Ruyſch*, and ſince by other anatomiſts, divided into two; the outermoſt of which retains its old name, and the innermoſt is called † *tunica ruyſchiana*. Both theſe coats are often conſidered as one, under the old name choroeides; the back part of which in the human eye is of a ruſty dark colour, occaſioned by a black *mucus* wherewith it is covered ||.

Uvea and iris.

4. Adjoining to the choroeides at BB, is ‡ a flat membrane called by moſt authors the *uvea*, and by ſome the *iris*; but generally by the *iris* is meant only the fore and circular coloured part of the

* It is not to be expected, that the account we are to give, ſhould be altogether perfect. For as it is impoſſible to examine the internal parts of the eye, whilſt it is in its natural and vivid ſtate; ſo is it difficult after the eye is taken out of its ſocket, to preſerve the figures of its ſeveral parts entire: and this is an excuſe for ſome diſagreements we find among *anatomiſts*. However, the accounts we have, are ſufficiently accurate for our purpoſe.

† This coat is deſcribed as leaving the choroeides at B, B, and as paſſing behind the proceſſus ciliares, and round the edge and back part of the cryſtalline humour.

|| Anatomiſts obſerve, that in brutes of prey the middle part of this membrane is perfectly white; and in brutes that feed on graſs, the ſame part is of a bright green. The whiteneſs in the one is ſuppoſed to enable them to diſcover their prey better in the night; and the greenneſs in the other, is alſo ſuppoſed to be a help to them to find their food: whilſt men have the advantage of the dark colour ſuffocating the rays, to prevent an indiſtinctneſs of viſion by their reflection back. But theſe Hypotheſes for explaining the Uſes of the different colours of the choroeides in the eyes of different animals, are not ſo ſatisfactory as at firſt they may ſeem to be. For if viſion is owing, as I think it is now univerſally acknowledged, to the impulſe of light upon the retina (or to its impulſe upon the choroeides, as ſome have imagined) there does not ſeem to be any neceſſary connection betwixt viſion, and the colours of either the choroeides or retina; unleſs it be ſuppoſed, that according to their colours, they are more ſenſible of the impulſes of the rays of light of theſe colours, than of any other: But ſuch an hypotheſis ſeems very precarious. Perhaps the eyes of ſome animals tranſmit more of ſome kind of the coloured rays, than of others; and that this may be the cauſe of the diverſities of colours above obſerved: And admitting this, it will appear very probable, that theſe different conſtitutions of the eyes of different animals, are intended for the purpoſe above aſſigned.

‡ This membrane was formerly ſuppoſed to be ſpherical, and of much the ſame curvity with the choroeides; but Monſ. *Petit*, a ſkilful anatomiſt at *Paris* in the *Mem. de l'Acad. Royale*, for the Year 1728, ſhews that in the human eye, the uvea is flat. In delineating the eye, I have followed this gentleman's meaſures, as he ſeems to have taken more than ordinary pains in the examination of its ſtructure.

the ſaid membrane. This membrane is opaque, and of different colours in different eyes, as *grey*, *black*, *hazel*, &c. and being ſeated behind the *cornea*, its colour is ſeen through it; whence the eye is accordingly denominated *grey* or *black*, &c. The uvea ariſes from a circular ridge or protuberance running all along the inſide of the cornea, at its juncture with the ſclerotica at BB.

5. In the uvea, a little nearer the noſe than the middle, is † a round hole *aa*, for the admiſſion of light into the eye, called the *pupil*. ‖ The pupil may be enlarged or contracted for the admitting of more or leſs light, as the circumſtances of viſion require: for which purpoſe the uvea is furniſhed with ſtrait muſcular fibres, proceeding from its circumference towards the center, and terminated in the orbicular limb of the pupil; which limb is alſo ſuppoſed to be muſcular **, that ſo by the contraction and dilatation of theſe ſtrait and annular fibres, like the cloſing and opening of a purſe, the pupil may be dilated and contracted. Pupil.

6. At the back part of each eye, a little nearer to the noſe than the middle, there enters a nerve as V, called the *optic nerve* ‡. Optic nerve.

7. The fibres of the optic nerve, after their entrance into the eye at V, ſpread themſelves like a fine web or net over the choroeides or *Ruyſchiana*; and ſo form a very thin membrane, called *Tunica Retina*, repreſented in the figure by the etched circle BFB. The retina is reckoned to extend all round as far as BB; and is of a * darkiſh colour, occaſioned by its immerſion in the [a] black *mucus* adhering to the *tunica ruyſchiana*. But the optic nerve is white like the brain, from whence it is propagated. Retina. [a] 3. II.

N 2 8.

† *Anatomiſts* obſerve, that in animals which are the prey both of birds and beaſts, the pupil is round as in man, which fits them to ſee every way alike. But in large animals which feed on graſs, the pupil is oblong horizontally, which ſuits them for ſeeing a large circular ſpace upon the earth. And in animals of the cat-kind, which climb trees, the pupil is oblong the contrary way, which fits them beſt for looking both upwards and downwards: And it is farther remarkable, that cats in the dark have their pupils prodigious large, which enables them to ſee their prey with a very little light.

‖ The latitude of contraction and dilatation of the pupil is pretty conſiderable; and it is very admirable, that while the pupil changes its magnitude it preſerves its figure.

** It is ſuppoſed neceſſary that the uvea ſhould be furniſhed with theſe two ſorts of fibres. Anatomiſts are agreed as to the ſtrait ones; and it is allowed that there is a muſcular ring round the pupil, into which the ſtrait fibres are inſerted: but the exiſtence of this ring is rather inferred from the reaſon of the thing, than proved from experiments. *Albini* thinks, that the cryſtalline by preſſing againſt the uvea, changes its figure and widens the pupil.

‡ The ſclerotica and the choroeides enter the orbit of the eye along with the optic nerve: the origin of the ſclerotica is the *dura mater*, a thick tough membrane lying immediately under the *ſkull*; and the choroeides is derived from the *pia mater*, which is a fine thin membrane adhering cloſely to the brain. Through the middle of the optic nerve, there runs a pretty conſpicuous artery, which goes ſtrait through the vitreous humour, and ſpreads itſelf upon the membrane that covers the back-ſide of the cryſtalline.

* This membrane is ſaid to be quite black in infants; not ſo black at the age of twenty; of a greyiſh colour about the thirtieth year; and in very old age almoſt white.

Cryſtalline humour.

8. Within the cavity of the eye, a little behind the pupil, is a tranſparent ſubſtance EE, for the moſt part of about the conſiſtency of hard jelly, which it nearly reſembles; but from the middle outwardly, it grows gradually ſofter. This is called the *cryſtalline humour*; it is contained in a thin membraneous and tranſparent capſula †. The figure of the cryſtalline reſembles that of a double convex lens of unequal convexities, the flatteſt ſide being that next the pupil. The circular edge E E is flattiſh, or as it were rounded off, as repreſented in the figure. The cryſtalline till about twenty or thirty years of age is ſaid to be colourleſs; after which time it begins to be ſlightly tinged with yellow, which increaſes with age, till about eighty it is as yellow as amber.

Proceſſus Ciliares.

9. The cryſtalline is ſuſpended within the eye by a muſcle B*b b*B called ‖ *Proceſſus Ciliares*, and by ſome authors ‖ *ligamentum ciliare*. This muſcle is a little convex towards, and much weaker than, the uvea, and ariſes behind it at B B where the ſclerotica and cornea join together; at *bb* it enters the capſula, and thence ſpreads over a great part of its anterior ſurface. The anterior ſurface of the proceſſus ciliares is beſmeared with a black unctuous matter, as is alſo the back ſide of the uvea: this black is, with good reaſon, ſuppoſed to be for preventing the reflection back of any of the rays of light, that may happen to reach ſo far.

Aqueous and vitreous humours.

10. The cryſtalline humour with the proceſſus ciliares divide the cavity of the eye into two unequal portions; in the firſt and ſmalleſt of which BAB*b*C*b*, is contained a tranſparent fluid like water, called thence the *aqueous humour*. The hinder and larger part of the concavity of the eye B*b*D*b*BF is filled with a tranſparent ſubſtance called the *vitreous humour*: This humour is not ſo hard as the cryſtalline, or ſo fluid as the aqueous *. Again, the cavity containing the aqueous humour,

† That part of the capſula which inveſts the hinder ſurface of the cryſtalline humour, adheres to the membrane incloſing the vitreous humour, yet ſo as to be ſeparated from it without cutting; but all along the limb or edge E E of the cryſtalline, theſe two membranes adhere ſo cloſely together, as not to be parted without the knife. The cryſtalline has no viſible communication with its capſula, and conſequently with no part of the body; for as ſoon as the capſula is opened, the humour within ſlips clean out: whence it is imagined by ſome that the cryſtalline has only a vegetable kind of life; it having, as above obſerved, no viſible continuity with the adjacent parts by any fibre or blood-veſſels: but although no ſuch fibres or ſmall veſſels have been yet diſcovered, there ſeems to be but little room to doubt of their exiſtence. M. *Petit* in the *Mem. de l'Acad. Royale* for 1730 has given a table of the dimenſions, *&c.* of the cryſtalline humour in 26 human eyes.

‖ Both theſe names are uſed for the ſame thing by different authors, even of good repute. Mr. *Sharp*, in his treatiſe of ſurgery, *p.* 169, chuſes the firſt, and calls the circular ridge where all the coats unite at B B (which he deſcribes as whitiſh, and denſer than any of the coats) *Ligamentum Ciliare*.

* The vitreous and aqueous humours, are each contained in ſo very thin and tranſparent films or bags, as ſcarce to be perceivable till they are broke, and the liquid let out. The aqueous humour is more ſpirituous and viſcous than common water: and the vitreous appears ſomewhat like to ſoft jelly; but is ſuppoſed nevertheleſs to conſiſt of a liquid like the other, contained in a great number of delicate veſicles or cellules, which are eaſily broke; for upon touching it always appears moiſt and wateriſh. It is not doubted, but that both theſe humours are nouriſhed through proper veſſels communicating with the adjacent parts.

humour, is divided by the uvea into two unequal portions; the first and largest of which next the cornea is called the *anterior chamber*; and the other next the crystalline, is called the *posterior chamber*.

11. It is agreed, that the several coats, and the surfaces of the different humours of the eye, are so situated as to have one strait line perpendicular to them all. This imaginary line ACDF is called the *axis of the eye*. And the axes of both eyes produced, are called the *optic axes*. Axis of the eye. Optic axes.

12. Having now described those parts of the eye, that are more immediately concerned in vision †; I shall next subjoin a table of their dimensions, at a medium, from several observations of Monf. *Petit*. The *French* measures are reduced to my hand to the *English*, by the learned Dr. *Jurin*, in his curious *Essay upon distinct and indistinct vision*, published at the end of Dr. *Smith*'s Optics. In the table the tenth of the *English* inch is made the unit; and tho' the parts are given to the hundredth part of this unit, or the thousandth of an inch; yet it is not to be imagined, that we have the dimensions of the eye to that exactness; these small fractions having their rise only by the reduction of the *French* measures (which I do not find given to less than $\frac{1}{4}$ of a line) to the *English*.

13.

† For the sake of having a description of the eye all together, I shall borrow from the anatomists a brief account of some of the other parts.

The sclerotica is embraced on its out-side with six muscles, which serve to roll the eye in its orbit all the different ways, for the sake of seeing objects not directly before us without moving the head; and to facilitate these motions, it is observed that all the vacant places, between the orbit and the sclerotica, are filled with a loose fat. Four of these muscles are strait, and the other two oblique; of the strait muscles two of them are situated vertically opposite to one another; and the other two horizontally. Each of the six, according to its situation or office, has a proper name.

In the upper and outer part of the orbit, is seated the *lachrymal gland*, whose use is to furnish at all times water enough to keep the outer surface of the eye moist, without which the cornea would be less pellucid. And that this liquor may be rightly disposed of, we frequently close the eye-lids, even when we are not conscious of it. At the inner corner of the eye between the eye-lids stands a caruncle, whose office seems to be to keep that corner of the eye from being totally closed, that any tears or gummy matter may flow from under the eye-lids when we sleep, or into the puncta lachrymalia, which are little holes one in each eye-lid near this corner, for carrying into the *ductus ad nasum* any superfluous tears.

Both the eye-lids are lined with a membrane, which also folds so much of the eye as is called the white. This membrane is named *conjunctiva*, also *adnata*; and besides other uses it may have, one very considerable is, to hinder any dust or extraneous particles from getting behind the eye into the orbit.

To give a more minute account of these, and some other parts of the eye, which have yet been here mentioned, is the proper province of anatomists.

13. A table of the * dimensions of some of the parts of the human eye, at a medium from several eyes.

	Tenths.
The axis AF of the eye from out to out, is about	9.5
The radius of the outer surface of the cornea	3.35
The radius of the *sclerotica*	4.61
And the distance of its center from the cornea	4.89
Thickness of the sclerotica	0.25
Whence the diameter of the eye from the outer surface of the cornea to the choroeides	9.25
Distance between the outer surface, of the cornea and crystalline	1.06
Radius of the anterior surface ECE of the crystalline	3.31
Thickness or axis CD of the crystalline	1.85
Radius of the hindermost surface EDE of the crystalline	2.50
Breadth or chord EE of the crystalline	3.73

Let r be the center of the cornea, s the focal center of the crystal-
207. I. line; then we shall have $Cs = {}^{a}1.05$, $Ds = {}^{a}0.8$; and consequently $As = 2.11$, and $sr = 1.24$

14. The diameter *aa* of the pupil, varies according to the degree of light the eye is exposed to; † it being bigger or less, as the light is fainter or brighter, that so in all cases the quantity of light entering the eye, may be the better suited for the purpose of vision. The pupil also contracts a little in the same degree of light, when we look near at a small object; but the limit of its aperture, in the eyes of adult persons, is, I think, from about $\frac{1}{4}$ to $\frac{1}{10}$ or $\frac{1}{12}$ of an inch.

15. *Of the refractions of the different humours:* It is very difficult, I think, to determine accurately the measures of these refractions; but from such ‖ experiments as could be made, it has been found, that the refractive

* I found it necessary to qualify some of these numbers a little, that the parts of the eye might the more uniformly join; some incoherence in the measures unavoidably arising from some distortion at the time they were taken: however, upon the whole, their agreement is greater than could be expected.

† Of this any one may be satisfied, by looking in a glass whilst a lighted candle is moved transversely before him. At the approach of the candle the pupil will contract, and enlarge as the candle is moved sideways. If instead of the looking glass be used a small concave speculum, the change in the pupil will be more sensible, as the eye will be then magnified. The experiment may be also made, without the candle, by turning to and from a window.

‖ This experiment was made, by pressing the humours of an ox's eye into a hollow prism, formed by two glass planes cemented together in a given angle; and this seems to be the best method of making it: or it might be made by pressing the humour into the shape of a lens, between two glasses properly figured. But the difficulty lies, to know what changes these humours may undergo after the animal is dead, and also after they are bruised, and taken out of their proper vehicles; and whether the humours of different animals refract alike. If an eye be placed in a small hole in the window-shutter of a darkned chamber, and the bottom of the sclerotica be pared till it becomes transparent, the pictures of objects that are on the outside will be seen on the bottom of the eye: and if while the eye is in this position, the crystalline be couched or depressed, and a paper be held behind, the focus of the two humours that remain may be found: and thence knowing the figure of the cornea, the refractive powers

refractive powers of the aqueous and vitreous humours, are each of them much the same with that of common water; and that of the crystalline is a little * greater: That is, the proportion between the sines of incidence and refraction out of air into the cornea or † aqueous humour, is as 4 to 3; out of the aqueous humour into the crystalline, as 13 to 12; and out of the crystalline humour into the vitreous, as 12 to 13.

16. The cornea and the aqueous and vitreous humours being supposed to have the same refractive power; all three may be looked upon as one dense medium, whose refractive surface is the cornea; and the crystalline humour may be considered as a convex lens, placed in a given position within the said medium; whence the humours of the eye all together make a kind of a compound lens, whose effect in refracting rays, having a given focus of incidence, may be found as by the following

PROPOSITION I.

Having the focus of incidence of rays upon the cornea of the eye, and the refractive powers, radii, and positions of the cornea and of each surface of the crystalline humour; to find the focus of the rays, after their last refraction, by the posterior surface of the crystalline.

Let the refractive surfaces of the eye be as above supposed in *Art.* 13, and 15; and let GF be the axis of the eye; F the principal focus of the cornea; G the focus of rays parallel to the axis within the crystalline, after refraction by the surface C at their entrance into the aqueous humour; and H the focus of rays coming parallel to the axis out of the vitreous, upon the surface D of the crystalline humour, after refraction by that surface. Then we shall have Fig. 2, 3.

AF = [a] 13.4; CG = [b] 39.72; and DH = [a] 32.5. [a] 126. I. [b] 127. I.

17. CASE 1. *To find the focus of parallel rays, after all their refractions by the three refracting surfaces of the eye.*

1. Having found the focus F of parallel rays refracted by the cornea, it will be, Fig. 2.

2.

powers of the aqueous and vitreous humours taken together will be known; and supposing the refractive powers of these two to be the same, that of the crystalline may now be easily found.

* It is observed, that those who have been couched for a cataract (which is a depression of the crystalline humour to the bottom of the eye) are obliged to use convex glasses for obtaining distinct vision; which is a proof that the crystalline refracts light more than the adjacent humours.

† It is supposed that the refractions of the cornea and aqueous humour, are much the same: But if it was otherwise, no sensible difference could thence arise, because the cornea is nearly of the same thickness.

142. I. 2. FG : FC :: FC—3.31 : Fb; (see *fig.* 47. Book I.) that is, 52.06(=39.72+13.4—1.06 or CG+AF—AC) : 12.34(13.4—1.06) :: 9.03 : 2.14.

3. To find the focus Y, after the last refraction by the surface D;
144. I. say Hb : HD — 2.5 :: [d]Db : DY. See *fig.* 45. Book I.

Now Db (=AF—AD—Fb=13.4—2.91—2.14) = 8.35; and Hb (=HD+Db=32.5 + 8.35) = 40.85.

Wherefore 40.85 : 30 :: 8.35 : 6.13 = DY; whence AY (=AD+DY) = 9.04. *Q. E. I.*

18. Case 2. *Having the focus of rays diverging or converging upon the cornea, to find their focus after the last refraction by the posterior surface of the crystalline.*

Fig. 3. Let Q be the focus of incident rays, and f the focus of rays coming parallel to the axis out of the aqueous humour upon the cornea; say,

1. Qf : QA :: [c]QA + 3.35 : Qa. Let QA be 8 inches or 80 tenths; then 69.95 (=80—10.05) : 80 :: 83.35 : Qa=95.325. See *fig.* 43. Book I.

Now Ca (=95.325—80—1.06) = 14.265, and aG (=14.265 + 39.72) = 53.985.

2. Say aG : aC :: [c]aC—3.31 : ab. (see *fig.* 47. Book I.) that is,

53.985 : 14.265 :: 10.955 : ab = 2.895—.

Now Db (=Ca—CD—ab=14.265—1.85 — 2.895) = 9.52 and Hb (=32.5 + 9.52) = 42.02.

3. To find the focus q after the last refraction at D, it will be, Hb : HD—2.5 :: [d]Db : Dq. (see *fig.* 45. Book I.) that is, 42.02 : 30 :: 9.52 : 6.797 = Dq. *Q. E. I.*

After the same manner, by 142 to 144. I. may be found the focus q of the refracted rays, wherever the focus of incidence Q is placed. *Vide fig.* 43 to 48, Book I.

[a] 13. II. [b] 17. II. [c] 18. II. 19. *Schol.* 1. Let t be the bottom of the retina; then Dt=[a]6.34, Yt=[b]0.21, tq=[c]0.45, and Yq=0.66. And hence if it be supposed that parallel rays, and rays diverging from a focus at the distance of 8 inches from the cornea, are equally suited for distinct vision; the retina should bisect Yq; and therefore the axis of the eye should be either a little longer, or its refractive power a little greater, than what has been above

above supposed. However, if we argue from the places of Y, t and q, the measures of the eye are pretty consistent.

20. *Schol.* 2. By supposing t a focus of incidence, we may, after the manner shewed in the preceding *Prop.* find a focus Q, from whence rays diverging upon the eye would be united at the retina; and according to the numbers in *Art.* 13, 15, AQ = 23.956 or 24 inches nearly. Whence, if our measures of the eye could be depended upon, it might reasonably enough be concluded, that 24 inches is the natural distance of distinct vision; or that distance at which the eye would see an object perfectly distinct, without any straining or effort of any of its parts. And if the axis of the eye be supposed a little longer, so as the retina may bisect Yq, the said distance AQ will be less than 24 inches. But to assign that distance from the measures of the eye, would be a vain attempt; for a small error in these measures, would produce a very great one in the situation of the point sought, Q.

21. *Schol.* 3. If it be supposed that the pictures of objects, when seen distinctly, are formed upon the retina; it is manifest that a different conformation of the eye is necessary, for obtaining distinct vision at different distances, either as to the length of its axis, or as to the figure or position of the cornea or crystalline. But which way soever this different conformation is effected, as small a change in the eye as will suffice for procuring a distinct vision of objects, from a moderately small distance, to double that distance; will also suffice for all distances, from that double distance to infinity.

Let AQ be the axis of the eye, AB its semi-aperture; p the place of the nearest object, Q a second object; so that pQ = pA or pB, and let kB be a ray parallel to the axis. Now because Bp = pQ, the ∠ pBQ = BQp = kBQ; whence it is sufficiently manifest, that if the eye is so adapted as to unite a ray pB with the axis, at the point where the retina intersects it; as great a change in the eye is requisite to bring a ray QB to unite with the axis and the retina, as will afterwards be necessary to bring the ray kB to an union with the axis of the eye and retina. Fig. 3.

22. *Schol.* 4. The focal center of the eye, (or that point O in its axis, at which an object, and its image upon the retina, form both the same angle) is nearly, where the hindermost surface of the crystalline intersects the axis. Fig. 2, 8.

The focal center of the cornea alone, is its center r; and the focal center of the crystalline alone is the point s, so that sD = [a] 0.8, and sr = [a] 1.24. Whence it is sufficiently manifest, that the focal center of the [a] 13. II.

eye, or of the cornea and cryſtalline taken both together, muſt be ſomewhere betwixt s and r. Let f be the principal focus of rays coming out of the aqueous humour upon the cornea, g the principal focus of the cryſtalline humour, and let the ſpace sr be ſo divided in O, that $sO : Or :: sg : rf$; it is pretty evident that the point O thus found, will be the focal center of the eye, at leaſt nearly *; to find which we ſhall have $rf = 13.4$, $sg =$ [a] 17.56, and $sr = 1.24$; whence $sO = 0.7$. But $sD = 0.8$, and $Dt = 6.34$, and therefore $Ot = 6.44$, and $AO = 2.81$. *Q. E. I.*

[a] 185. I.

Fig. 4. 23. *Corol.* 1. In every oblique pencil which enters the eye, that ray is to be eſteemed the axis, which interſects the axis of the eye in O, as repreſented by the black lines in the annexed figure. And therefore in an oblique pencil, the axis is not the middlemoſt ray; and the greater is the obliquity of the pencil, the more remote will its axis be from the middle; and when the obliquity is very great, the axis will be excluded from entering into the pupil.

24. *Corol.* 2. Having the angle ſubtended by any object at the eye, by making Ot radius, we ſhall have the diameter of its image upon the retina. Thus, a circle facing the eye, and ſubtending an angle thereat of 1 *min.* will have for its image a circular ſpot upon the retina, whoſe diameter is about $\frac{1}{1340}$ part of an inch.

§. 2. *Of the manner of viſion.*

The nerves the organs of ſenſation.

25. It is ſuppoſed, without any controverſy, that the nerves (which are diſtributed all over the body, and center in the brain) are the immediate organs of ſenſation. And there ſeems no room to doubt, but that our ſenſations are immediately cauſed by impulſes from external bodies, either by touching our bodies in the groſs, or by ſome ſmall material particles ſome how propagated from them. Of the firſt ſort, is the ſenſe of *feeling*, ſo called; and not unlike that, is alſo the ſenſe of *taſting*, the bodies we taſte being in immediate contact with the palate, tho' diſſolved into ſmaller particles, which either by their different texture, figure or ſizes, give us the ideas of different taſtes. *Smelling* is cauſed by the impulſes of ſmall particles, or effluvia, propagated from the odoriferous body: While the idea of ſound, or of *hearing*, is excited by

* Authors differ very much about the ſituation of the focal center O; ſome placing it in the center of the eye, others in the vertex of the cornea; ſome in the middle of the cryſtalline, and others in the middle of the pupil; but for the reaſon above aſſigned, the point O cannot be far from where we have placed it. The method of finding the focal center of two or more lenſes, or refracting ſurfaces any how combined, will be ſhewed hereafter; but for our preſent purpoſe, the ſummary explanation above is ſufficient.

by impulses of the air (put into an undulating motion, by a tremor or vibrating motion of the sonorous body,) on the drum of the ear *.

26. And we can make no doubt, but that light propagated from visible bodies, by striking upon the proper organ, is the mediate cause of vision.

Light, the mediate cause of vision.

Thus, for example, the pencils of light BPB, BQB, BRB propagated from the points P, Q, R of an object PQR, are first refracted by the cornea, so as to belong to the focus's *a*, *b*, *c* respectively, at some certain distance behind the eye; then by the anterior surface of the crystalline humour, they are again refracted towards the focus's *h*, *i*, *k* nearer to the eye than before; and lastly, in going out of the crystalline into the vitreous humour, they are refracted so as to unite in the points *p*, *q*, *r*. In like manner, the pencils of rays coming to the cornea from every physical point of the object PQR, are, by the different refracting surfaces of the eye, brought orderly to unite upon the retina, and there form as it were an image *pqr*, of the object; but in an inverted position, the rays belonging to P uniting at *p*, &c.

Fig. 4. Light, how refracted by the eye, a picture formed upon the retina.

Thus, the cavity of the eye is a kind of † *Camera Obscura*; the cornea and crystalline making a sort of compound lens, whose aperture is limited by the breadth *aa* of the pupil.

27. Sir *Isaac Newton* seems to think that sensations are excited in the brain by a vibrating motion of the nerves. *Newton*'s *Optics*, Q. 12. pag. 319. "Do not the rays of light in falling upon the bottom of the eye excite vibrations in the *tunica retina*? Which vibrations, being propagated along the solid fibres of the optic nerves into the brain, cause the sense of seeing. For because dense bodies conserve their heat a long time, and the densest bodies conserve their heat the longest, the vibrations of their parts are of a lasting nature, and therefore may be propagated along solid fibres of uniform dense matter to a great distance, for conveying into the brain the impressions made upon all the organs of sense ‖."

Sir *Isaac Newton*'s thoughts concerning the manner of vision.

* How exquisitely small must be the effluvia of bodies, and how delicate our nerves, to be moved by such tender impressions; and not only that, but to be affected differently by such an innumerable variety of them! How vast are the variety of smells, and what a long time will some odoriferous bodies, preserve their odour, without a sensible diminution of their weight! What an immense space an inch of tallow is capable of filling! For a lighted candle may be seen at the distance of some miles, and the motion of light is in a manner instantaneous. What an infinite variety of sounds can we distinguish; and how surprising is the effect of musick! Besides these effluvia's of bodies that immediately affect our senses, there may be a great many of other sorts that are constantly emitted: Magnetism is one remarkable instance; and electricity is a wonderful phænomenon. The flashes of lightning from rubbing a glass tube in the dark; the spray of the sea against the sides of a ship under a fresh gale; the shining of touchwood; are instances, among several others, that bodies may emit light without being in what we call a burning state. And how does that light differ from that of fire, except in degree?

† For the theory of the *Camera Obscura*, see *Art.* 243, to 246. Book I.

‖ See also Q. 16. latter end.

Of colours, how perceived; by the same author.

28. In the next *Query* (13.) the same great author proposes the following most beautiful hypothesis, as a solution of the phænomena of colours. "Do not several sorts of rays make vibrations of several bignesses, which according to their bignesses excite sensations of several colours, much after the manner that the vibrations of the air, according to their several bignesses, excite sensations of several sounds? And particularly, do not the most refrangible rays excite the shortest vibrations for making a sensation of deep violet; the least refrangible, the largest, for making a sensation of deep red; and the several intermediate sorts of rays, vibrations of several bignesses, to make sensations of the several intermediate colours?"

Sir *Isaac Newton*'s opinion, that visible sensations are communicated to the brain by the mediation of a fine æther.

29. In a manuscript of Sir *Isaac Newton*'s, a copy of which I was favoured with by the curious and learned *William Jones* Esq; that great author's opinion is, that the sensations caused by the impulses of light upon the retina, are communicated to the brain or sensorium, by the mediation of a fine æther: His words see below *.

Structure of the eye wisely contrived.

30. In order to form distinct vision, the structure of the eye itself plainly indicates, that a certain quantity of rays from every visible point of an object, are necessary to be united at the fund of the eye; and that the points of union of the rays of the different pencils, must be distinct and separate from one another, at least that there should be but as little mixture of them as possible. For the effecting of these purposes, the structure of the eye is admirably contrived: All the rays coming from any visible point of an object, that can enter the pupil, are united closely together upon the retina, and thereby make a vastly greater impression, than a single ray alone could do; and for this end, the retina is placed at a proper distance behind the refracting surfaces. In like manner, each pencil of rays is refracted orderly into distinct focus's,

* 'Light seldom strikes upon the parts of gross bodies, (as may be seen in its passing through them;) its reflection and refraction is made by the diversity of æthers; and therefore its effect upon the retina can only be to make this vibrate: which motion then must be either carried in the optic nerves to the sensorium, or produce other motions that are carried thither. Not the latter, for water is too gross for such subtile impressions; and as for animal spirits, tho' I tied a piece of the optic nerve at one end, and warmed it in the middle, to see if any airy substance by that means would disclose itself in bubbles at the other end, I could not spy the least bubble; a little moisture only, and the marrow itself squeezed out. And indeed they that know how difficultly air enters small pores of bodies, have reason to suspect that an airy body, tho' much finer than air, can pervade and without violence (as it ought to do) the small pores of the brain and nerves, I should say of water; because those pores are filled with water: and if it could, it would be too subtile to be imprisoned by the *dura mater* and skull, and might pass for *æther*. However, what need of such spirits? Much motion is ever lost by communication, especially betwixt bodies of different constitutions. And therefore it can no way be conveyed to the sensorium so entirely, as by the æther itself. Nay, granting me, but that there are pipes filled with a pure transparent liquor passing from the eye to the sensorium, and the vibrating motion of the æther will of necessity run along thither. For nothing interrupts that motion but reflecting surfaces; and therefore also that motion cannot stray through the reflecting surfaces of the pipe, but must run along (like a sound in a trunk) entire to the sensorium. And that vision thus made, is very conformable to the sense of hearing, which is made by like vibrations.'

focus's, that the whole object may at once be distinctly seen. If the rays of light were not thus refracted, very few of them would strike upon the least sensible point of the visionary nerve, wherever that was placed, and the rays from different objects, or from different parts of objects, would strike on the same place at once, which must have created an indistinctness equal to blindness.

When the light is weak, or strong, the pupil is accordingly enlarged or contracted, for the admission of more or fewer rays; that the impressions upon the retina, in both cases, may be the more suitable.

The crystalline humour being denser towards the [a] middle than elsewhere; it is reasonable to suppose, that this humour is not equally refractive, the denser, or middlemost part, being more refractive than the rest *. This different density, of the constituent parts of the crystalline, is admirably contrived for correcting the aberration from its figure, as well as that of the cornea †. The rays in each pencil that are more and more remote from the axes, by passing through a medium gradually diminishing in density, from the middle towards the extremes, have their focus's gradually lengthened, which corrects the aberrations of the figure, that so they may all unite nearer together. The concave figure of the retina is also serviceable in this case. [a] 8. II.

31. It is indisputably true, that objects are seen by means of the impulses of the rays of light, propagated from them upon the visionary nerve; and we have the concurrent testimony of all our senses, that visible objects, or such of which we have visible ideas, do really exist somewhere, that is, at some, may be uncertain, distance ||, in the direction of those rays, or of the principal ray in each pencil. And hence it necessarily Why objects are seen erect.

* It is constantly observed, that in mediums of the same kind, but of different densities; that which has the greater density, has the greater refractive power.

† This conjecture appears so natural, that it looks as if this important end be one main design of the crystalline. For if we consider it as a mere lens, for shortening the focus's of rays refracted by the cornea; that end might have been obtained by a much less apparatus, by only increasing a little the convexity of the cornea, and the density of the humour behind it: Perhaps it also helps to accommodate the eye for distinct vision, at different distances. In the eyes of fishes the crystalline is globular, having in the middle a small nucleus or kernel, much harder than the rest. This figure causes a greater aberration than that of a thinner lens, to correct which, the kernel in the middle is accordingly harder in proportion, than the middle of the crystalline in the eyes of other animals. A due consideration of the structure of the eye, may furnish useful hints for the improvement of optical instruments.

|| It is almost unnecessary to advertise the reader, that what is here meant, is light coming directly from the object to the spectator, without being bent by reflection or refraction. Images, as those in a looking-glass, &c. excite the same kind of ideas, and that by the same laws of vision, as real objects do; and it is very true, that our ideas themselves have no similitudes, or perhaps any necessary connection, with material bodies. And hence somewhat like an argument has been advanced, to shew that a material world does not exist, or at least does not, as to any thing we know, necessarily exist. But to wave metaphysical arguments, which perhaps are more serviceable for maintaining of feigned hypotheses, than for discovering truth; it is sufficiently manifest that something does really exist, which is inactive, ponderous, endued with some peculiar qualities, which men have learned to modify various ways for their convenience. But if these refined logicians will not allow us the testimony of our senses, they will not, 'tis to be hoped, deprive us of our religion too; and does not that suppose the existence of a material world?

neceſſarily follows, that we ſhould ſee objects erect, and all their parts in their due order and ſituation; that is, thoſe parts that are to the right and to the left, we perceive in theſe ſituations. For though inverted images of objects are, as it were, formed at the bottom of the eye, yet it cannot be ſuppoſed that the eye, or the mind ſees theſe images, as ſuch: For the eye ſees no part of itſelf, and even external objects muſt be at a competent diſtance from it, to become diſtinctly viſible.

What is usually meant by an image upon the retina.

32. The impreſſions upon the retina, of the ſeveral pencils of light propagated from an object, will be neceſſarily more or leſs diſtinctly perceived, according as the imaginary image there formed by them, is more or leſs perfect; and ſo for ſhortneſs of expreſſion, an image upon the retina is often ſpoke of, to denote the degree of diſtinctneſs of a viſible object; and alſo to denote in general, the impreſſions of the pencils of light upon the retina, when the diſtinctneſs of viſion is not particularly conſidered.

Ideas accompanying different ſenſations acquired by experience.

33. As all our ſenſations are ſome how cauſed by impreſſions from external bodies, ſo particular ſenſations never fail to excite in us the ideas of particular bodies; and theſe ideas, or ſomething at leaſt in the compoſition of them, we undoubtedly acquire by experience. How elſe could a particular ſound produce the compound idea of a bell, a drum, a man, an acquaintance? And language is a ſurpriſing inſtance of the great ſtrength and quickneſs of our retentive faculties, in aſſociating together a vaſt variety of ideas, which before were learned and treaſured up, as the proper ſignifications of ſuch particular words or ſounds. In like manner, different ſmells and taſtes are accompanied with ideas, which before had been treaſured up in the memory. Our ſenſations themſelves are probably coeval with us, and perhaps they differ but little all our lives, except in degree of quickneſs or perfection: It is the ideas that accompany them, which are acquired; and theſe for the moſt part are got ſo early, that we forget our having ever learned, as it may not improperly be called, to ſee and to hear, *&c.* And yet ſo true it is, that we have as it were learned ſeeing and hearing, *&c.* that an adult perſon, who had been blind or deaf from his infancy, would not be able immediately to make any great uſe of his new acquired ſenſe, till he had time and opportunities to make his proper obſervations, as others had done in their childhood *. The theory of viſion, as little as we know of it, is yet better underſtood than that of either of our other ſenſes. And it is very manifeſt, that the ſame object placed in a different light, in a different poſition with reſpect to the optic axes, or at a different diſtance from the eye, will have a different image upon the retina;

* See *Phil. Tranſ.* N° 402; and *Tatler*, N° 55.

retina; and yet innumerable variety of these circumstances will cause no difference in the visible idea; experience having taught us, that these different sensations proceed from those different circumstances, we instantly, and imperceptibly, form the same idea of the object that we were used to do.

34. There seems but little room to doubt, but that like sensations do always excite like ideas, although the causes of these similar sensations may be different. The flashes of lightning sometimes perceivable from a sudden blow upon the eye, or other part of the head, are probably owing to the same kind of motions being excited in the optic nerve by the blow, as by the impressions of real lightening. After the same manner we may make some probable conjectures, as to the causes of other phænomena of the like sort. As the circle of colours, like those in the feather of a peacock's tail, that appear upon pressing in the dark one corner of the eye with the finger, are probably owing to the like motions being excited at the bottom of the eye, by the pressure of the finger, as at other times are excited there by light for causing vision *. And these phænomena do somewhat corroborate the hypothesis, of vision being caused by some motion excited in the optic nerve, by the impulses of light upon the bottom of the eye.

Like sensations, tho' from different causes, excite like ideas.

35. The ideas we have from the impressions of light, do not instantly vanish, but remain for some small time. This must undoubtedly be true, with respect to all our sensations; for all motions must be performed in time, though imperceptible to us. The phænomenon of a burning coal appearing like a ring of fire, when whirled nimbly about in the circumference of a circle, seems to prove that visible sensations have a sensible duration. For the impressions of light from the coal in every part of the circle, do probably remain entire on the eye, till they are renewed by the arrival of the coal at the same part again; otherwise, instead of a ring of fire, we should see the coal distinctly in the successive places, as the case is when it is moved but slowly. In like manner, in going suddenly out of a strong light into the dark, the impressions of the light are perceptible for some time; and the uneasiness we feel at such times, shews that a sudden change, even from light to darkness, is hurtful to the eyes: The same kind of uneasiness is likewise felt, in going suddenly out of a dark room into a strong light. These phænomena do also shew, that the eye requires some time to adapt itself to different degrees of light. These and some other phænomena, † Sir *Isaac Newton* brings to corroborate his hypothesis, that the motions excited by the impressions of light upon the retina, are of a vibrating nature;

Visible impressions have some duration.

And last about a *second* of time.

* *Newton*'s Optics, *Quer.* 16.

† Ibid.

ture; and moreover, he deduces from them, that motions once excited at the retina, continue there about a *second* of time before they ceaſe.

§. 3. *Of apparent motion.*

Motion how perceived.

36. In like manner, as the different ſituations of ſeveral bodies, or of the ſeveral parts of the ſame body, whilſt at reſt, are perceived by the impulſes of the light proceeding from them upon correſpondent parts of the retina; ſo alſo are the motions of bodies, perceived by the ſucceſſive progreſs of theſe impulſes over different parts of the retina. Thus, if an object moves from P along the line PQ, its image upon the retina will alſo move from *p* along the line *pq*. Fig. 6.

A ſpectator in motion, ſees an object at reſt move the contrary way.

37. The ſame phænomenon, as above, will happen, if while the object P is at reſt, the eye be carried parallel to PQ the contrary way; for the image of the object P will in both caſes be moved alike along the line *p q*. And in the two caſes, if the velocity of P and the eye be the ſame, the apparent velocity of P will be the ſame alſo *.

This accounts why a paſſenger carried in a coach or boat, ſees, or imagines he ſees, the adjacent trees or buildings move, juſt at the ſame rate, as a perſon at the place he looks at would ſee his vehicle move. And this deception, if it may be ſo called, will be the ſtronger, as he is the leſs ſenſible of his own motion, and the quicker it is; for then he has the leſs incitement, and the leſs time, to inform his judgement of the true cauſe †.

Fig. 6. Phænom. of a ſpectator and an object both in motion.

38. *Corol.* An object moving along a line PK, will appear at reſt, to a ſpectator moving with the ſame velocity along the line QG, parallel to PK. And if the motion of the object, is quicker or ſlower than that of the ſpectator, it will have an apparent motion, which will be direct or retrograde accordingly. If the two motions are contrarily, the apparent motion of the object will be ſo much the greater, in proportion to the real motion of the ſpectator. To paſſengers in a boat on a broad river, the phænomena here explained do frequently occur; and they are perhaps more agreeable to theory, becauſe we are leſs uſed to them, than to objects at reſt.

PROP.

* Hence, if the earth be ſuppoſed to move round its axis from weſt to eaſt, while the heavenly bodies are at reſt, they will appear to us, to move the contrary way from eaſt to weſt; there being nothing in this caſe to indicate to us our own motion. And therefore no argument drawn from the apparent diurnal motions of the ſtars and planets, can be of any ſupport to either the *Ptolemaic* or *Pythagorean Syſtems*.

† Thus a perſon walking or riding ſlowly, tho' he perceives the change of ſituation of adjacent bodies, yet being ſenſible of his own motion, and having time ſufficient to reflect in the intervals of theſe apparent changes, thoſe bodies appear nevertheleſs to keep in their places. But if he runs or rides ſwiftly, he cannot help fancying the bodies he looks at, to move towards him. The deception is ſtill ſtronger, when one ſits at eaſe. In coming up the river on board a ſhip, I could not help fancying that the veſſels a-head came at a good rate to meet us, and being in ſome dread of their running foul upon us; tho' their want of ſails, was an indication to undeceive me.

PROP. II.

39. *The same degree of velocity appears greatest, when the motion is in a line perpendicular to the optic axis; and when the motion is in other directions, the apparent velocity will be, as the cosines of the angles of inclination to the said perpendicular.* Apparent velocity in oblique directions to the optic axis.

Thus, if two bodies set out at the same time from P, the one moving along the line PQ perpendicular to the optic axis Q*q*, and the other along the line PS obliquely to it; and if their velocities be such, as to pass over the lines PQ, PS in the same time, it is manifest that their apparent velocities will be the same; for the images of each, will pass over the same space *pq* at the bottom of the eye, in the same time. Their real velocities in this case being as PQ to PS; it is manifest that when the velocities are equal, *the apparent velocity of the body which moves in* PQ : *the apparent velocity of the body moving in* PS :: PS : PQ :: *Radius: Cosine of the* ∠ QPS. Therefore, *&c.* *Q. E. D.* Fig. 6.

40. And we can no otherwise perceive motion directly towards or from the eye, than by the change of the apparent magnitude or brightness of the object: For in that case, the image of the moving body, keeps its situation upon the retina invariable. Motion in the optic axis how perceived.

PROP. III.

41. *If two or more objects at different distances from the eye, move in parallel lines nearly at right angles to the optic axis; and if their velocities are proportional to the said distances, their apparent velocities will be all equal: And if their real velocities are equal, their apparent velocities will be reciprocally as their distances from the spectator* *.

1. Let an object move from Q to P, in the same time that another moves from G to H, and it is manifest that their apparent velocities will be equal; for the space *qp* upon the retina, will be passed over in the same time, by the image of each; and their real velocities are, as the lines QP, GH; but QP : GH :: QO : GO. *Q. E. D.* 2. Let the velocities of the objects G and Q, be equal; then will the object G arrive at K, and its image describe the space *qk* upon the retina, in the same time that the image of the object Q describes the space *qp*; and therefore the apparent velocities of these two objects are as *qk* and *qp*, or as GK (=QP) and GH; that is, the apparent velocity of the object G, is to that of Q; as QO to GO. *Q. E. D.* Fig. 6.

P PROP.

* It is here supposed, that the spectator makes no allowance for the different distances; when he does, the phenomena may disagree with the theory.

PROP. IV.

Apparent velocities at different distances.

42. *The apparent velocities, of bodies moving in parallel lines at different distances from the eye, are directly as the real velocities, and reciprocally as the distances; that is, as the velocities divided by the distances.*

Fig. 6.

Let two bodies move from Q, G, in the parallel lines QP, GH; and let the velocity of the object Q, be called V, and that of G, v; and let their apparent velocities be called A, a. I say, that

$$\frac{V}{QO} : \frac{v}{GO} :: A : a :: V \times GO : v \times QO.$$

If the two objects be conceived to move in the same line, suppose GK, then whatever be their velocities, it will be $V : v :: A : a$; and supposing the velocity of the object Q to be the same in QP, as before in GK, it will be A : A in GK :: [a] GO : QO;

[a] 41. II.

And A in GK : a :: V : v; and compounding these, it will be $A : a :: GO \times V : QO \times v :: \frac{V}{QO} : \frac{v}{GO}$. *Q. E. D.*

Objects at rest may have different apparent motions.

43. *Schol.* 1. After the same manner, as objects in motion will have different apparent velocities, according to their different distances from a spectator; so to a spectator in motion, objects at rest will have different apparent velocities. Thus, a passenger in a coach sees the trees in the next hedge move swiftly backwards, whilst those in the field beyond move slower, and those beyond these still slower; and so on, those that are very remote being scarce perceived to move at all. And if a spectator in motion keeps his eye fixed upon an object at some distance, objects which are pretty near that, will appear at rest, as in this case, their images upon the retina will remain fixed; whilst nearer objects will seem to go backwards, and more remote ones progressively forwards, the same way with the spectator.

Objects at rest may seem in motion, and the converse, when the spectator is at rest.

44. *Schol.* 2. If two or more objects, having the same apparent velocity, move all the same way; an object at rest, by which they pass, may appear to move the contrary way, whilst the objects in motion may seem at rest. For no motion among themselves is perceivable, since their images upon the retina keep the same distances from each other; and if the spectator insensibly moves his eye, so as to keep these images in the same place, the image of the object at rest will pass successively over them, after the same manner as if that object had been in motion the contrary way. The same phænomenon may happen, if the single object be in motion either the same or contrary way; only its apparent motion will be quicker or slower, direct or retrograde, according to the different circumstances. Thus, when the clouds move successively over the

the moon, ſhe ſeems to go with their velocity the contrary way, whether that be eaſtward or weſtward.

45. *Schol.* 3. From the preceding propoſitions, it appears how difficult it is, to form a juſt eſtimation of the real velocities of objects, from the apparent ones; ſince we ought to know both the directions and diſtances of the moving objects, neither of which in many caſes, can be gueſſed at with any tolerable accuracy; and tho' we had them both given, the calculus is too perplexed to be made *extempore*. We are in ſome caſes, as above obſerved, even liable to miſtake the way towards which an object ſteers. Degrees of velocity difficult to be eſtimated.

46. Apparent motion, or the motion of the images of a moving object upon the retina, muſt have a certain limited degree of velocity to become perceptible; that is, the ſpace deſcribed upon the retina in a given time, muſt be neither leſs than ſome given ſpace, nor greater than ſome other determinate ſpace. Viſible motion limited within certain degrees of apparent velocity.

47. If the motion of the ſun's ſhade, at the diſtance of five feet from the gnomon that caſts it, be obſerved at about a foot diſtance from the ſhade, when the ſun changes his azimuth about ten degrees in an hour; the apparent motion of the ſhade will be then as ſlow as can well be perceived, and the ſpace deſcribed by it upon the floor, will be about * $10\frac{1}{2}$ Inches in an hour; and the motion of the image of the ſhade, upon the retina of an eye looking at it at the diſtance of 11 Inches, will be at the rate of about $\frac{6}{10}$ Inch in an hour, or .01 of an inch in a minute. Of the ſloweſt apparent motion.

48. By the above ſuppoſition, the ſpace deſcribed in an hour by an object having the ſloweſt perceivable motion, and the diſtance of the eye from the object, are nearly equal: And therefore the ſpace deſcribed in a minute of time, by an object having ſuch a degree of motion, ſubtends an angle at the eye of about one degree: Or the ſpace deſcribed in a min. in a line perpendicular to the optic axis, by an object having the leaſt perceivable motion, is about $\frac{1}{60}$ of the diſtance between the object and the ſpectator. Hence, we ſee why the heavenly bodies are not perceived to move; the ſpaces deſcribed by them in a minute, not ſubtending an angle of above a $\frac{1}{4}$ degree, when their apparent motion is greateſt. The angular ſpace deſcribed in a given time, by an object having the leaſt perceptible motion.

49. It is difficult to aſſign the greateſt angular ſpace, that can be deſcribed in a given time, by an object having the greateſt perceptible motion; and the difference may be very great, betwixt that degree of velocity with which ſomething may be perceived to croſs the optic axis, Of the quickeſt perceptible motion.

 and

* The motion of the ſhade would not be perceivable, if we did not attend to fix marks upon the floor; and when we cannot compare the ſituations of moving bodies with others at reſt, a motion much ſwifter than that of the ſhade will be imperceptible.

and that, with which the ſame object may be diſtinctly ſeen, as it moves: Of this latter ſort, let that of a falling body at the diſtance of 20 feet from the ſpectator, and in that part of its fall when its velocity is at the rate of 19 feet in a ſecond, be the quickeſt perceivable motion; the motion of the image upon the retina at that inſtant, will be at the rate of about $\frac{6}{10}$ of an inch in a ſecond; and therefore upon this ſuppoſition, the quickeſt perceivable motion is to the ſloweſt, as an hour is to a ſecond of time, or as about 3600 to 1.

50. We do not here pretend to aſſign the preciſe limits of either the leaſt or greateſt perceivable velocity; what has been obſerved above, is ſufficient to ſhew that they have their limits, and perhaps theſe limits are not far from thoſe above ſuppoſed; and perhaps alſo, the ratio of the greateſt to the ſloweſt of velocities upon the retina, that are diſtinctly perceivable, is not greater than about 2000 to 1. We have ſeen
48. II. [a]above, that a prodigious great velocity, ſuch as is the diurnal motions of the heavenly bodies, may be yet too ſlow to be perceived; and the flight of a ball out of a gun, is much too quick for an eye not far off.

§ 4. *Of viſion with both eyes.*

What is ſaid below about the manner of communication between both eyes, and of viſion (diſtinguiſhed by the inverted comma's) is taken *verbatim* from the manuſcript of Sir *Iſaac Newton*'s, refer'd to in *Art.* 29.

Deſcription of the optic nerves and their juncture in the brain, by Sir *Iſaac Newton.*

51. "The *tunica retina* grows not from the ſides of the optic nerve "(as the other two * which riſe one from the *dura*, the other from the "*pia mater:*) but it grows from the middle of the nerve, ſticking to it all "over the extremity of its marrow. Which marrow, if the nerve be any "where cut croſs-wiſe betwixt the eye and the union of the nerves, appears "full of ſmall ſpots or pimples, which are a little prominent, eſpecially "if the nerve be preſſed, or warmed at a candle; and theſe ſhoot into "the very eye, and may be ſeen within ſide, where the retina grows to
Fig. 5. "the nerve; and they alſo continue to the very juncture EFGH. "But at the juncture they end on a ſudden into a more tender white pap, "like the anterior part of the brain: and ſo the nerve continues after "the juncture into the brain filled with a white tender pap, in which "can be ſeen no diſtinction of parts as betwixt the ſaid juncture and "the eye †."

52.

* Meaning the choroides and ſclerotica.

† The object PQRS, with the pencils of rays ſuppoſed to flow from it, together with the fore-part of the eyes, are here added to the figure in the Manuſcript.

52. "Now I conceive that every point in the retina of one eye, hath "its correspondent point in the other *; from which two very slender "pipes filled with a most limpid liquor do, without either interruption "or any other unevenness or irregularity in their process, go along the "optic nerves to the juncture EFGH, where they meet either be-"twixt G,F or F,H, and there unite into one pipe as big as both of "them; and so continue in one, passing either betwixt I,L or M,K into "the brain, where they are terminated perhaps at the next meeting of "the nerves betwixt the *cerebrum* and *cerebellum*, in the same order that "their extremities were situated in the retina's. And so there are a "vast multitude of these slender pipes which flow from the brain, the "one half through the right-side nerve IL, till they come at the junc-"ture GF, where they are each divided into two branches, the one "passing by G and T to the right side of the right-eye AB, the other "half shooting through the space EF, and so passing by X to the "right-side of the left eye $\alpha\beta$. And in like manner the other half "shooting through the left-side nerve MK, divide themselves at FH, and "their branches passing by EV to the right eye, and by HY to the left, "compose that half of the retina in both eyes, which is towards the "left side CD and $\gamma\delta$ †."

Sir *Isaac Newton*'s explanation of the manner of vision with both eyes.

Our author proceeds, and draws the following conclusions.

53. "Hence it appears, 1. Why the two images of both eyes, make "but one image *abcd* in the brain."

Why objects appear not double.

54. "2. Why, when one eye is distorted, objects appear double; "for if the image of any object be made upon A in the one eye, and "β in the other, that object shall have two images in the brain at *a* and "*b*. Therefore the pictures of any object ought to be made upon the "corresponding points of the two retina's; if upon A in the right eye, "then upon α in the left; if upon B, then also upon β. And so shall "the motions concur after they have passed the juncture GH, and "make one image at *a* or *b* more vivid than one eye alone could "do."

Why objects appear double when one eye is distorted.

55.

* An hypothesis somewhat like this we have in *Rohault*'s *Physicks*, Part I. chap. 31. *Art.* 20. that is, Mr. *Rohault* supposes with our Author, that the two optic nerves have their correspondent fibres which unite in the brain; and so accounts why objects seen with both eyes appear not double. But these two authors differ in their anatomy of the optic nerves; *Rohault* continuing them separate, without any union at EF, till the sympathetic fibres, as he calls them, of each nerve, ultimately meet in the brain in so many points.

† The substance of this elegant hypothesis, is contained in *Qu.* 15. annexed to our Author's Optics. It is agreed, that the union of the optic nerves at EGFH is so close, that their substances seem to be confounded: But notwithstanding this seeming confusion, some anatomical observations render it at least very probable, that the substance of each of the two nerves, continue distinct without being any ways mixed or united together. *Vide Edinb. Med. Ess.* Vol. 3. pag. 198. We have other hypotheses by other authors; nor is it a wonder that men should differ in opinions, where they have so little light to guide them. However, the reader cannot be displeased to have Sir *Isaac Newton*'s opinion upon this difficult subject, in his own words.

Why two things cannot appear in one place.

55. "3. Why, tho' one thing may appear in two places by distorting "the eyes, yet two things cannot appear in one place. If the pic- "ture of one thing fall upon A, and of another upon α, they may both "proceed to p, but no farther; they cannot both be carried on the "same pipes pa into the brain; that which is strongest or most helped "by phantasy will there prevail, and blot out the other."

56. "4. Why, if one of the branches of the nerve beyond the "juncture, as at GF or FH, should be cut, that half of both eyes "towards the wounded nerve would be blind, the other half remaining "perfect *."

Sir *Isaac* proceeds to account for the structure of the optic nerves, as in the note below †.

57.

* Such an experiment, if it could be made, would very much corroborate the author's hypothesis; or, what would come to the same thing, if upon dissection it be found, that a person who saw only at the right or left side of each eye, had the nerve cut or otherwise rendered ineffectual beyond FH or GF; it would go a great way towards proving a kind of coalescence, or union of the fibres of the two optic nerves, somewhere between that section and the eyes.

† 5. "Why the juncture is almost as broad again betwixt G and H, as between E and F; because all the tubuli of both eyes pass between G and H, and but half of them betwixt E and F."

6. "Why the nerve GILF buts not directly upon the nerve XEHY, but deviates a little towards TV; because its tubuli are to pass only into that side of the nerve EHXY towards EX. The like of FMKH."

7. "Why the marrow of the nerve TVEG grows soft on a sudden, when it comes at the juncture EF, and more suddenly on that side towards G than towards E. And the like of the nerve EXYH: For it being necessary that the nerve TVEG should be stretched and bended several ways by the motion of the eye; therefore the tubuli are involved or wrought up within the substances of several tough skins, which being folded up together, compose the marrow of the nerve pretty solid and flexible, lest the tubuli should be prejudiced by the several motions of the nerve. And those small pimples or prominencies which appear in the nerve cut crosswise, I conceive to be made by the foldings of those crasser skins. But the nerve at the juncture EGFH, being well guarded from all violence and motion by the bones into which it is closely adapted; 'tis not necessary the said membraneous substance should be continued any farther than EG: therefore the tubuli there on a sudden unsheath themselves, that those on the inner side of the nerves towards VE and XE may severally cross 'twixt E, F, and be united with their correspondents on the other sides YH and TG. Now because the inner tubuli must first cross, before they can concur with the outmost tubuli of the opposite nerve; hence it is, that the nerves grow soft sooner on the inner side at E, than on the outer side at G and H."

"8. Why the two nerves meet a second time in the brain: because the two half images carried along IL and MK may be united in one compleat image, in the sensory. *Note*, that the nerves at their meeting, are round about disjoined from the rest of the brain; nor are they so thick there, as a little before their meeting. But by their external figure, they seem as if the *capillamenta* concentered like the radii of a hemisphere to a point in the lower part of the juncture. And 'tis probable, that the visive faculty is there: for else why do the nerves swell there to so great a bulk, as it were preparing for their last office? Why do they run directly cross from either side the brain to meet there, if the design was to have the motions conveyed by the shortest cut from the eye to the sensorium, before they grew too weak. If they were to proceed farther, they might have gone a shorter cut, and in a less channel. There is indeed a marrow shoots from under them towards the *cerebellum*, to which they are united; but the greatest part of their substance, if not all of it, lies above this marrow, and also shoots cross beyond it to the center of the brain where they meet. Lastly, the substance here is most pure,

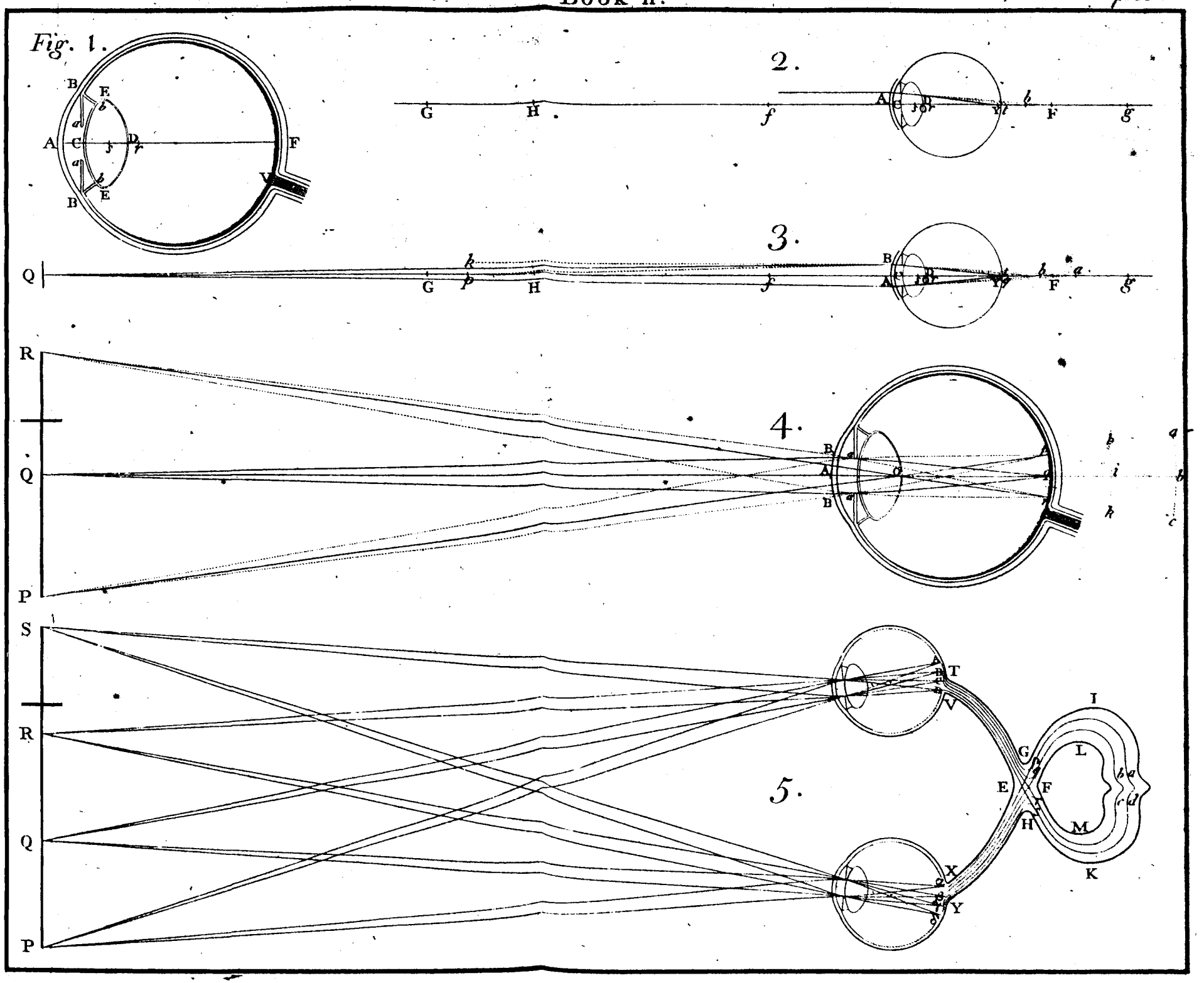
Fig. 1.
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3.
4.
5.
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57. *Defin.* Let *a*AF, *b*BF produced till they meet in F, be the axis of the right and left eyes A, B respectively; the angle AFB contained between these two axes, is called the *optic angle*. And a plain PR touching the point F, so as to be perpendicular to the plain of AFB, and also make equal angles with the axes AF, BF, is called the *Horopter*. The angle P*v*R, *fig.* 7. or POR, *fig.* 4. contained between two lines P*v*, R*v*, or PO, RO drawn from the extremities of an object to the focal center of the eye, is called the *visual angle* of that object; the visual angle then is that, which any object, however situated, subtends at either eye of the spectator. Fig. 7, 8, 9. Optic angle, horopter, and visual angle.

58. So far as observation can reach, it is universally confirmed, that when we look attentively upon any object, the axes of both eyes are directed towards it, or towards that part of it which we more particularly look at; and we are so habituated to turn both our eyes towards the same place, that if one eye be shut, its motion will follow that of the other: As in the figure, the axes of both eyes are directed to the object F, where they both meet and concur. This concurrence of the optic axes, is the reason assigned by some, why an object seen with both eyes appears single; it being impossible for us to conceive two objects existing in the same place, at the same time. To this it may be answered, that the eyes see not the concurrence of these imaginary axes; and tho' we can make some estimation of distances by sight, yet it is scarce allowable that each eye can determine the distance of an object F so precisely, as the case before us requires. Whatever be the cause of it, we are assured of the fact, that when both eyes look at the same object, the mind sees that object single. And it is very certain, that in at least abundance of cases, the visive faculty is assisted by a kind of reasoning, drawn from former experience, tho' perhaps unattended to at the time we use it. And whether the phænomenon before us, is owing to experience, or to some sympathetic correspondence in the parts of the eye, is a very difficult query. But whatever kind of reasons are admitted, to shew why the point F appears not double, may be also easily enough admitted, to shew why the lateral points P, R appear likewise single. Another cause assigned why objects appear not double. Fig. 7.

59.

pure, the situation in the midst of the brain, constituting the upper part of that small passage 'twixt all the ventricles, where all superfluous humours have the greatest advantage to slide away, that they may not incumber that precious organ."

This elegant account is an instance of the great sagacity and penetration of its author, and very much strengthens his hypothesis. Our author proceeds next to give his opinion, of the manner that the impressions of light upon the retina, are communicated or carried to the brain, as in *Note**, *Art.* 29.

When objects appear double.

Fig. 8, 9.

59. When the axes of both eyes are directed to some determinate point F, an object G within the optic angle, *fig.* 8, or its opposite, *fig.* 9, will appear double; that is, instead of being seen at G, it will appear somewhere in the lines *r*G, *s*G; as at *m* and *n*, either farther or nearer than its true place G, according as G is nearer or farther than the horopter PR. And if either eye A or B be shut, while the other is kept open, the object *m* or *n* will accordingly vanish, and so alternately. Whatever hypothesis be admitted, for solving the phænomenon of the single appearance of an object F, to which both the eyes are directed, the same will also serve to solve that of the double appearance of G. The images *r*, *s* of the object G, are now made on contrary sides of the optic axes A*a*, B*b*; and this does not ordinarily happen, but when there are really two objects, as *m* and *n*, on a different side of the mark F. Thus, in *fig.* 8, the image *r* being to the right of the axis A*a*, the mind judges the object to be to the left of that axis, as in the like case of ordinary vision; and for the same reason, the left eye sees the object to the right of its axis. The same argument is also applicable to the case of *fig.* 9; and generally, as near as can be estimated, the apparent distances of *m* and *n* from F, in both cases, are measured by the visual angles FAG, FBG; or the apparent distance *mn*, is as the sum of these angles, or as the sum of the arches *ar*, *sb*. A double appearance may also happen, when G is in one of the optic axes, or even without them both: In the one case, that eye in whose axis G is, will have but one image; and in the other, both the images *r*, *s* will be to the same side of the axes A*a*, B*b*, but the arches *ar*, *sb*, will be unequal; and the apparent distance *mn* will be as the difference of the visual angles, or of the arches *ar*, *sb*.

Seeing two objects, and seeing one double, a different sensation.

60. The double appearance of an object, does perhaps as frequently happen, as we see an object before us that we do not look at, or before we have directed our eyes properly towards it. But we are in no danger of mistaking one object for two, for the sensations we have when we see two objects, and see one double, are very different: In the one case, there are two images as *p*, *r*, fig. 7. formed upon each retina; in the other, there is but one image in each eye, and these images being on parts of the retina's which do not correspond, may be the cause of that painful sensation we generally have, when we see an object double. The two false objects are always indistinct, have a kind of a tremulous motion, approach nearer together, retreat back, or advance forwards, as the case is; and at length, as soon as the eyes have conformed themselves properly for seeing the real object distinctly, the two false objects vanish or coalesce into one, at that determined place where the real object is.

61.

61. Two ſimilar objects, not far aſunder, if they happen to be one in each optic axis, may alſo appear as one object; and that for a like cauſe, that one object may appear double. But this phænomenon ſeldom happens; and when it does, the miſtake is ſoon found, as in the caſe of an object ſeen double; and the eyes preſently adapt themſelves, to the proper diſtance of the two objects. Two objects may appear in one.

62. When the eyes are directed to a determinate mark, as F, we generally regard only thoſe objects that are pretty near to F, or to the plane of the *horopter* PR, and not thoſe that are remote from that plane. And hence it is, that double appearances of objects do not more frequently happen. Why objects do not often appear double.

63. An object that is a little out of the plane PR, may yet appear ſingle: But the object will not be ſo diſtinct, as after the eyes have properly conformed themſelves to it; and till then, it will alſo ſhift its place by winking either eye, and looking at it with both; but afterwards, it will remain fixed to either eye alone, or to both open. An object may appear ſingle, tho' out of the horopter.

64. By an inſpection of the figures, it is manifeſt enough, that a larger ſegment of a round object can be ſeen with both eyes open, than with one eye alone: And the reaſon is likewiſe manifeſt, why when we would bring a third object into a ſtrait line with two given ones, as in *levelling*, *ſhooting*, &c. we muſt take our aim, or look by two of them with one eye only; and to prevent miſtakes, it is beſt to ſhut the other eye, as people in ſuch caſes generally do. Fig. 8, 9. Why an aim muſt be taken with one eye only.

65. Perſons that ſquint have not the axes of both eyes directed to the ſame object, and yet they ſee objects ſingle. Squinting may proceed from ſeveral cauſes: one may be, that the correſpondent fibres in the two retina's are differently ſituated, which makes it neceſſary to turn the eyes different ways, that the images of objects may be received upon the correſponding fibres in both: And this alſo accounts, why perſons having ſuch eyes, ſee objects ſingle. It is ſuppoſed, and that with great probability, that there is a ſmall part in the retina of each eye, which is more ſenſible than the reſt; and that we naturally turn our eyes ſo, that the impreſſions of the light may be received upon this moſt ſenſible part: Thoſe who have this part awry, or to one ſide of the optic axes in one or both eyes, neceſſarily ſquint; and yet ſuch Perſons ſee objects ſingle like other men, either becauſe they have learnt by experience, that ſuch is the natural direction of their eyes for looking at the ſame object; or becauſe correſpondent fibres of the optic nerves, proceed to thoſe moſt ſenſible parts in both eyes. It is manifeſt, that ſquinting from either of theſe two cauſes, is neceſſary, and incurable. Of ſquinting.

Sometimes a cold in the head, a lowness of spirits, or long poreing with the eyes, will cause people to squint, that never do at other times. This is a fact I am well satisfied of; and the cause of it probably is, a kind of soreness or tenderness of the retina, which make the impressions of light offensive, or hurtful; to avoid which, the person so troubled turns his eyes about different ways, without being able to fix them attentively to any one object: that is, he avoids as much as he can seeing at all, and then he sees but very obscurely. This distemper vanishes with the cause, and may be remedied generally by using green spectacles with flat glasses, or some other contrivance, to lessen the light coming to the eyes. This soreness may be a fixt distemper in one eye only, and then the person troubled with it, turns this eye awry, not for the sake of seeing better with it, but to avoid seeing with it at all. People may acquire a habit of squinting, especially children, when there is no defect in either of their eyes, by using themselves to look at particular objects with one eye only, till by degrees they acquire the habit of always turning obliquely the other eye. The best method of cure in this case, is for them to look at some object stedfastly with the distorted eye, while the other is shut, then try to look at it with both eyes: And so repeat these trials, till they can look with both eyes at objects in different positions. Fowlers can take their aim better with one eye, than the other; and but few people can see equally well through a telescope with one eye as they can with the other: If they look with the eye they have not been accustomed to, they are obliged to shut the other, before they can see at all. This is entirely owing to practice, as one hand becomes more expert or dextrous than the other.

A circular space at the bottom of each eye insensible of light.

66. Monsieur *l'Abbe Marriote* is the author of a curious experiment, by which he discovered, that there is a certain space at the bottom of each eye, where the optic nerves enter, which is insensible of light. The experiment is to be made in this manner: Upon an obscure wall, at about the height of the eye, fasten two round pieces of white paper; or if the wall is of a bright colour, let the papers be dark; the papers may be a foot or two, more or less asunder: Then placing your self before one of the papers, shut the correspondent eye, that is, if you stand before the left-hand paper, shut the left eye, and looking at this paper steadily with the right eye; if the other paper is visible, move slowly to or from the paper you look at, and at a certain station the other paper will totally disappear, tho' objects, that are more remote from the axis of the eye, are visible at the same time. Thus, if the left eye be shut, the observer must place himself so as to bring the left object F into the axis *a*A of his right eye; then moving to and fro in the line AF, at a certain station the object P will disappear, that is, when its image falls upon the optic nerve *p*.

Fig. 10.

The paper looked at ſhould be ſmall, that the eye may be the more ſteadily fixed upon it; the other may be two or three or more inches diameter, according as the papers are leſs or more aſunder. Or the experiment may be made with only one paper, and from any ſtation, by turning the axis of the open eye gradually towards the noſe. In this manner, I have loſt a ſegment of a ſteeple, whilſt I could ſee the top and bottom of it.

This experiment, which ſucceeds univerſally, proves undoubtedly that there is a certain ſpace in the eye, which is inſenſible of the impreſſions of light; and this inſenſible ſpace being ſituated about the places where the optic nerves enter, it is very probable that it is ſituated at thoſe very places; or, that the optic nerves, or ſome part of the middle of them, at their entrance into the eyes, are inſenſible.

Situation and magnitude of this inſenſible ſpace determined.

67. In order to form an eſtimate of the ſituation, and magnitude, of the inſenſible ſpot in the eye, Mr. *Short* (well known for the excellent performance of his reflecting Teleſcopes) and myſelf, made the following experiment, thus: Having fixed upon a light wainſcot, at a proper height, a dark-blue circular paper of three inches diameter, and another paper of the ſame colour of about $\frac{1}{4}$ inch diameter, to the right ſide of the firſt, at the diſtance of 30 inches, reckoning from their centers: At a medium of ſeveral obſervations, which agreed pretty near together, looking with the left eye on the ſmall paper, the other being ſhut, Mr. *Short* continued to loſe ſight of the large paper, from the diſtance of 7 feet 8 inches, to the diſtance of 11 feet 2 inches, from the wainſcot; and I loſt ſight of the ſame paper, from the diſtance of 9 feet 4 inches, to the diſtance of 11 feet 2 inches. The ſame papers being placed 18 inches aſunder, Mr. *Short* loſt the biggeſt paper, from the diſtance of 4 feet 9 inches, to 6 feet 5 inches: And being placed $7\frac{1}{3}$ inch aſunder, he loſt the biggeſt at the diſtance of 2 feet 3 inches; but by the leaſt motion from this place, recovered part of it again. A paper of 4 inches diameter, was loſt by Mr. *Short* at the diſtance of 3 feet from the wainſcot, and recovered again at the leaſt motion of the head from this place; the diſtance between the centers of the papers being 10 inches. The paper being placed one diameter higher or lower, was ſeen at all diſtances, which ſhews that the inſenſible ſpot in the eye is round.

Taking a medium of the ſeveral obſervations; the $\angle avp$ contained between the optic axis, and a line paſſing through the focal center of the eye, to the center of the inſenſible ſpot, is in Mr. *Short*'s eye about $15\frac{1}{3}$ *deg.* and in mine about $13\frac{1}{2}$ *deg.* The greateſt ſpace that can be loſt by Mr. *Short*, ſubtends an angle of about 6 *deg.* or the diameter of the greateſt circle that can be loſt by him, is ſomewhat greater than Fig. 10.

the $\frac{1}{10}$ part of its diſtance from his eye. The greateſt angle I can loſe does not exceed 4 *deg*. If we ſuppoſe the diſtance *vp* in each eye to be $6\frac{1}{2}$ *tenths*; the diameter of the inſenſible † hole will be, in Mr. *Short*'s eye about 0.68 *tenths*, and in mine about 0.45 *tenths*.

Why we are not ordinarily ſenſible of this defect in our eyes.

68. In looking with one eye, at a large object of an uniform colour and ſurface, we are not ſenſible of this defect, or of the loſs of one part: The reaſon may be, becauſe objects near the place where the defect of viſion happens, being remote from the optic axes, are but obſcurely ſeen; and ſo, a particular part is not miſſed, till it becomes a remarkable object, by its being, as it were, detached from the reſt by a diſtinct figure and a different colour: Nor in viewing objects of uniform ſurfaces and of like colours, do we uſually attend to their parts, but take the whole figure in the lump. To theſe reaſons may be added, that we ſeldom fix our eyes long to one point, but we nimbly, and inſenſibly, turn them towards different parts.

An object cannot be loſt to both eyes at once.

69. It is manifeſt, that an object cannot be loſt to both eyes, at the ſame time; for when the image of an object falls upon the optic nerve in one eye, it will fall at ſome diſtance from it, in the other. See *fig*. 7.

Objects appear a little brighter when ſeen with both eyes than with one.

70. An object ſeen with both eyes appears a little brighter, or more luminous, than it does when ſeen with one eye alone; as will be evident, by looking alternately with both eyes, and with one only: And the difference of brightneſs will be ſtill more manifeſt, if at the ſame time that a part of a flat object, of an uniform colour, is ſeen with both eyes, the light from the adjacent part is excluded from one of them; which may be done by applying a book to one ſide of the head, and ſo as it may reach a little forwarder than the face. But although the difference of brightneſs, in the two caſes, is very perceptible, yet is it not very conſiderable; nor is it eaſy to determine it accurately. Dr. *Jurin* ‖ has given a pretty method of making this experiment, thus: In a room illuminated only with two candles, placed at about the ſame height, and kept burning as near as may be with equal brightneſs, fix a ſlip of white paper flat againſt the wainſcot, at about the ſame height with the flame of the candles, as they ſtand upon the tables: Having placed one of the candles at a determinate diſtance from the paper, as a yard; place the other at a greater diſtance, as ſuppoſe two yards, and a little ſide-ways of the firſt; then place a book, or ſome ſcreen, ſo before this hindermoſt candle, as to prevent its light from falling

† M. *Marriote*, from the bigneſs of this hole, thinks that viſion is performed in the choroides, and not in the retina; and gives ſeveral ingenious arguments to ſupport his hypotheſis: But herein he has had but few followers, and the argument drawn from the obſervation of the nerves being the organs of all our other ſenſations, makes againſt him. But after all, this queſtion ſeems too difficult, to be determined with certainty.

‖ Dr. *Smith*'s Optics, Remarks p. 108.

falling upon one half of the paper. In this caſe the light of the firſt candle being reckoned 1, the light of the ſecond candle, which is at twice the diſtance from the paper, will be only $\frac{1}{4}$; and therefore the light upon that half of the paper which is illuminated by both candles, is to the light of the other half illuminated by one alone, as 5 to 4. Things being thus diſpoſed, apply a book to one ſide of the head, ſo as to hide that half of the paper, which is illuminated by both candles, from one eye; and looking at the paper, if both halves of it appear equally luminous; the degree of brightneſs of an object ſeen with both eyes, and with one eye alone, will be as 5 to 4. But if, as will be the caſe to moſt people, the half illuminated by both candles appears brighter, than the other half of the paper; remove the ſecond candle ſtill farther and farther, till by repeating the experiment, the ſcreen being placed before the ſecond candle, and to one of your temples, as before directed, the two halves of the paper appear equally luminous: Then compute as before; as if the ſecond candle is diſtant three yards, and the firſt one yard, the light of the brighteſt half of the paper will be to the light of the other, as $1\frac{1}{9}$ to 1, or as 10 to 9; and that will be the proportion of the brightneſs of an object ſeen with both eyes, to its brightneſs when ſeen only with one.

71. The ingenious author above quoted, found that an object ſeen with both his eyes, appeared brighter than when ſeen with one eye alone, by about a thirteenth part. By this experiment I find, that my eyes are not both equally good, for the difference will be greater or leſs as I exclude the light from one or t'other eye. But at a medium, I gueſs the difference above to be about $\frac{1}{10}$ or $\frac{1}{12}$, it being very difficult to make the experiment to great exactneſs.

72. In viſion with both eyes, and with one eye alone, the apparent magnitudes of objects are, as near as can be perceived, the ſame in both caſes. Of the truth of this any one may eaſily ſatisfy himſelf, either by looking alternately at the ſame object with both eyes, and with one only; or clapping a book to his temple, as in the preceding experiment, and looking at two or three parallel lines, placed ſo before him as to be equidiſtant from both his eyes; if the lines appear ſtrait throughout, it is a proof that the ſpace between them, appears of equal breadth to both eyes, and to one alone. Objects appear of the ſame bigneſs to both eyes, and to one alone.

73. It may be wonder'd, that there ſhould be ſo little difference betwixt viſion with both eyes, and with one only. Perhaps we beſtow a greater attention in one caſe, than what ſuffices in the other; and that this may be the reaſon, of the ſo little difference in the apparent brightneſs: And the idea of the diſtance of the object, being previouſly fixed. Advantage of both eyes.

fixed in the mind, looking at it with one eye, or with both, does not, it ſeems, alter this idea; which may be the reaſon, of the apparent magnitude being the ſame in both caſes. Theſe are only conjectures; however, the fact is rather advantageous to us, than otherwiſe; that if one eye be hurt or loſt, the other may, as it were, perform the office of both, with very little diſadvantage.

Field of viſion.

74. *Defin.* That conical ſpace indefinitely extended before us, and ſpreading ſo wide, as to include all objects as can be ſeen at one view, or without turning the eyes, is called the *Field of viſion.*

Its extent.

75. It is not eaſy to aſcertain the preciſe limits of the field of viſion, or the greateſt angle that can be ſeen at one view. When we look attentively at a ſmall diſtance before us, we have an imperfect glimpſe of objects for almoſt the ſpace of half a ſphere, or at leaſt for above 60 degrees each way from the optic axes. But towards the extremity of this ſpace, objects are ſeen but very imperfectly; and they are gradually diſtincter, the nearer they are to the point we look at. About this point there is ſome ſmall ſpace, that ſeems at the ſame time equally diſtinct; but to aſcertain exactly the quantity of this ſpace is difficult, becauſe the indiſtinctneſs round it, is ſo very gradual: And this difficulty is increaſed by that of keeping our eyes fixed for any time, on the ſame object; we are apt to move them nimbly, even when we are not conſcious of ſo doing, over the circumjacent objects, and the impreſſions upon the retina[a], having ſome duration, we thence imagine the field of diſtinct viſion to be much larger than it is. But, I think, the diameter of this field does not ſubtend an angle of above five degrees at moſt; whence the diameter of a diſtinct image upon the retina, is leſs than .06 of an inch. A ſmall object, as a ſtar ſeen by twilight, is loſt by a ſmall motion of the eye from it; and if we look attentively upon a letter in the middle of a word, conſiſting of capitals of a middling ſize, the adjacent letters on either ſide will be ſomewhat indiſtinct. Theſe are proofs that the field of diſtinct viſion is very ſmall, and probably it does not exceed one degree, or two degrees at the moſt. The limitation of this field may be owing to the retina's being more ſenſible near the optic axes; or, becauſe the points of union of the rays of the oblique pencils fall ſhort of the retina; or both theſe cauſes may conſpire together.

[a] 35. II.

76. To me there is no ſenſible difference in the bigneſs of the field of diſtinct viſion, when I look with either eye alone, and when I look with both together; only the field is a little brighter in one caſe than in the other.

SECTION

SECTION II.

Of the minimum visibile; *distinct and indistinct vision; of long and short-sighted eyes, and of spectacles; of apparent distances, and magnitudes.*

§. 1. *Of the extent or limits of vision.*

77. AS in a dark chamber, a very slender beam of light is perceptible, so in all cases, when the medium about us is very dark, objects are perceivable by small quantities of light; and the darker is the medium, or the freer the eye is from extraneous light, the less light suffices for vision. Thus, in a dark night, the feeble light of a candle is perceptible at a great distance; and the fixed stars, though they have no sensible diameters, are yet very visible; and the darker the night, the more of them are seen: The eye being now free from other impressions, the mind perceives, and attends to, these delicate ones of those very slender pencils. But even in this case, a certain quantity of light is requisite for vision, without which objects cannot be discerned. The impressions of light from the Satellites of *Jupiter* and *Saturn*, are too feeble to be perceived without the assistance of telescopes, although then these Satellites have sensible diameters; and a small flame will not visible beyond a certain limited distance. Luminous objects by night.

78. At the approach of day, and as the twilight increases, the eye begins to be enlightened by the reflection of the atmosphere, and then gradually more and more; so the stars grow fainter, and disappear by degrees; first the smaller ones, and so on. As the day advances, the moon herself loses of her lustre, and at length her light is overpowered by that reflected from the clouds, and she is no longer seen. Thus, weaker impressions upon the retina, are obliterated and rendered insensible, by those that are much stronger; and this is an easy explication of the above phænomena, and of others of the like kind. Very small particles are seen floating in a beam of light let in to a darkned room; but as soon as the room is enlightened, these particles disappear. A man in a darkish room easily sees those that are in broad day-light, but cannot be seen by them; the light reflected out from the room, compared to the open day-light, being insensible. Stars, why invisible by day-light.

79.

Objects distinguished by their diversities of colour and splendour.

79. The light of the ambient medium, constantly intrudes into the eye, along with the pencils of light proceeding from bodies, having sensible magnitudes; but such bodies, if they are not too far off, are nevertheless very perceptible, as well by their different colour and splendour, or by the different degrees and sort of light reflected by them; and also by their intercepting the light of the ambient medium. A shade becomes an object, or objects are perceivable by their want of light, as a hole in a board, or a dark spot on a light body, are perceived by the unenlightened chasm in the eye, within the illuminated field; and the figure and magnitude of the hole or spot, are distinguished by those of the said chasm. So a steeple afar off is distinguished by its covering a part of the sky, of such a figure and apparent magnitude.

Opacity of the atmosphere, a cause of indistinctness and invisibility of remote objects.

80. It is manifest from many observations, that there are heterogeneous particles constantly floating in the atmosphere, and that these by their opacity and reflexibility, make a kind of veil that obscures the vision of remote objects: And the more the medium is loaded with these particles, and the more remote an object is from the spectator, the more indistinct and obscure it will be, and the less will be the latitude of vision, or that distance beyond which no objects can be perceived. Every one knows that objects at a given distance, are more distinct, and others are visible at a greater distance, in clear than in foggy weather. And early in a clear morning, when the air is pretty free from vapours, and not much enlightened, a hill or head-land is visible at a great distance, but as the day advances, the land becomes more obscure; and so by degrees, by the greater opacity of the long tract of vapour between, and the greater light reflected by it to the eye, the object becomes less and less perceptible, and at length totally disappears.

Another cause that limits the extent of vision.

81. Another cause of the invisibility of objects, and to remove which optical instruments are more properly adapted, is their smallness in proportion to their distances. For, excepting in the cases of luminous objects seen in the dark, an image upon the retina, to become perceptible, must not be less than some determinate magnitude. To prove this, we do not want for instances: A house may be seen at a considerable distance; but we must approach nearer, before the windows are discernible; and nearer still, to discern the bricks and tiles. At a certain distance, I see a white paper stuck to the wall; coming nearer, I perceive it is printed; at a nearer appproach, I can read the large letters; and at length the smallest letters are legible.

The *minimum visibile*.

82. The quantity of the *minimum visibile*, or the angle subtended by the least visible object, cannot very precisely be ascertained; but in order to form some estimate thereof, I made the following experiment. A parallelogram

parallelogram one inch long and half an inch broad, drawn upon clean white paper, I divided into little squares, whose sides were each $\frac{1}{10}$ of an inch; then blacked every other square, very black, with ink. This chequer'd paper being stuck to the wainscot, I retreated a good distance from it, and then advancing slowly, I began to discern some imperfect specks of black and white, at the distance of 25 feet; and therefore from this place, the sides of each square subtended an angle of 1′ 9″ nearly. But the squares were not defined and perfect, till I came within 7 feet of the paper; and therefore I could not see them distinctly, till they each formed angles of 4′ 5″. I placed by the first another white paper, which was divided into squares of the same dimensions, as above; and the alternate squares, instead of being very black, were only tinged lightly with *Indian* ink: The different colours of the squares upon this paper, were not perceivable at a greater distance than 16 feet; but at the distance of 7 feet, they were as well defined, as in the former paper.

Upon the same paper, was drawn a black square by it self; a white square within a black circle; a square, and a circle, with *Indian* ink; each the $\frac{1}{10}$ inch as before: Also a black ring and parallelogram, each of the same breadth as above.

The white square within the black field, and the black square, were each perceivable as some white and black speck, at the distance of 40 feet; but were not well defined, till I came within 7 or 8 feet. The square shaded with *Indian* ink, and the white one within the circle shaded with that ink, could not be perceived at a nearer distance than 25 feet; but were well defined at the same distance with the rest. The black ring and parallelogram, could be perceived at a greater distance than I had room to retreat, which was 51 feet: But a parallelogram of the same breadth, lightly shaded with *Indian* ink, became invisible at the distance of 40 feet. The papers were placed in an advantageous light, and the experiments succeeded, much the same, to other spectators.

83. Hence it appears, that a simple object, as the white or black square upon the opposite colour, can be seen under a less angle than the parts of a compound object, as the chequered paper; and that this least angle, in most cases cannot be less than 40″; and at a medium, I think, it is not less than 2 minutes. But by the above experiment, it appears that several contiguous objects, are scarce discernible one from another, unless they each subtend angles that are not less than about 4 minutes. The difficulty of keeping the eye perfectly steady, may be one cause why a simple object can be discerned under a less angle, than the parts of a greater one; and it is very natural to suppose, that the fewer objects we contemplate, and the more they differ in colour, the easier

A simple object is perceivable under a less angle, than the parts of a compound one.

we can diſtinguiſh their ſeveral impreſſions upon the retina. But there is no aſcertaining exactly, the leaſt angle under which an object can be ſeen; for the ſame object ſhall be viſible at a leſs or greater diſtance, as the circumſtance of light is different.

84. Some have imagined, that the impreſſions of viſible objects, are received upon *villi* of the optic nerve, ſtanding erect upon the retina, like the pile upon cloth or velvet; and that the *minimum viſibile* is determined by the angles ſubtended by theſe *villi*, at the focal center of the eye. There is no difficulty, in ſuppoſing the retina to be covered with ſenſible *villi*, having inſenſible pores between them: But from the above argument, it would ſeem, that the *minimum viſibile* ſhould be the ſame in all caſes, which is contrary to experience. But, however the retina is conſtituted, it is manifeſt that the cauſe of the non-viſibility of objects, is the want of ſufficient light in the pencils proceeding from them; and that more or leſs light is requiſite for viſion, according as the eye is more or leſs illuminated.

A long ſlender object, is viſible under a ſmaller angle, than a ſquare object of the ſame breadth.

85. Upon a white paper, I drew black lines or parallelograms of different lengths, and each $\frac{1}{20}$ inch broad; and the paper was paſted to the wainſcot, ſo that ſome of the lines were vertical, and ſome horizontal. The longer lines were viſible at the diſtance of 50 feet: A line $\frac{1}{4}$ inch long was juſt perceptible, at the diſtance of 45 feet; and one of half that length, was viſible at no greater diſtance than about 20 feet. I was not ſure of any difference, in the appearance of the vertical and horizontal lines, of the ſame length; but was inclined to think, that the vertical lines were the diſtincteſt. Two parallel black lines, with a white line between them, each the $\frac{1}{20}$ inch broad, did not appear ſeparate at a greater diſtance than 20 feet. In like manner, a piece of fine thread or wire is viſible at ſuch a diſtance, that a globulous object of the ſame diameter is totally inviſible.

A ſlender object, as a phyſical line, may be conſidered as conſiſting of ſeveral ſquares joined together; and though one of theſe ſquares, or a parallelogram conſiſting of two or more of them, may be too ſmall to be ſeen; yet the pencils of light coming from each of them, being contiguous and ſtriking at the ſame time upon the retina, are capable by their united ſtrength to awaken the viſive faculty, and ſo to render viſible the object from whence they came; juſt as in mechanics, the united force of many agents will have its effect, when the effort of a few can avail nothing. And from what experiments I have made, I am inclined to think, that the areas of the leaſt viſible objects are pretty much the ſame, though they differ in figure, all other circumſtances being

being the ſame alſo; that is, the area of the leaſt viſible line, whoſe length does not extend without the field of diſtinct viſion, is nearly equal to the area of the leaſt viſible ſquare or circle; and indeed this ſeems natural enough to be imagined. The phænomenon of the two black lines, with the white line between them, is analogous to that of the chequered table before explained.

86. *Schol.* The ſhadows of ſmall bodies, as hairs, wires, threads, or ſcratches in poliſhed glaſs, *&c.* placed in a beam of the ſun's light, will be much broader than they ought to be, if the rays of light paſſed by theſe bodies in right lines: And a beam of light, let through a ſmall ſlit not above the $\frac{1}{100}$ of an inch broad, will be ſplit into two, ſo as to have a dark ſpace or ſhadow in the middle. The cauſe of both theſe phænomena is, undoubtedly, an inflection of light out of its rectilineal courſe, at or near the ſides of bodies, in its paſſage by them *. Sir *Iſaac Newton* hath alſo ſhewed, that light which differs in refrangibility, differs likewiſe in flexibility.

It has been obſerved, that ſmall wires or threads, placed on or before white paper, or ſuſpended in the air in certain ſituations with reſpect to the light, will be viſible under very ſmall angles, as only 2 or 3 ſeconds: But in theſe caſes, is not the light in paſſing by the ſides of the wire, inflected ſo as to paint upon the retina, the images of theſe ſides much wider aſunder, than according to the angle which the wire ſubtends at the eye? And ſo an image is formed in the eye, equal to that of a *minimum viſibile* in other caſes; or an image that ſubtends, at the focal center of the eye, an angle not leſs than a minute: and if ſo, then in the preſent enquiry, theſe extraordinary inſtances are not to be regarded.

87. A ſmall object in motion is eaſier diſcerned, than if at reſt; and may be viſible in the one caſe, though it would not be ſo in the other. A ſmall ſtar by day or twilight, that cannot be eaſily ſeen through a teleſcope directed to it, will be the ſooner perceived, if the teleſcope be a little ſhaked or moved; but after it is once diſcovered, it will continue very perceptible. A ſmall object, the eaſier diſcerned when in motion.

The reaſon of this phænomenon, ſeems to be much the ſame, with that of the viſibility of a long ſlender object. The image of the ſtar upon the retina, although too feeble to be perceived, while the eye is at reſt, and the point where it falls is not yet diſcovered; yet by its gradual motion over different parts, and the impreſſion upon each having [a] ſome duration; theſe ſeveral impreſſions being all joined together, before the firſt is obliterated, they form a kind of a linear image, [a] 35. II.

 which

* See *Newton*'s Optics, Book III. Part I.

which being once perceived, we quickly discover the star; and having now found its true place, we afterwards keep sight of it with ease. In like manner, a small hair shaked by the air, is visible at a greater distance, than the hair at rest could be seen.

Extraordinary quickness of some eyes.

88. There are great differences to be observed, in the degrees of quickness, or sensibility, of the eyes of different people; some having them more acute, and others less sensible than common. We have heard of persons seeing a satellite of *Jupiter*, without the aid of glasses, which to many seems incredible. But when we consider, how much the various circumstances of light affect vision, and how much farther our vision is extended at some favourable opportunities, than it is at other times; the extraordinary accounts above, may the easier gain belief.

Nearest distance of distinct vision.

89. If an object be held too close to the eye, it becomes indistinct; and the more so, the closer it is held, notwithstanding its apparent magnitude is thereby increased; and a very slender object will become totally invisible. To the generality of eyes, the nearest distance of distinct vision is about 7 or 8 inches: At this distance, they commonly read a small print, and examine all sorts of minute objects; as the divisions upon a plotting scale, the fineness of the threads of linnen, *&c.* It is true that some eyes can see small objects best, at the distance of six, four, or even three inches; and some again, at no less distance than twelve, fifteen, or twenty inches: But these are uncommon cases, and do not affect our present enquiry.

Smallest visible object.

90. Hence, and from *Art.* 83, a globular object that is less than $\frac{1}{650}$ inch in diameter, is, to the generality of eyes, totally invisible: And excepting in few cases, an object cannot be seen that is less than $\frac{1}{430}$ inch in diameter; an object of that breadth subtending an angle of 1 *min.* at the distance of 8 inches from the eye. But an object, placed on a field differing insensibly from it in colour, is not perceivable under a less angle than about 4 *min.* and in such circumstance the smallest visible object, is not less than about $\frac{1}{100}$ inch in diameter. At a medium, the breadth of the least globular object that is discernible by the naked eye, is perhaps about the $\frac{1}{300}$ of an inch.

91. Thus, we see naked vision is limited within certain boundaries, and that by various causes. Remote objects become invisible, as houses, trees, cattle, *&c.* because of their smallness, in proportion to their distances; and others, tho' they subtend angles sufficiently big, as mountains, *&c.* are visible or invisible by turns, as the circumstances of light vary. Very minute objects, again are never visible to the naked eye, because we cannot approach them sufficiently near, to increase their magnitudes.

§. 2.

§. 2. *Of distinct and indistinct vision.*

92. *Defin.* An object is said to be *bright* or *clear*, when the eye receives from it a sufficient number of rays; and *obscure* or *dark*, when it receives too few, or when the light is faint and weak. And an object is said to be *distinct*, or *seen* distinctly, when its out-lines appear clear and well defined; and the figures, sizes, and colours, of its several parts, if not too small, are plainly distinguishable, so as they may be easily compared one with another. It has been already observed, that in order to obtain distinct vision, it is requisite that the object be sufficiently bright, or have a distinguishable colour; and that the several pencils on their falling on the retina, should be separate, and not mixed one with another: And when this is not the case, the out-lines of the object, and likewise its parts, will appear faint, hazy, and ill-defined; and this is called *indistinct*, or *confused* vision. Just as in the common *camera obscura*, the pictures of the objects without, will be more or less distinct, as the objects are more or less illuminated; and as the paper-screen is nearer to, or farther from the focus's of the respective pencils: So in a duskish light, the letters of a book will be confused and not legible; and in a clear light they will be also confused, if the book be held too near the eyes.

Objects when bright and obscure, distinct and indistinct.

93. From the above general observations, it appears, that in order to have distinct vision, the following are the chief requisites:

Chief requisites for distinct vision.

1. The objects should be sufficiently illuminated: And all other things being the same, the nearer is an object, and the brighter is its colour, the more light the eye receives from it; and this is one cause, why near objects, are seen more distinctly, than those that are remote.

2. The geometrical images of objects, should either fall upon the retina, or very near it: And these images should be sufficiently large, otherwise the parts of the object cannot be distinctly perceived; and this is another cause of the indistinctness of remote objects.

3. A good constitution of the eye is a requisite, without which the others will have less effect; the more transparent are the cornea, and the humours of the eye, the more light will be transmitted upon the retina, and the more lively will the impressions there be; and the more tender and sound is the retina, and also the whole optic nerve from thence to the brain, the more distinct will the impressions of light from objects be perceived.

When all the above requisites conspire together, vision is the most distinct; and it will be the less so, as one or more of these is less perfect.

94.

94. In a given eye, and a given disposition of that eye, an image upon the retina will be most perfect, when the object is at some determinate distance from the eye; and it is at about that distance therefore, that objects, if they are not too small or obscure, will be seen most distinctly: An object at a lesser, or a much greater distance than this, will have its image accordingly either behind, or before the retina; and in both cases, if the distances of the images from the retina be considerable, vision will be indistinct. See *fig.* 12, 13, 14. in which *q* is supposed the focus of rays, diverging from a point Q, after their refraction by the humours of the eye.

Distinct vision, when the images are not precisely upon the retina.

[a] 30 & 93. II.

95. The chief requisites for distinct vision, have been already [a] considered: But in order thereto, it is not necessary that the images of objects, or the points of union of the rays in the several pencils, be precisely upon the retina; there being some small latitude both before and behind the retina, within which, wherever the images are formed, vision will be equally distinct; and this latitude will be greater or less, according as the visual angles subtended by the respective objects, are greater or less. Let a printed page, in which there are letters of three or four different sizes, be placed at such a distance, that every sort of print may, without any straining or effort of the eye, appear perfectly distinct: In this case it may reasonably be presumed, that the images of the several letters do fall upon the retina: If the printed leaf be brought gradually nigher and nigher, the smallest print will first begin to be confused, whilst all the larger sort are yet as distinct as before; and by advancing nearer, the smallest print will be more confused, the next size above that will be a little confused, whilst the larger print is still as legible as at first; and so through several degrees, till at length the whole is in confusion. The experiment may be made the contrary way, by using a pair of spectacles of a proper convexity: Thus, first place the paper at such a distance, that all the several sorts of print, may appear as distinct as possible; then moving the paper farther and farther, the smaller prints will, as before, one after another, according to the several sizes, become confused, whilst the larger still preserve their distinctness. By the two sorts of experiments, it is manifest, that we may have distinct vision when the focus's of the pencils are at some distance, either beyond or before the retina; and also that the larger the object, the greater will be the latitude the image has to move from the retina, before we can be sensible of any indistinctness *.

96.

* It has been thought by others, that a precise union of the respective rays upon the retina, is not necessary to distinct vision; but, I think, the first author that published the above observations, which, seem to be decisive, and to set this affair above all doubt, is Dr. *Jurin*, in the *Essay* already quoted.

96. Let BAC be an object, whose image after refraction by the humours of the eye, is *bac*: Now if the image *bac* coincides with the retina, the object BAC must needs appear as distinct as it is possible; and so must also all objects that are at the same distance, if they are sufficiently bright; the image of every physical point, being also a physical point upon the retina. But if *fg* or *hk* be the retina, that is, if the distance of the object and the constitution of the eye be such, that the image falls beyond the retina, as when objects are too near; or before the retina, as when the rays have not a sufficient divergency; in either case, the rays of each pencil will occupy a circular space upon the retina. This circle may be called the *circle of aberration*; and all other circumstances being alike, the said circle will be greater or less, according to the greater or lesser distances from the retina, of the focus's of the refracted rays Fig. 11. Circle of aberration.

97. From *Art.* 95, it is manifest, that a circle of aberration causes no perceptible difference in the distinctness of vision, while that circle is not greater than some determinate magnitude; but as soon as it exceeds that magnitude, we begin to perceive an indistinctness, which increases as the circle of aberration increases, and at length the object is lost in confusion. The greatest circle of aberration, that causes no perceptible indistinctness, is also greater or less, according to the greater or less magnitude of the visible object: It is difficult perhaps, to assign the diameter of the said circle; but it seems very probable, that vision continues distinct for all such distances, or so long, as the circles of aberrations of the least visible objects, or the pencils of light from them, do not touch one another upon the retina; and that indistinctness begins, when the said circles begin to interfere. By the least visible object here, is meant the least object we regard at the time, and not the least object that we are capable of perceiving. To a person that reads, the thickness of a letter is properly then his *minimum visibile*; and so long as the pencils of light from the several letters are not mixed upon the retina, so long he reads with ease, and the letters appear distinct. The larger are the least visible objects, the larger will be the circles of aberration before they touch; and hence it is, that a larger print is legible at a nearer distance, than a smaller. Greatest circle of aberration, that impedes not the distinctness of vision.

Let the visual angle of the least visible part of an object, be equal to the angle AEC, and let *bac* be the retina: Now, upon the above supposition, the object will continue distinct at all such distances, as its image will fall between *hk* and *fg*; for between *hk* and *fg* the pencils of light proceeding from the points A, C do not touch one another. If the focus's of the pencils are farther from the retina than *hk* or *fg*, the object will be indistinct, and the indistinctness will be the greater, Fig. 11.

according

according to the greater diſtance from the retina of the ſaid focus's. If the leaſt viſual angle be much leſs, as the caſe generally is, than the angle AEC, as the $\frac{1}{2}$ or the $\frac{1}{4}$, *&c.* of AEC; then will the limited diſtance of a diſtinct image, be accordingly the $\frac{1}{2}$ or the $\frac{1}{4}$, *&c.* of the diſtance between *hk* and *fg*.

Why a ſingle object ſhall be diſtinct, when ſeveral objects are in confuſion.

98. Hence we have the reaſon, why a ſingle object ſhall be viſible at ſuch a diſtance, that the parts of another object, each as big as the ſingle object, ſhall be in confuſion; as the caſe of the ſingle ſquare, and chequer'd table, *Art.* 82.

99. Perhaps the humours of the eye cannot, in any caſe, perfectly unite, into a point, all the rays in one pencil; but that there will always be ſome aberration from the focus, as in the caſe of glaſs lenſes: If this be the caſe, it may be a reaſon why the *minimum viſibile* is limited to a particular angle; and there will be no need of ſuppoſing the retina to be covered with *villi* of a certain ſize, but the whole ſurface of it may be equally ſenſible.

Viſion equally diſtinct within a certain limit.

100. But whatever it is that limits the *minimum viſibile*, it ſeems very probable, as has been before obſerved, that in all caſes viſion will be diſtinct, ſo long as the circles of aberrations upon the retina, are not greater than the geometrical image of the *minimum viſibile*: And it is alſo very probable, that in all ſuch caſes, viſion is equally diſtinct; for within the above limits, the pencils of light from the leaſt viſible parts of the object, are ſeparate one from another; and the quantities of light received upon the retina, are alſo the ſame. The principal, and may be the only, perceptible difference here will be, that the nearer the geometrical image is to the retina, the ſmaller will be the *minimum viſibile*, or the angle ſubtended by the leaſt viſible object.

Natural diſtance of diſtinct viſion.

101. It is difficult to aſcertain preciſely, the natural diſtance of diſtinct viſion; or that diſtance, at which the eye when in a ſtate of reſt, without any ſtraining or effort of any of its parts, is ſuited to ſee objects diſtinctly. If we ſuppoſe this diſtance to be that, at which we uſually read a fair large print; this to the generality of eyes will be about 15 or 16 inches: And leſs than that, I think, it cannot be; for we are rather more concerned, with objects that are bigger than the letters of a book, than we are with thoſe that are leſs; and when we view objects nearer, it is on account of their ſmallneſs, that they may ſubtend larger angles: Nor is it probable, that the ſaid diſtance is many feet; for we chuſe to view objects pretty near, excepting when they are very large, as buildings, *&c.* and then we retreat to a greater diſtance, for the ſake of comprehending more of them at one view. I am inclined to think that to moſt eyes, the natural diſtance of diſtinct viſion is about 2 or 3 feet.

By

By the preceding [a]articles, it appears, that the ſame conformation of the eye, will ſuit for diſtinct viſion at different diſtances, within ſome certain limits; excepting perhaps only in that caſe, when the object we look at is a *minimum viſibile*. [a] 95 to 100. II.

The eye changes its conformation, for viſion at different diſtances.

102. None can be ignorant, that when ſeveral objects are at different diſtances before us, they will not all appear equally diſtinct, at the ſame time. But, leſt it ſhould be ſuſpected, that the indiſtinctneſs in this caſe, may be owing to the impreſſions not being made upon correſpondent fibrils of the two retina's; let us make a trial with one eye alone, while the other is ſhut: Thus, place two ſmall objects, as two pins, one behind the other; and let the one be at about a foot, and the other at about ſix foot diſtance from the eye: Either of theſe objects that we look at, will appear diſtinct; but the other, at the ſame time, although it be in the axis of the eye, will be confuſed. And this, I think, is an undeniable proof, that the ſame conformation of the eye is not ſuited for diſtinct viſion at all diſtances; and alſo that the eye ſome how changes its conformation, ſo as it may be the better ſuited for viſion at different diſtances. In like manner as above, if the eye looks attentively upon the little ſcratches, or particles of duſt upon a window-glaſs, the objects without doors will be indiſtinct; and when we look at the external objects, the opaque particles upon the glaſs, which before were diſtinct, will now be confuſed. It frequently happens, that when we firſt look at an object, it will appear very confuſed, which confuſion will vaniſh by degrees, and preſently the object will become quite diſtinct: And how can this phenomenon be better explained, than by ſuppoſing that at firſt, the eye was not properly adapted for viſion at ſuch a particular diſtance, and that afterwards, a ſuitable change was wrought in it accordingly.

103. In like manner, if after poreing ſome time on a book, we ſuddenly look at objects farther off, they will at firſt appear confuſed, and become diſtincter by degrees: The like indiſtinctneſs happens, when from looking at remote objects, we ſuddenly look at one that is near. This phenomenon will happen, when both objects are in the ſame room, and have the ſame degree of light; and what elſe can this be owing to, but that the eye changes its conformation for viſion at theſe different diſtances, and takes ſome ſmall time ſo to do? It cannot be altogether owing, to the laſt impreſſions on the eye being yet not quite obliterated; otherwiſe the ſame confuſion would be obſervable, upon ſhifting the eye from one page of a book to the other, which does not happen, at leaſt in the ſame degree. Theſe phenomena are ſtronger, when they happen without our thinking of them; for when we make the experiment on purpoſe, and the mind is already prepared for what is to happen, it has time in part to fruſtrate our deſign; theſe things being a little

painful to us. The ſame obſervation is juſt, with reſpect to ſome other experiments of this kind.

It may be owing, at leaſt partly, to the ſame cauſe, that when we ſuddenly ſee an object which we did not before look at, we ſometimes have a *painful ſenſation; and we immediately ſhut our eyes, till we have time, as it were, to prepare them for ſeeing at ſuch a diſtance. It is probable, we may have theſe ſenſations when we do not attend to them; and that they ſerve as monitors to inſtruct the mind, that the eyes are not properly ſuited for viſion at ſuch a diſtance.

104. Before we proceed any farther, it may not be amiſs to take notice of the following experiment, and the inferences that have been drawn from it. Let *r*, *s*, be two ſmall holes made in a card, ſo near to each other, that when the card is held cloſe to the eye, the light coming through both holes may enter the pupil; and let Q be any ſmall object, whoſe focus or image, after refraction by the humours of the eye, is *q*; the two ſlender pencils Q*r*, Q*s*, which enter the eye through the holes in the card, being both united at *q*; if *q* falls upon the retina, as in *fig.* 12, the object will appear ſingle, and in its true place; the only difference cauſed by the interpoſition of the card in this caſe, is, that the object is fainter for want of the light contained in the ſpace between Q*r* and Q*s*. But if *q* falls either beyond or before the retina, in both caſes, the object ſeen through the two holes in the card, will appear double: Thus,

Fig. 12, 13, 14.

Fig. 13, 14. The ſmall pencil of rays Q*r*, being refracted to *a*, and the pencil Q*s*, to *b*; the object inſtead of being ſeen at Q, is ſeen in the two places A, B; on different ſides, and a little wide, of the true place Q. For the impreſſions at *a*, *b*, being received on the ſame parts of the retina, on which would be formed the images of two objects, ſituated in the directions *a*A, *b*B, and ſeen by diſtinct viſion without the interpoſition of the card; the mind has the ſame perception, as it would have, if two objects were really placed in theſe directions; and therefore the object Q appears double. The phantaſms A, B, are ſimilar to the object Q, and alſo pretty diſtinct; becauſe the rays contained in the ſlender pencils Q*r*, Q*s*, are united ſufficiently cloſe at *a*, *b*; only they appear faint for want of light.

That the cauſe above aſſigned, for the double appearance of the object, is the true one, is farther evinced from hence: If when the object is very near, as in *fig.* 13, the right hole *s* be covered, the left object B will vaniſh; and if the left hole *r* be ſtopped, the object A on the right hand, will vaniſh: And if, in *fig.* 14, where the object is ſuppoſed to be at

* This ſort of painfulneſs in the eyes, does alſo ſometimes proceed from a different cauſe; as when we ſuddenly go out of a light room into the dark, or out of darkneſs into a ſtrong light.

at a considerable distance from the eye, either of the holes *r* or *s* be stopped, the object A or B on the same side, will accordingly vanish. And hence it is manifest, that the focus *q* in the first case, is beyond the retina; and in the latter, before it; that is, the rays in the one case diverge too much, and in the other too little, for the humours of the eye to unite them all upon the retina. If three holes be made in the card, within the compass before limited, then the object will be seen in three places.

105. *Schol.* 1. If the lens of a *camera obscura*, be covered with an opaque paper, having in it two holes or slits at a little distance from each other; and a white paper be held behind for receiving the sun's image, or that of any bright object, without doors; by moving this paper along the axis, we may have all the phenomena of the preceding experiment. When the paper is in the proper focus, there will be only one image of the object; the pencils of light transmitted through each hole, uniting in this case, upon the paper: But as soon as the paper is moved from the focus, the pencils through the different holes will begin to separate, and the image will begin to widen; and presently it will be split into two, which will be the wider asunder, according as the paper is more remote from the focus. When the paper is before the focus, the images will be on the same side of the axis, with the respective holes; and on the contrary side, when the paper is beyond the focus. The same experiment may be made, by covering the lens only with patches.

106. If the above lens be covered only with one large patch, and a paper be held in the focus, the images of small objects without, will be free from any speck; but if the paper be without the focus, each image will have a dark spot in it. In like manner, if a small opaque object be held near the eye, before which is placed many candles, at the distance of 10 or 20 yards; if the eye looks attentively on the small object, it will be seen multiplied, according to the number of candles; that is, it will be seen like a dark spot in each candle. The reason of which is, because in this case the eye having conformed itself, as much as it could, to the near object, the geometrical images of the candles are before the retina; and so the rays from each candle, spreading themselves into wide spaces upon the retina, the shade of the opaque object falls within them, and causes the dark spots above noted. But when the eye looks at the candles, no spots will be seen in them, and the opaque body will not be multiplied: For the light which passes by the opaque body, being now united into the proper focus's upon the retina, there is no other imperfection in the images of the candles, excepting their being less bright,

on account of the loſs of the intercepted light; and the ſmall opaque body itſelf, ſeems tranſparent.

107. *Schol.* 2. The ſmaller is the object, and the wider the holes in the card are aſunder, ſo that they are not wider than the breadth of the pupil; the leſs ought to be the diſtance between thoſe two places, at which the object begins to appear double, from a too great and too little divergency of the rays. A needle held vertically, or a black line drawn upon white paper, makes a better object than a point of the ſame diameter, becauſe they are eaſier ſeen; alſo two parallel ſlits in the card, will do better than the holes, becauſe they admit more light to paſs through them. A ſlit made in the ſide of a dark lanthorn, having a lighted candle in it, is a proper remote object. In making the experiment, care muſt be taken, that the object is ſeen through both holes or ſlits, which may be known by moving the card a little ſide-ways.

But after all the care that can be taken, in making this experiment, it will be frequently attended with inconſiſtencies; and an object will appear ſingle or double, as it were caſually: Sometimes it will be double at all diſtances; ſometimes it will be ſingle for all diſtances beyond a certain one; and frequently an object will appear double at ſuch a diſtance, that with an open eye we might be ſure of having diſtinct viſion.

[a] 104. II. 108. From the [a] experiment with the card, M. *de la Hire* concludes, *that no change is made in the eye for viſion at different diſtances*, excepting only as to the ſize of the pupil: But herein, I think, this author has had no followers. Dr. *Jurin* * obſerves, that if M. *de la Hire*'s opinion was juſt, the *minimum viſibile* would, in moſt caſes, be much greater than what we find it by experience. And Dr. *Porterfield* †, who has wrote very largely about the above experiment with the card, ſhews, that when the ſmall object appears double, at a proper diſtance for diſtinct viſion, it is becauſe the eye is not properly ſuited for diſtinct viſion at that diſtance; which he proves by ſhewing, that if in ſuch a caſe we look at the object with the other eye, it will appear double, on account of its being out of the plane of the horopter. And this ingenious author gives good reaſons to ſhew, why we are ſo liable to miſtake the diſtance of an object, ſeen through the two pin-holes in the card: But nevertheleſs he proceeds afterwards to aſſign, from this experiment, the leaſt and greateſt diſtance of diſtinct viſion.

Experiment with the card not concluſive, for ſettling the limits of diſtinct viſion, &c.

109. The above experiment, I think, is very little to be depended upon, either for proving that the eye does, or does not, change its conformation.

* Eſſay upon diſtinct viſion, p. 136.

† See *Edenburgh* Medical Eſſays, Vol. III. and IV.

formation, for vision at different distances; or, for assigning the limited distances of distinct vision. For in this case, the slender pencils of rays, transmitted through the pin-holes, are united so close upon the retina, that no perceptible difference is observable in the distinctness of the object, by changing the conformation of the eye; and foreign rays, which otherwise, by mixing with these pencils, might have created an indistinctness, are here excluded: And so in the present case, we have no incitement to change that conformation, which the eye happens to have, when the card is placed before it; for why should we alter our eyes, unless it be for seeing more distinctly? If it be said that the double appearance of the object, is it self in some cases, an indication that the eye is not adapted to the proper distance; yet this being an uncommon phenomenon, and not attended with such a sensation as we have, when an object is either too near or too remote; we are at a loss what motion we should give our eye, and also where to stop. What passes in our eye, with respect to vision at different distances, is rather involuntary, than dependent on our will; the eye adapts itself by a kind of habit, unknown to us, so that we may see an object the most distinctly. And this accounts for the inconsistencies of the experiment; and also shews, that the conclusions drawn from it must be weak and precarious. An object much too near for distinct vision, seen through the two pin-holes, always appears double; but for the reasons above assigned, I think, the experiment is not conclusive for shewing even the least distance of distinct vision, and much less for shewing the greatest; for in this latter case, the card being near the eye, may be a prejudice that deters us from exerting our utmost, as we ordinarily do, when we look at remote objects.

But, without regarding the experiment with the card, we may rest the argument for proving that the eye changes its conformation, for vision at different distances, upon what has been already offer'd, in *Art.* 102, 103.

110. Authors are divided in their opinions, concerning that change which is made in the conformation of the eye, in order to obtain distinct vision at different distances: Some, thinking it to be a change in the length of the eye; others, a change in the figure or position of the crystalline humour; others, that it is a change in the cornea. These opinions by being split, make no less than six different ones; according as the eye when in a state of rest, and inaction, is supposed to be accommodated for distinct vision, at the least or greatest distance, it is capable of. The authors of each opinion have their objections to the rest, and perhaps the truth may lie in a mean betwixt them all. Dr. *Jurin** suppose

Of the change of conformation in the eye.

* Essay upon distinct and indistinct vision, p. 136, to 144.

ſuppoſes, that the change which is made in the eye, for obtaining diſtinct viſion at different diſtances, is in the figure of the cornea; and this learned author, ſupports his opinion by very ingenious arguments. And as the rays of light ſuffer a greater refraction at the cornea, than they do afterwards; it is manifeſt, without entering into particulars, that a leſs change, as to quantity, in the radius of the cornea, will effect the buſineſs, than will ſuffice in any other part of the eye. But if it be true, as ſome authors affirm, that perſons who have been couched for cataracts, are obliged to have convex-glaſſes of different focus's, in order to have diſtinct viſion at different diſtances; it ſeems neceſſarily to follow, that the cryſtalline humour is ſome how concerned, in changing the conformation of the eye: And the manner that this humour is ſuſpended in the eye, ſeems to intimate as if it might be ſubſervient for ſuch a purpoſe. Perhaps, the cornea and the cryſtalline, if not ſome other parts of the eye beſides, may contribute their ſhares together: And that in order to obtain diſtinct viſion at a nearer diſtance, at the ſame time, as the cornea is rendred more convex; the axis of the eye may be a little lengthened; the cryſtalline may be rendred a little more convex, and alſo brought a little forwarder; all which changes conſpire to the ſame end: And the contrary, for obtaining viſion at a greater diſtance. But this matter ſeems to be too far out of our reach, to be decided with certainty.

Of indiſtinct viſion.

111. An object that is far without the limits of diſtinct viſion, will not only be confuſed, but alſo its figure, unleſs it be circular, will appear too broad in proportion to its length.

Fig. 15.

Let ABC be an object, ſo near the eye, that its geometrical image is at *pqr*, beyond the retina. Now the light contained in the pencils proceeding from the phyſical points A, B, C, is ſpread upon the retina, over the circular ſpaces *a*, *b*, *c*, reſpectively; which circular ſpaces, anſwer in bigneſs to the images of circles A, B, C, ſeen by diſtinct viſion; as appears by the dotted lines drawn through the focal center of the eye. And as the rays from every phyſical point of the object, are, in like manner, ſpread over circular ſpaces upon the retina, theſe circles run into one another; and ſo the object, inſtead of appearing lineal, is confuſed, and as broad as the circles A, B, C, as repreſented in *fig.* 16.

Circle of aberration.

112. *Defin.* The leaſt circle upon the retina, into which all the rays coming from a phyſical point of an object ſeen by indiſtinct viſion, can be collected, we ſhall call the *circle of aberration*; as the circle *a*, or *b*, or *c*, fig. 15.

PROP.

PROP. V.

113. *A circular object, seen by rays diverging either too much or too little for distinct vision, has also a circular image upon the retina; and the diameter of this image, exceeds the diameter of the true image, by the diameter of the circle of aberration.*

Let the circle RABC be the true image, or that circle formed upon the retina, by the axes of all the pencils that come from the object; and let the circles of aberrations be each equal to the circle *rabc*, whose center *r* is here in the periphery of the circle RABC; and let RDEF be a circle, whose periphery touches the outermost part of the periphery of the circle *rabc*. I say, that the whole image upon the retina is the circle RDEF. Fig. 17, 18.

For, the axes of all the pencils of light coming from the periphery of the object, do [a]fall in the periphery ABC; and the circles of aberrations of these pencils, are [a]each equal to *rabc*. Therefore, &c. *Q. E. D.* [a] Hypoth.

PROP. VI.

114. *In the middle of the image of a circular object, seen by indistinct vision; there is a circular space, whose diameter is the difference of the diameters of the true image and of the circle of aberration, which is all over equally enlightened; and from the circumference of this equally enlightened circle, the light continually diminishes towards the periphery of the image, so as at that periphery to be infinitely rare.*

Retaining the same references as in the preceding *Prop.* let RGHI be a circle touching the circle of aberration *rabc*, in that point *c*, which is nearest to the center R; let the radius of the true image RABC be called R, and the radius of *rabc*, call *r*; then it is manifest that the radius of RGHI $=$ R $\backsim$ *r*. I say, that the circle RGHI is all over equally enlightened, and that without this circle, the light is continually diminished from GHI to DEF. Fig. 17, 18.

1. Let R be greater than *r*, and in the line *r*R, take $cs = r = cr$. Now, if the lines *rc*, *sc*, be alike divided from *r* to *c*, and from *s* to *c*, and these divisions be supposed the centers of circles of aberrations, whose radii are each equal to *r*; it is manifest then, that an indefinite small sector R*c*, will receive as much light, from the circle of aberration whose center is next to *r*, as it will be deprived of, by the extension beyond *c* of the circle whose center is next to *s*: And so the correspondent circles will all the way countervail each other, till their centers unite in *c*; so that the point *c* loses none of its light, and all the points from Fig. 17.

from *c* to *s*, are equally enlightened: And this is true, of all the indefinitely ſmall ſectors, whoſe center is R. Thoſe circles of aberrations whoſe centers are not more remote from R, than *s*, are all included within the circle RGHI; and by the preceding argument, it is manifeſt, that the light is equally denſe in every point of the ſaid circle. But as no circle of aberration can have any return, for ſo much of its light as falls without ABC, the light without GHI muſt continually decreaſe, till at the periphery DEF it is infinitely rare; the ſaid periphery, receiving only one ray from each of the outermoſt circles of aberration.

Fig. 18. 2. When R is leſs than *r*. It is manifeſt that every point in the circle RGHI, is equally enlightned; for the whole of that circle, falls within every one of the circles of aberrations: And by the ſame argument as above, it appears that the denſity of the light continually diminiſhes, from the periphery GHI to DEF. Therefore, *&c.* *Q. E. D.*

115. *Schol.* 1. When R is greater than *r*, as in *fig.* 17. the circle RGHI receives the ſame quantity of light, as it would if there was no aberration: This is manifeſt from the preceding. When the circle of abérration as CKRL, is equal to the true image, only the center of that image has its full ſhare of light; and in all other places the light decreaſes in denſity, according to the diſtance from the center. But when R is leſs than *r*, as in *fig.* 18. no part of the image has its full quantity of light: For ſince every pencil, whoſe axis falls within GHI, diffuſes its light into a circle as large as *rabc*, and the points where theſe axes fall, receive light only from pencils whoſe axes fall within the circle RABC; it follows, that the circle GHI diſperſes more light than it receives, and that in proportion of r^2 to R^2; and by the preceding, the light in every point without that circle, is diminiſhed ſtill more.

116. *Schol.* 2. The image at the circumference DEF will be imperceptible, becauſe there the light is [a] infinitely rare: And to a given circle of aberration; the more luminous the object is, the image will be ſenſible, ſo much the nearer to the circumference DEF.

[a] 114. II.

For the denſity of the light decreaſes in equal proportion from GHI to DEF, whatever be the quantity of light in the whole image [a]; and therefore the more luminous the object is, the more ſenſible muſt the image be, at any given diſtance from the edge DEF; and it will be equally ſenſible, ſo much the nearer to that edge.

117. *Schol.* 3. The three preceding articles, may be alſo applied to objects of other figures, as parallellograms *, *&c.* The principal difference in the phenomena of circular and oblong objects, is, that the falſe images of the latter, will be [b] broader in proportion to their lengths, than their

[b] 111. II.

* This Subject has been conſidered very minutely by Dr. *Jurin*, in the *Eſſay* before quoted.

their true images, and alſo blunt at the corners. But the phenomena above ſeldom happen, excepting when we look at objects that are too near, or through lens's at objects that are out of the proper focus; and when the aberration is conſiderable, the confuſion is generally ſo great, that the appearance, at leaſt in many caſes, will not anſwer to the theory; and it would be needleſs, here, to purſue this ſpeculation any farther.

118. A ſmall bright object, by cloſing the eye-lids a little, will have two ſimilar images incroaching one upon another; and the cloſer the eye-lids are brought together, the more detached will be the images, or the farther aſunder will be their centers: Both the images will be diſtinct, but one will be brighter than the other; and that part, which is common to both, will be brighter than either. Thus, let A be a round hole in the window-ſhutter of a dark room; if this hole is not too big, upon cloſing a little the eye-lids, there will appear another hole, as B; and ſometimes there will be three images, one above, and the other below the true image, as in *fig.* 20. The true image A will be brighter than B; and the part *ab*, as receiving light from both images, will accordingly be brighter than the reſt of A. There will be the ſame appearance, if one eye be totally ſhut; and therefore, it cannot be owing to the object being out of the plane of the horopter, and beſides the images are here always in a vertical ſituation; nor can it be owing to indiſtinctneſs of viſion, becauſe in that caſe, inſtead of two images, [a] we ſhould only have one broad and confuſed image.

A ſmall bright object, by cloſing the eye-lids, becomes double.

Fig. 19.

[a] 113. II.

119. If the eye-lids be pretty much cloſed, the two falſe images above deſcribed will vaniſh; and they will be ſucceeded by luminous beams, as ABC, ADE, *fig.* 21, projecting towards the eye; juſt after the ſame manner with thoſe beams, that, in a like diſpoſition of the eyes, appear to ſhoot from the flame of a candle. If the head be inclined upwards or downwards, the upper or lower beam will accordingly vaniſh; and ſo, by a like inclination of the head, will alſo vaniſh the reſpective falſe images in the preceding *Art.*

Beams of light ſhooting from candles.

120. The phenomena of the falſe images, and the beams, muſt undoubtedly be both owing to a like cauſe; and that cauſe, muſt be ſome inflection of the rays at the edges of the eye-lids. If a ruler be held againſt the face below the eye, and then moved upwards, the uppermoſt beam or image will firſt vaniſh; and if the ruler be placed above the eye, and moved downwards, it will be the lowermoſt beam that will vaniſh: And this ſhews, that the rays which cauſe the appearance of the upper and lower beams, are inflected reſpectively at the lower and upper eye-lid. M. *Rohault* ſuppoſes, that the rays are re-

Cauſe of the above phenomena.

 flected

flected at the edges of the eye-lids, as by a ſpeculum; but the phenomenon of the ruler juſt mentioned, invalidates this hypotheſis. M. *de la Hire* thinks, that the water which ſerves to moiſten the eye, is ſo ſqueezed by the preſſure of the eye-lids, as to form between them a concave ſurface; and by this ſurface, the rays which fall on the lower lid, are refracted downwards; and thoſe falling on the upper, are refracted towards the upper part of the retina: And ſo by the greater refraction of the rays, as they fall nearer to the edges of the eye-lids, the beams of light appear extended to a greater diſtance from the object.

Fig. 21. Thus, one of the outermoſt rays, in the pencil of light proceeding from A, is refracted by the concave water adhering to the eye-lid G, ſo, that after its refraction by the cornea and cryſtalline humour, it falls upon the retina at the point *e*; and ſome of the other rays that are nearer the axis, after all the refractions, fall at the point *d*, between *e* and *a*. In like manner, the extreme ray which comes to the eye-lid H, is, by the concave ſurface of the water adhering to it, ſo refracted as to fall upon the retina at *c*; and the reſt of the rays, from this laſt ray to the axis, ſtrike orderly upon the retina from *c* to *a*; as if they proceeded from ſome luminous beam of light ABC. And ſo the rays which fall betwixt *a* and *e*, ſeem to have proceeded from another luminous beam ADE.

121. *Schol.* 1. From the above ſolution, it ſeems neceſſary it ſhould follow, that the beams of light ſhould appear coloured, as in the caſe of viſion through a priſm: But this not being the caſe, it may ſeem more likely that the rays inſtead of being refracted, are inflected at the eye-lids, as in *Art.* 86 *; the upper eye-lid, inflecting the rays upwards, and the lower lid inflecting them downwards upon the retina, ſo as to cauſe the appearance of the luminous beams, for the reaſons aſſigned in the preceding *Art.* Perhaps both the above cauſes may conſpire together, as each of them is equally ſuited for diſperſing the rays over the ſame parts of the retina: But by either of them, it ſeems difficult to account for the falſe images in *Art.* 118. The middlemoſt and true image, is undoubtedly cauſed by rays that freely enter into the eye, as uſual; and the rays that cauſe the falſe image, muſt be ſome how regularly or equally inflected. If this is done by refraction, it muſt be by a thin wedge of water, whoſe thickeſt ſide adheres to the eye-lid; and this refracting but little, the falſe image is not ſenſibly diſtorted, nor moved far out of its true place.

The phenomenon above conſidered, does not happen, but when the object is much brighter than the medium round it; which ſhews, that the inflected rays are but thinly ſpread upon the retina, and that perhaps

* See *Newton*'s Optics, p. 300.

haps may be one reason, why the false images do not appear coloured. The rays which enter the pupil laterally, or at the sides, meet with no obstruction from the eye-lids; and this is the reason why, the false images, and the luminous beams, appear always one above, and the other below, the object.

122. *Schol.* 2. Two physical points, or small objects, seen by indistinct vision, will have two large circular images, as A,B; and if they are so near as to have a part *ab* common to both, that part will be more luminous than either A or B, and the whole will form a kind of an oblong image. But in this case, these images are confused, ill defined, and very unlike the phenomenon above explained. Fig. 19.

PROP. VII.

123. *All other things being the same: The indistinctness of an object whose geometrical image falls either behind or before the retina, will be, as the area of a circle of aberration; or, as the area of the least circle upon the retina, into which all the rays of a single pencil can be collected.*

Let RABC be the geometrical image, or that space upon the retina which contains the axes of all the pencils of light coming from the object, and let the circles of aberrations be each equal to *rabc*: I say the indistinctness of the object, will be as the area of *rabc*. Fig. 17, 18.

For every sensible point upon the retina, within the space RABC, will be a center of a circle of aberration, and will at once be affected by a mixture of the rays of as many different pencils, as there are sensible points within that circle: But the indistinctness is necessarily, as the number of pencils which at once affect the same point of the retina, and that number is as the area of a circle of aberration. Therefore, *&c* *Q. E. D.*

PROP. VIII.

124. *All other things being the same: The indistinctness of an object having its geometrical image at a given distance from the retina, will be as the area of the pupil.*

Let *q* be the focus of a pencil of rays, after refraction by the eye; now the situation of the point *q* being invariable, it is manifest that the circle of aberration *b*, will be as the circle whose diameter is *st*, that is as the area of the pupil. The same demonstration takes place, if the focus *q* be before the retina; and in both cases, it is supposed that the pencils of light at their entrance into the eye, are each at least as broad as the pupil. But the indistinctness, is [a] as the area of the circle *b*. Therefore, *&c.* *Q. E. D.* Fig. 15. [a] 123. II.

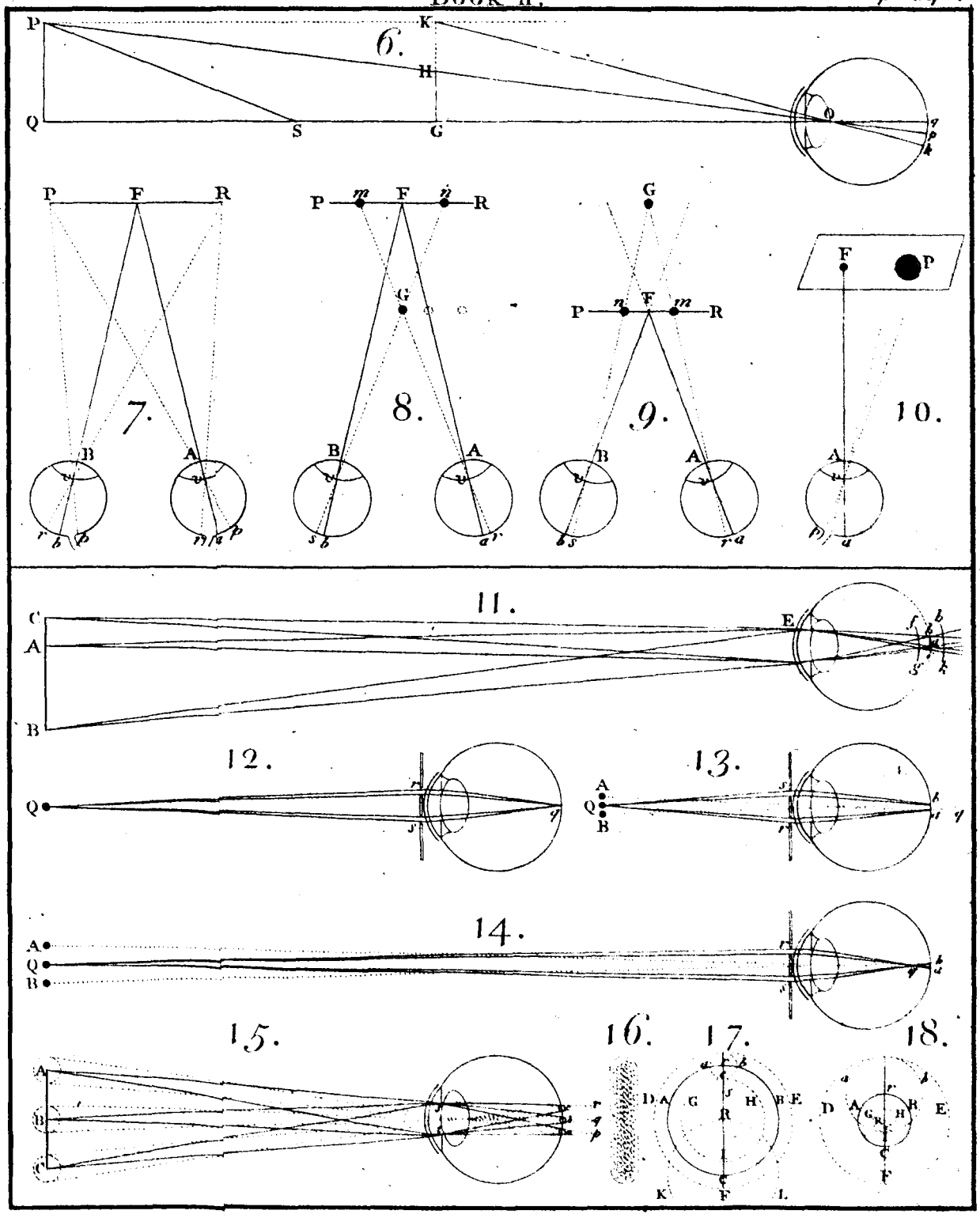
125. *Schol.* 1. If we had a power at will of contracting the pupil, an indiſtinctneſs of viſion might be thereby leſſened: But this is not the caſe; the motions of our eyes are rather ſubſequent to particular ſenſations, than ſubject to our direction; and a greater degree of light is a much more powerful cauſe of contracting the pupil, than an indiſtinctneſs of viſion. The eye looking at a ſtar will have its pupil much wider, than when it looks at an object at a moderate diſtance by day-light. And a book may be read at a nigher diſtance by a ſtrong light, than it can by a faint light; the ſtrength of the light contracting the pupil into a ſmaller ſize, than the ſenſation of indiſtinctneſs alone could do. It is eaſy to make a kind of artificial pupil, of what ſize we pleaſe; but this will not help us much, as is ſhewed in the following.

Viſion through a pin-hole.

Fig. 22.

126. *Schol.* 2. If we look through a pin-hole, made in a card or thin plate, held cloſe to the eye, an object will appear diſtinct at a ſomewhat leſs diſtance than it will to an open pupil. Thus, if AQ is the leaſt diſtance at which the eye with an open pupil, can ſee a ſmall object as Q, diſtinctly; if the eye approaches nigher to the object, as to B, and no more light be admitted through the hole *a*, than what before entered the pupil at A, the object will be ſtill pretty diſtinct; becauſe the outermoſt rays in the pencil, have now no greater degree of divergency than they had before. In like manner, if the hole in the card be leſſened, in proportion to the diſtance, the eye may approach ſtill nigher to the object, as to C: However, there is a limit as to this approach, otherwiſe we ſhould have a ready way of making a kind of microſcope; and although it ſeems, as if the eye received the ſame rays of light from the object, at the different diſtances; yet the object does not appear equally bright, and diſtinct, at thoſe ſeveral diſtances. It is to be obſerved, that an image upon the retina is more or leſs vivid, according to the quantity of light received from the leaſt viſible part of an object, and not according to the quantity received from the whole object: And the diameter of the *minimum viſibile* is increaſed, as the diſtance from the object is leſſened. And therefore in the caſe above, if the hole be leſſened in proportion to the diſtance, the object, as it is approached nigher will be darker and darker, and ſo by degrees grow indiſtinct for want of light; and if the hole be bigger than in the above proportion, there will be an indiſtinctneſs from a too great divergency of the rays: And in both caſes, the indiſtinctneſs may be increaſed, by [a] an inflection of ſome of the rays at the edge of the hole; and the ſmaller the hole, the greater proportion will theſe inflected rays bear to thoſe that are freely tranſmitted to the eye. So that the above experiment,

[a] 86. II.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periment, though it may help a little in some cases, yet is it of much less use than some have imagined *.

127. *Schol.* 3. In all that has been said about an indistinct image upon the retina, no regard has been had to any aberrations of the humours of the eye; for although it is very probable that there may be such aberrations, yet it is impossible to discover them by any experiment; we cannot see what passes in the eye, and light after it is once entered there, can be traced no farther.

128. Before we conclude this head, we should make some enquiry about the extent of distinct vision, so far as distinctness is dependent on the different distances of objects, or the degrees of divergencies of the rays of light proceeding from them. It is agreed that, to the generality of eyes, the nearest distance of distinct vision is [b] about 7 or 8 inches. But the greatest distance of distinct vision to a common eye, is perhaps not so easily settled: Dr. *Porterfield* determines this greatest distance for his eye, to be 27 inches; and this conclusion he draws from the experiment, *Art.* 104. But that experiment is [c] too precarious to be relied on: And I see no room to doubt of our being able to see objects distinctly, if they are sufficiently large, at a much greater distance than two or three feet. A large print at the distance of five or ten feet, is, I think, every whit as legible, as a print that is five or ten times smaller, at the distance of one foot. And cannot a person, having common eyes, see a large object, as the front of a house, *&c.* as distinctly at the distance of 20 or 30 feet, as if that object was nigher to him?

Latitude of distinct vision.

[b] 89. II.

[c] 109. II.

I am inclined to think, that a common eye can adapt itself so, as to have distinct vision at the greatest distance, or with parallel rays; or that in such a case, the geometrical image is not so remote from the retina, as to cause any indistinctness upon that account. For,

1. It is natural to suppose, that the eye can alter its figure as much, from that conformation it naturally has when in a state of rest and inaction, one way, as it can another; or, that the eye can change its conformation as much for vision with rays diverging less, as it can for rays diverging more, than that degree of divergency which is proper for vision when the eye is in its natural state of rest. Upon this supposition, and admitting the natural distance of distinct vision is not less than 16 inches; it will follow, that the image of an object at an infinite distance will be as near the retina, [a] as the image of an object at no greater distance from the eye, than about 8 inches; and so in both cases, the images will be equally distinct.

[a] 21. II.

2.

* A slender object, as a needle, placed between the eye and a small hole in a card held pretty near, will appear inverted; as will be manifest, by moving the needle slowly over the hole; but in this case, it is not the needle, but its shadow on the other side, that is seen, and that is the cause of the inverted appearance.

2. Without regarding what is the natural diſtance of diſtinct viſion, let us ſuppoſe 10 feet and 8 inches to be the greateſt and leaſt diſtance, at which a common eye can ſee an object diſtinctly, with eaſe, and without much ſtraining of its parts. Now if 2 tenths be ſuppoſed the breadth of the pupil, no greater change in the eye will be requiſite for procuring diſtinct viſion from 10 feet to an infinite diſtance, than is neceſſary from 8 inches to $7\frac{1}{2}$ inches. Let AB be the ſemi-aperture of the pupil $=$ 1 tenth, AC the axis of the eye produced, EB a ray parallel thereto, DB the axis of a pencil whoſe focus is 10 foot from the eye, and let AC $=$ 8 inches. The $\angle$ EBD is $= 2' \; 52''$, and by making the $\angle$ CBF $= \angle$ EBD, AF will be $7\frac{1}{2}$ inches. It cannot be doubted, but that we can ſee an object diſtinctly at the diſtance of 10 feet; and it is as certain, that we can have diſtinct viſion, at leaſt half an inch nearer than that diſtance, at which we commonly look at ſmall objects.

Fig. 23.

3. But if it be doubted, whether we can change the conformation of our eyes, as much for procuring diſtinct viſion farther, as we can for procuring it nigher: Let a perſon having common eyes, after looking attentively at a remote object, as the moon, look at the ſame object through a concave lens, having what focal length he may think beſt; and I believe by this trial, he will be convinced that a parallelifm of the rays of light, is not an impediment to diſtinct viſion, and that therefore the indiſtinctneſs of remote objects is to be referred to ſome other cauſe. By looking through a convex lens having a long focus, it will appear, that a common eye may even have diſtinct viſion with rays a little converging.

Cauſe of indiſtinctneſs of remote objects.

129. *Schol.* 1. The chief cauſes of the indiſtinctneſs of remote objects, I take to be thoſe [a] already aſſigned: And it may be farther obſerved, that we uſually diſtinguiſh near objects, by certain marks, as the figures, ſizes and colours, of ſome eminent parts; and when theſe marks, by reaſon of their ſmallneſs, are loſt; the object, by wearing a quite different face from what it uſed to have at a nearer view, becomes neceſſarily indiſtinct. For in this caſe, we not only loſe ſight of our principal marks, but alſo the out-line becomes blunt, faint, and imperfect, as the exterior parts of the object are not diſtinguiſhable one from another; and ſo there being nothing viſible, but the groſſer parts, the whole appears confuſed, and ill defined. The features of a man, the buttons on his coat, *&c.* are not viſible beyond a certain diſtance: And at a greater diſtance, his head and ſhoulders will ſeem contiguous, and no marks will be left, whereby he may be diſtinguiſhed from the ſtump of a tree.

[a] 80, 81, & 93. II.

130.

130. *Schol.* 2. Let AB be the ſemi-diameter of the pupil, *b*AC produced the axis of the eye, EB a ray parallel thereto, *e*C*g* an indefinite line perpendicular to AC; and let the conformation of the eye be ſuch, as to ſee moſt diſtinctly at the diſtance AC, or let *b* be the geometrical image of the point C: Now, if innumerable pencils of rays, be ſuppoſed to enter the eye, from innumerable focus's taken in the axis AC, from C onwards to infinity; the diameter of the leaſt circle upon the retina, which will contain all the rays that enter the pupil, will be equal to that of the geometrical image of a circular object, whoſe ſemi-diameter is the line CE. Fig. 23.

In the line *eg*, take CG=CE, and let *a* be the geometrical focus of a pencil of rays flowing from E: Now it is manifeſt, that a ray coming parallel to the axis, from an infinite diſtance, through the point E, will thenceforward be carried to the eye, along with the ray EB, and that both theſe rays will fall upon the retina, at the ſame point *a*; *ba* being the geometrical image of CE: And all the rays, whoſe focus's are any where in the axis beyond C, will fall ſomewhere within the circle whoſe diameter is GE; and conſequently will fall upon the retina, within the image of that circle. Therefore, *&c.* *Q. E. D.*

131. *Corol.* Let BG be a line cutting the axis AC in H: Now the focus of a pencil of light diverging from H, will fall at a point *y*, about as much beyond the retina, as the focus *x* of parallel rays, is before the retina; and all the pencils of light, whoſe focus's are any where in the axis betwixt G and H, will be contained upon the retina, within the circle whoſe ſemi-diameter is *ba*; for theſe rays produced backwards, will fall within the circle whoſe diameter is GE.

And univerſally, if C be ſuppoſed that point in the axis, whoſe geometrical image falls upon the retina; and innumerable pencils of rays be ſuppoſed to enter the pupil, ſome whereof are converging and others diverging, to or from, certain focus's in the axis of the eye: Produce backwards that ray, as IB, which converges the moſt, and that ray as *b*B which diverges the moſt, till they meet with the perpendiculars to the axis C*e*, C*g*, in *e* and *g*; and the longeſt of theſe two perpendiculars C*e* or C*g*, will be the ſemidiameter of that circle, whoſe image is the leaſt circle upon the retina, which can comprehend all the rays that enter the pupil.

§. 3.

§. 3. *Of long and ſhort-ſighted eyes, and of ſpectacles.*

Of long-ſightedneſs. Fig. 24.

132. If the cornea and cryſtalline, or either of them, be too flat, a pencil of light coming from an object at an ordinary diſtance, will have its focus at ſome point *y*, beyond the retina; and therefore viſion will be indiſtinct, as in the caſe of an object too near for a common eye. This defect commonly comes on with old age, and is thought to be occaſioned by a ſhrinking of the humours, which flattens the cornea, and the coats of the cryſtalline: And hence, theſe kind of eyes are called by authors, *præſbytæ*, *viſus ſenilis*; alſo *long-ſighted*, becauſe they cannot ſee near objects diſtinctly. See alſo *fig.* 13.

133. Long-ſighted people, in reading, place the book at a greater diſtance than ordinary, as two or three feet; that being their neareſt diſtance of diſtinct viſion: They alſo commonly write a large hand, and always chuſe a book with a large print; for being obliged to read at a great diſtance, ſmall letters will not ſubtend angles ſufficiently large, and upon that account will be confuſed, as to a common eye, at a like diſtance. The long-ſighted alſo require more light for reading a given print, than others do for reading the ſame print; for they receive leſs light than others, in proportion as the object or book is held at a greater diſtance.

Of ſhort-ſightedneſs. Fig. 25.

134. When the cornea and cryſtalline are ſo convex, that the images of objects, at an ordinary diſtance, fall before the retina, as at *y*; the pictures upon the retina, and viſion cauſed thereby, will be confuſed: Perſons having ſuch eyes, can ſee no objects diſtinctly, but ſuch as are very near; and hence they are called *ſhort-ſighted*; alſo *purblind*, and *Myopes* *. See *fig.* 14.

135. Short-ſighted people, commonly chuſe a ſmall print, and write a ſmall hand; for to them, the letters are magnified by their nearneſs, ſo as to appear as big as thoſe of a middling ſize do to a common eye; and the letters being ſmall, they have the advantage of ſeeing more of them at one view. It has been remarked, that *Myopes* do not uſually look at the perſon they converſe with; for as they cannot obſerve his looks, and the motions of his eyes and features, they are attentive to his words only.

136.

* There are other cauſes that limit the extent of viſion: As, when the pupil is more than ordinary ſmall, the humours turbid, or leſs tranſparent than common; the retina ſomewhat callous or inſenſible: People labouring under theſe diſorders, require more light for viſion, than ordinary; and therefore they cannot ſee ſo far as other people, although their eyes be of proper figures, for uniting the rays from remote objects upon the retina.

136. Those who are short-sighted, can see smaller objects, and also see clearly a small given object with less light, than other people can: For by viewing them nearer, they increase their visible magnitudes, and also receive more light from a given object, in proportion to that nearness. And hence it is, that *Myopes* can read by moonshine or twilight, when a common eye is not able to distinguish one letter from another: They also can see somewhat farther off, by a strong light, than they can by a weak light; for the strength of the light contracts the pupil, and that lessens the indistinctness. Some advantages of short-sightedness.

137. *Schol.* If to a given eye, the nearest distance of distinct vision be four inches; as great a change of configuration will be requisite to such an eye, for procuring distinct vision from four inches to eight inches distance [a]; as will suffice a common eye for obtaining distinct vision at all distances, from 8 inches to infinity. [a] 21. II.

There are some people that are shorter-sighted than as above supposed: It is no wonder then, that such cannot see a small object at arm's length from them, or to avoid a stone at which they are just going to stumble.

138. It is a common observation, that the eyes grow flatter with age; and hence it is, that people who were purblind in their youth, see better as they grow in years, and continue to read without spectacles to the very last. On the other hand, the long-sighted eye grows worse and worse; and at length its refractive coats may become so flat, as not to be able to collect even parallel rays upon the retina. And hence all objects will necessarily be confused; those that are small, as the letters of a book, *&c.* will be altogether imperceptible; and at length by a still greater decay of the humours of the eye, the benefit of naked vision will be in a manner totally lost. The eye grows flatter with age.

It cannot be doubted, but that the parts of the eye, like the rest of the body, grow stiffer with years, and less pliable; and hence aged persons, have less latitude of distinct vision, than when they were younger. Also it is probable, that the optic nerve grows less sensible with years; and this may be another cause of the decay of sight in old age.

139. It has been observed, that people who are frequently used to look at remote objects, as country-men, travellers, sailors, and the like, are generally long-sighted; and so want spectacles the soonest: On the other hand, the greatest number of short-sighted persons, are found among students and mechanics, who are daily conversant with small Short and long sightedness may be acquired.

and near objects. The eye, it seems, in both cases endeavouring to keep that configuration, to which it is most accustomed.

Sometimes come by accidents.

140. Sometimes both long and short-sightedness, come on suddenly by accidents, or illness: Colds especially have been productive of both these effects; and there have been instances of aged persons, who had used spectacles many years, having by an accidental cold become so short-sighted, as never to want them again.

Some persons both long and short-sighted.

141. There are many instances of people who seem to be both long and short-sighted; that is, they can see near objects, and seemingly also remote objects, distinctly; but those that are at intermediate distances, appear confused: This case will admit of an easy solution, if it be supposed, that such people have one eye short-sighted, and the other long-sighted, and that they see only with that eye, which is suited for distinct vision at the near or remote distance *.

Vision of remote objects by short-sighted people.

[a] 129. II.

142. But when we consider what are the [a]chief causes of the indistinctness of remote objects to a common eye; it will seem probable, that with respect to vision of distant objects, the short-sighted are not under such great disadvantages, as at first might be imagined. The planets and all the brighter stars, are as visible to the most short-sighted, as they are to any other; and probably their appearances are much alike to all sorts of eyes. And the greater quickness of some in discerning the smaller stars, may perhaps be more owing to a better constitution of the retina, *&c.* than to any advantage in the figure of the eye. But because of the aberrations upon the retina, a short-sighted person cannot distinguish beyond a certain small distance, such small parts of an object, as another can: Thus, he cannot distinguish the features of a face a-cross the room; and as objects in general are distinguished by their visible parts, so with respect to vision at moderate distances, the disadvantages of the short-sighted are very great and manifest. But although such a person cannot distinguish parts, unless they be very large, or very near him; yet he can perceive an object in the gross, if it be not too small, at a considerable distance: He can perceive a man at the distance of several paces, though perhaps he must advance within one of him, before he can call him by his name. A large tree he can distinguish yet farther; and from his experience in like cases, the large obscure object which he perceives at a great distance off, he concludes

* Some have imagined, that we never see but with one eye at a time: If this be the case, it may be wondered why in looking we always turn the axes of both eyes towards the object. It is very likely, that some people either see altogether with one eye only, or see better with one eye than they do with the other: But that this is not universally the case, I think, is very plain, from the following observation, *viz.* If, whilst looking at any object, either eye be covered ever so nimbly, we continue still to see the object, without any the least intermediate stop; which seems difficult to be accounted for, if the now naked eye had before been idle.

cludes to be a house, to the great surprize, perhaps, of his acquaintance who knows his case. And hence, without supposing one eye to be short and the other long-sighted, we may solve that seeming paradox; why a person who is so purblind as not to be able to see a small object at arm's length from him, is yet, in many cases, not apparently under any such great disadvantages, with respect to vision of remote objects.

143. An eye, that has no other defect but that of being long or short-sighted, may be helped by a lens of a proper figure, so as to be able to see distinctly at any given distance. As, if A*q*, *fig.* 24. be the nearest distance of distinct vision to a long-sighted eye; A*q*, *fig.* 25. the remotest distance of distinct vision to a short-sighted eye; and AQ be that distance, at which each respective eye wants to see an object distinctly. Let O be a lens of such a figure, that rays diverging from Q will after refraction, diverge from *q*; an object Q seen through such a lens will appear distinct. For in both cases, [a] the rays, by the refraction of the lens, are made to enter the eye with such a degree of divergency as is proper for distinct vision: The want of plumpness in the long-sighted eye, being supplied by the convex lens, and the too great plumpness of the myopical eye is corrected by the concave lens.

Long and short-sightedness, how remedied

Fig. 24, 25.

[a] Hypoth.

144. *Defin.* Two glasses, having such figures as are requisite for procuring distinct vision at given distances, and fixed in a proper frame for holding them before each eye, are called *spectacles*.

Of spectacles.

145. Convex lenses magnify, and concave ones diminish, the objects that are seen through them; but these are accidental properties: The end or design of spectacles, being neither to magnify or diminish objects; but to alter the disposition of the rays at their entrance into the eye, as may be most suitable for procuring distinct vision. And this is an end of such vast importance, that the aged is thereby placed, in a manner, upon a level with the young; and enabled to read a common print, when by his naked sight he can hardly distinguish one letter from another. By the help of glasses, the purblind, whose sphere of distinct vision scarce extends the length of his arm, has his sight extended to the greatest distances: And it is owing to the same help, that many of the arts have been brought to that great perfection they now are in.

Their design and usefulness.

146. The long-sighted eye receives greater benefit from a convex lens, than the short-sighted does from one that is concave. For an object seen through a convex lens, is not only magnified; but also the eye receives a larger pencil of light from each visible point, because the rays enter it less diverging. On the other hand, a concave lens not only diminishes objects, but also renders them darker; for the rays coming

Presbytical eye receives more benefit by glasses, than the *Myopical*.

to the eye more diverging, the pupil cannot receive so many of them, as otherwise it would: And hence it is, that *Myopes* cannot see remote objects, unless they be very large and bright, so well through a concave lens, as the theory of the images being upon the retina, may seem to promise. For the principal impediments to distinct vision of remote objects, [a] being their want of light and magnitude; both these wants are, by the concave lens, still increased.

[a] 129. II.

The propereſt ſpectacles.

147. Those spectacles are the most proper, whose glasses are the least convex, or least concave, of any that will answer the purpose. For the least convex will accustom the long-sighted eye, to that conformation which is proper for seeing objects as near as it can; and so may prevent the eye from growing more long-sighted. In like manner, the least concave glass accustoms the short-sighted eye, for vision, at as great a distance as it can. Having fixed upon the proper focal length of the glasses, it is scarce needful to add, that the next thing is to chuse those that shew objects the clearest; and this is best determined by a comparison of several together. There are divers methods of fixing spectacles to the head; but it seems to me, that those which have springs for embracing the temples, are the most convenient.

Of reading-glasses.

148. A *reading glass* is a broad lens, fixed in a frame, having a convenient handle to hold it by: The theory of such glasses is scarce different from that of spectacles, excepting what arises on account of the one being commonly held about midway between the eyes and the object. But for all purposes, spectacles are undoubtedly the most convenient; not only because the space between the face and the object is left open and free, and also each hand is at liberty; both which are advantages in writing, and still more in working: but also they are insensibly moved, by the natural motion of the head, without any kind of trouble. Spectacles also shew objects clearer, and distincter, than a broad lens; because their glasses are thinner, and placed exactly before each eye.

Using spectacles, not detrimental.

149. It is a common received opinion, that looking through glasses is detrimental to the eyes: But, I think, this opinion is so far from being true, that to those who want them, spectacles having glasses [a] of proper figures, may contribute to preserve the eyes longer, as they take away the necessity of straining them too much. However, spectacles should not be used but in such cases, that distinct vision cannot be had without them. Without this caution, the eyes may become long or short-sighted, sooner than otherwise they would; and to prevent this inconveniency, the best remedy is frequently to look with the naked eye at objects at all such different distances, as we can see tolerably distinct; and also to look sometimes through glasses of different figures.

[a] 147. II.

PROP.

PROP. IX.

150. *To suit a given eye with proper spectacles: Or, to find a lens that will enable a given eye, to see distinctly at a given distance.*

Let Oq be the least distance of distinct vision to a long-sighted eye, and the greatest to a short-sighted eye; and let OQ be that distance, at which each eye would see distinctly. Fig. 24, 25.

1. To find a convex lens for the long-sighted eye; say, Qq : QO :: QO : QF; and place F from Q towards q. Fig. 24.

2. To find a concave lens, for the short-sighted eye; say, Qq : qO :: qO : qF; and place F from q towards Q. Fig. 25.

And in both cases, OF will be the focal length of the lens required. For rays diverging from Q, will after refraction [a] diverge from q. [a] 227. I. Therefore, &c. *Q. E. D.*

151. *Schol.* 1. In the above cases, OQ is the nearest distance of distinct vision through the convex lens, and the remotest through the concave. For Q and q [b] moving both the same way, and q being the nearest point of distinct naked vision in the one case, and the remotest in the other: If Q be brought nearer the convex lens, the refracted rays will diverge too much; and if Q be moved farther from the concave lens, the refracted rays will diverge too little; and so in both cases vision will be confused. [b] 229. I.

But, the *præsbytical* eye can see an object distinct, at a somewhat greater distance than Q; and even as far as F, or beyond F, if it can see distinctly with parallel or converging rays. In like manner, the short-sighted eye will be able to see distinctly, at a somewhat nearer distance than OQ; according to the power it has of changing its configuration.

152. *Schol.* 2. If a long-sighted person wants to see at no nearer distance than half Oq; a convex lens whose focal length is Oq will be the propereſt: for then Qq being equal to QO, the point F will fall upon q. If the above distance be greater than half Oq, OF will be greater than Oq. Fig. 24.

153. *Schol.* 3. If, as before, Oq be the greatest distance at which a short-sighted person can, with his naked eye, see an object distinctly; then a concave lens whose focal length is Oq, will enable him to see objects distinctly at the greatest distance. For such a lens will refract parallel rays so as to diverge from q. Fig. 25.

154.

154. *Schol.* 4. A ſhort-ſighted perſon may eaſily determine by experiment, the fartheſt diſtance at which he can ſee diſtinctly; and ſo, by the preceding theory, he may ſuit himſelf with a lens for any given purpoſe. But it is not ſo eaſy for a long-ſighted perſon to find his neareſt point of diſtinct viſion; for to him the ſmallneſs of objects is a cauſe of indiſtinctneſs, that neceſſitates him to approach them nearer, than according to the theory of aberrations: And therefore he'll be apt to imagine the point *q* to be nigher than it really is; and ſo the focal length OF, will be greater than that which is beſt ſuited for his purpoſe. In this caſe therefore, it is beſt to lay aſide theory, and to chuſe the glaſſes by trials only, remembring the cautions * before given.

Fig. 24.

* 147. II.

PROP. X.

155. *Having the neareſt diſtance of diſtinct viſion to each of two given eyes: To find a lens which ſhall put one eye in the ſame ſituation with the other, in reſpect of ſeeing objects at given diſtances; the power of changing the configuration in each being ſuppoſed equal, and neglecting alſo the conſideration of the loſs of light, and other imperfections of the lens.*

Fig. 24, 25. Let O*q* and OQ reſpectively, be the neareſt diſtance of diſtinct viſion to the eye A, and to any other given eye; and let it be required to change the condition of the eye A, into that of the other ſhorter or longer-ſighted given eye. By the preceding *Prop.* find the focal length OF, of a convex or concave lens, according as the given eye is ſhorter or longer ſighted than the eye A; and becauſe rays diverging from Q, will after refraction by ſuch a lens diverge from *q*; the point Q is alſo now the neareſt point of diſtinct viſion to the eye A: Whence A hath now the ſame degree of ſhort or long-ſightedneſs, with the other given eye without a lens. Therefore, *&c.* *Q. E. I.*

Thus, if O*q* is 8 inches, OQ in the one caſe 4 inches, and in the other 20 inches; the focal length of the convex lens that will cauſe the given ſhort-ſightedneſs, will be 8 inches; and the concave lens that will cauſe the given long-ſightedneſs will have its focal length $13\frac{1}{3}$ inches. But though the caſes of the two eyes above, with reſpect to the aberrations upon their retina's, will be the ſame; yet the loſs of light and other imperfections of the lens, which cannot be here accounted for, will make a very great difference.

PROP.

PROP. XI.

156. *Having the limits of distinct naked vision, to a given eye; to find the limits of distinct vision through a given lens, to the same eye.*

Let Aq and Ar, be the greatest and least distance of distinct vision, to the eye A; and let OF be the focal length of the lens O, placed close to the eye. To find QR the limited distance of distinct vision through the said lens; say, Fig. 25, 26.

1. qF : qO :: qO : qQ;

2. rF : rO :: rO : rR. And placing Q, R, the same way from q, r, that F lies from O; that is, if the lens is concave, place Q, R, farther from it, if it be convex place them nearer to it, than the given points q, r; and OQ, OR, will be respectively the greatest and least distance of distinct vision, through the given lens O. For rays diverging from Q, R, will after refraction by the lens, [a] diverge upon the eye, as from the focus's q, r, respectively; and therefore the space QR is the limited distance of distinct vision. *Q. E. I.* [a] 228. I.

157. *Schol.* 1. Take Of = OF: Now if the eye A can see distinctly, with parallel rays; it will be able to see distinctly through the lens O, as far as f: And if it can see distinctly with rays converging, as to a point s, beyond F; say sF : sO :: sO : sS; and RS will now be the limit of distinct vision through the given lens O: For rays diverging from S, will after refraction by the lens, [b] converge towards s. The nearer s is to F, the greater will be OS; and if s be in F, an object that is at an infinite distance, will appear distinct through the said lens. Fig. 26. [b] 227. I.

158. *Schol.* 2. If the nearest point r of distinct vision to a short-sighted eye, bisects the line Oq, the greatest extent of distinct vision; a concave lens whose focal length is Oq, will enable that eye to see distinctly at all distances, from q onwards to infinity. Fig. 25.

For parallel rays, will after refraction by the lens, diverge from q; and because rF (rq) = rO, the point R will fall in q.

And if when Oq : Or :: 2 : 1, OF be greater than Oq; then OR will be less than Oq.

159. *Schol.* 3. To the same eye, the extent of distinct vision QR, through a given concave lens, is greater than through a convex lens of equal focal length. Fig. 25, 26.

In *fig.* 25. suppose a point f taken beyond the eye in the axis Oy produced, so that Of = OF; then in both *fig.* it will be,

qF : OF :: [a]OF : fQ; [a] 227. I.

rF : OF :: [a]OF : fR; and therefore qF : rF :: fR : fQ; and qF : qr (rF ∽ qF) :: fR : QR (fQ ∽ fR).

Whence

Whence it is sufficiently manifest, that QR, *fig.* 25. must be greater than QR, *fig.* 26. *Q. E. D.*

160. *Schol.* 4. In all that has been observed in the preceding *propositions* about vision through lenses, the places of the geometrical images were supposed to coincide with the retina; but as a precise union of the rays upon the retina is [a] not necessary for distinct vision; so the extent of distinct vision through lenses, will be somewhat greater than according to the preceding theory.

[a] 95, 97. II.

§. 4. *Of apparent places and magnitudes of objects.*

Apparent distance and magnitude, what.

161. *Defin.* Distance and magnitude, are relative terms; that is, such a distance, and such a magnitude is great or small, according to the relation which each bears to that standard measure, to which it is compared. *Apparent distance* is that visible extension that seems to lie betwixt us and the object, compared to some standard measure; and as it appears in the simple act of vision, abstracted from the judgment we might otherwise make from prænotions of the real distance. The apparent distance of the moon, for instance, is the length of an imaginary line that would reach from that place where the moon seems to be, to the spectator: This apparent extent is certainly not many hundred yards, though we know the moon is many thousands of miles off: And the apparent distances properly so called of remote land-objects, are also much less than at first might be imagined. In like manner, *apparent magnitude* of an object, is its visible size or extension compared to some real measure. Thus, the diameter of the moon seems to be about a foot.

In vision a visible place and extension always joined together.

162. It is very certain, that every visible object appears in a determinate place, and as having a determinate magnitude as well as figure: So that the ideas of locality and extension, are inseparable from that of visibility; and these ideas are so intimately connected together, that a mistake in our estimate of the one, never fails to create a mistake in our estimate of the other. The greater, for instance, is the apparent distance of an object subtending a given angle, the greater also will be its apparent magnitude: For, of two objects subtending equal angles at the eye, we know, as it were intuitively, that the remotest object is the largest; and we instantly, and imperceptibly to ourselves, form an idea accordingly.

Apparent distance, an idea of a definite thing.

163. It has been rightly observed, that distance, of itself and immediately, is imperceptible; for it being only a mathematical line directed endwise to the eye, it can have no image upon the retina; or, if we consider it as a physical line, its image on the retina will be only a point,

point, which point will be invariably the ſame for all diſtances. But it is not diſtance in the abſtract, but the diſtance of objects that we are to conſider; and it is very certain, that all objects have their viſible or apparent diſtance, which we can compare groſsly to ſome known and ſtated meaſure. The ſaid viſible diſtance, during the act of viſion, either remains fixed and unalterable in the mind; or, if it ſeems to enlarge or contract, it will ſeem at the ſame time to have more or fewer parts of certain ſtated lengths. So that viſible or apparent diſtance is an idea of a definite thing; however inadequate it may be in ſome caſes, to the real diſtance.

164. It is yet very certain, that the ideas of viſible diſtances and viſible magnitudes, are not innate, but the reſult of experience. As ſuch a particular ſenſation had formerly been obſerved, to be cauſed by an object having ſuch a particular diſtance, magnitude, colour, brightneſs, *&c.* ſo when the like caſe happens again, we form an idea accordingly. Viſible magnitude depends upon the viſible diſtance and the viſual angle, both together: But viſible diſtance ſeems to be independent of both the other two; and as therefore it ſeems neceſſary, that in viſion the idea of viſible diſtance ſhould be formed previous to that of magnitude; it is very likely that the fact is ſo, though theſe two ideas croud ſo cloſely together, that it is impoſſible to diſtinguiſh which of the two was firſt produced. There is yet another reaſon, why we ſhould conſider diſtance firſt; and that is, that we may adapt our eyes properly for diſtinct viſion. But, as above obſerved, theſe things are too quick and tranſient, to be perceived by us. Ideas of diſtance and magnitude, acquired by experience.

The different appearances of objects, furniſh us with ſeveral means whereby to form our ideas of their diſtances; and we undoubtedly make uſe of all the helps that occur at the ſame time: But it is convenient to conſider them apart; and I ſhall begin with thoſe that reſult from the motions we give our eyes, in order to ſee as diſtinctly as we are able.

165. *Schol.* But before we proceed any farther, let it be obſerved here, once for all, that when we ſpeak of an *optic angle*, *focus of a pencil of rays*, or *viſual angle*. 1. It is not meant, that we ſee the point of concurrence of the two optic axes, theſe axes being nothing but imaginary lines: But nevertheleſs, as on the account of different diſtances, we give our eyes different inclinations; ſo by experience, we learn from the ſenſations accompanying thoſe motions of our eyes, to judge of ſuch diſtances as are not very great in proportion to the diſtance between the two pupils: And as the above inclinations are always analogous to to the *optic angle*, ſo we expreſs them by that angle. 2. In like manner,

the eye does not ſee a pencil of rays; but whilſt the breadth of the pupil, bears a ſenſible proportion to the diſtance of the focus from whence the rays diverge to the eye, we have peculiar ſenſations, which from the experience we have had, inform us pretty nearly of the place of that *focus*. 3. So when we ſpeak of a *viſual angle*, it is not meant that the eye meaſures that angle, as with an inſtrument: But the magnitude of an image upon the retina, being always proportional to that of the viſual angle of the object; ſo the difference of ſenſations accompanying different degrees of the one, enable us to judge of different degrees of the other. Thus, we ſee that lines and angles in optics, are not only uſeful for communicating our ideas to one another, but they are alſo not without their foundation in nature.

I. *The change of conformation in the eye, is one of the means whereby we are enabled to judge of ſmall diſtances.*

166. In viewing objects at ſuch ſmall and different diſtances, that the breadth of the pupil bears a ſenſible proportion thereto, or objects that are within about arm's length of us; at every ſenſible change of diſtance, the eye muſt alſo change its conformation for procuring diſtinct viſion. As if the eye is adapted for ſeeing diſtinctly at 8 inches diſtance; it muſt gradually change its figure, to bring upon the retina the images of objects that are more remote. And thus, if we were accuſtomed to look attentively at objects, with one eye only; it is very likely, that the changes made in the eye upon theſe occaſions, would be ſenſible enough, after repeated trials, to enable us to judge pretty accurately of the different degrees of ſmall diſtances. But for want of experience, a perſon at firſt making the trial, will be ſubject to ſome ſmall miſtakes; as has been obſerved, if a perſon, having the uſe of both eyes, attempts to thread a needle with one eye ſhut. However, it does not follow from that experiment, that a perſon in ſuch a ſituation cannot diſtinguiſh the place of a near object within an inch or two; and after a little practice, I think, that with one eye only, we can ſcarce ever miſtake $\frac{1}{4}$ or at moſt $\frac{1}{2}$ inch, in the ſituation of a near object.

Latitude within which we can thus judge of diſtances.

167. The above method of eſtimating diſtances, will not ſerve us beyond a certain ſmall extent, not greater perhaps than about 20 or 30 inches: For if beyond that diſtance, a line be ſuppoſed infinitely extended, and in this line ſeveral points be conſidered as the focus's of pencils of rays proceeding to the pupil; the difference of the degrees of divergency of the extreme rays in theſe pencils, will be inſenſible, and bear no proportion to the different diſtances of their focus's. And beſides in theſe caſes, there are other cauſes [a] that affect viſion, much more than

[a] 129. II.

than the different inclinations of the rays. Hence it is, that when the diſtance of an image, ſeen through a lens with one eye only, is greater than about a foot or a foot and a half from the lens, we are ſo liable to err in our judgment of that diſtance.

168. *Schol.* As an object that is approached too nigh [a] will appear [a] 89. II. confuſed, and the nearer it is approached, the greater will be the confuſion: So in theſe caſes, by the different degrees of confuſedneſs, we may make an eſtimate of the different nearneſſes. Remote objects may alſo have the ſame apparent confuſedneſs, though from other quite different cauſes: But, I think, we can ſcarce ever miſtake a remote object to be within a finger's length of us; and eſpecially if we uſe both our eyes, ſuch a miſtake will be next to impoſſible.

II. *Inclination of the optic axes, is another more certain means of diſtinguiſhing degrees of ſmall diſtances.*

169. In viewing an object attentively with both eyes, we always direct the axis of each towards it; and the nearer is the object, or the point of concurrence of the two axes, the more, conſequently, they will be inclined to each other; and the more remote the object is, the leſs will be the ſaid inclination: The ſenſations accompanying the different degrees, of theſe inclinations of our eyes, together with thoſe of their different conformations for ſeeing diſtinctly; ſeem to be ſufficient of themſelves, excluſive of other aid, to enable us to judge of ſmall diſtances, not exceeding a certain extent, pretty accurately.

It is very certain, that difference of brightneſs and ſhades of objects, are circumſtances, which in many caſes affect our ideas of diſtance. But notwithſtanding the painter's art, if one approaches within a certain diſtance of his picture, he will perceive very plainly, that the whole ſurface is ſmooth and flat. In the painted hall at Greenwich, for inſtance, the pilaſters viewed at ſome yards diſtance, appear fluted; but by approaching them nigher, at a certain diſtance, their ſurfaces appear ſmooth: Therefore one can perceive a ſmall difference in the diſtances of objects placed within this extent; and what is here to help him, but the ſenſations he has from the different motions of his eyes? To one that will make it, this experiment will enable him to determine that extent, within which he can judge to a given accuracy of different diſtances, by the above helps only. But the following experiment will be more deciſive.

Behind a ſcreen, place upright two or three objects a little ſideways of one another, and at little different diſtances from an horizontal ſlit made in the ſcreen. The objects may be cylindrical ſticks of different

thickneſs, and colours; and let them be ſo diſpoſed, as their ſizes, &c. may be moſt likely to deceive the ſpectator behind the ſcreen, in his notion of their diſtances. The ſlit in the ſcreen ſhould be ſo big, as to cauſe no manner of obſtruction to the light coming from the objects to the ſpectator; and it ſhould be alſo ſo limited, as to exclude all objects from between it and thoſe deſigned for the experiment; that is, neither the floor, or any covering above, or any objects to the right or left of the above-mentioned, ſhould be viſible. Things being thus diſpoſed, and repeating the experiment ſeveral times, by changing the places of the objects, without the privity of the ſpectator beyond the ſcreen; you may determine pretty accurately, within what degrees he will be able thus to judge of different diſtances. An experiment to this purpoſe, may be eaſily made with a ſmall looking-glaſs, inſtead of the above ſcreen.

The above farther confirmed, by the apparent places of images.

170. *Schol.* The ſudden and exact eſtimates, we make of the diſtances of images behind a ſpeculum, is a farther proof of what has been above advanced: For theſe images are generally attended with ſome circumſtances, that rather tend to miſlead than to help us, in our eſtimations of their diſtances. An image in a plain ſpeculum, is always darker than the object, and this might induce us to think it farther than it really is; but nevertheleſs, if it is not far, we never miſtake its diſtance, although it be detached from others, according to the conditions of the preceding experiment. The images of objects placed at ſome diſtance before a convex ſpeculum, [a] are always very much diminiſhed; and ſo they appear to a ſpectator: But notwithſtanding this diminution, if he is not above two yards from them, he will judge of their diſtances from the ſpeculum very accurately; or he will ſee them as near as can be eſtimated, in thoſe very places from whence the rays diverge. Let a plane ſpeculum be placed cloſe to the convex one; and the different diſtances of the images of the ſame objects in each, will be very remarkable; nor will it, from a near ſtation, be poſſible to fancy them ſo remote from the convex ſpeculum, as they are from the plane one.

[a] 111. I.

If I advance within 4 or 5 feet of an image, projected before a concave ſpeculum, I can define its place very preciſely; and the image itſelf, though much ſmaller than the object, will appear very perfect, and continue ſtill at the ſame place, as I advance nigher: But on my retreating farther back than my firſt ſtation, I begin to be leſs certain of its place; and the miſtake lies, in ſuppoſing it farther from me, than it really is: My faculty of diſtinguiſhing diſtance in this caſe, not being ſufficient to overcome the prejudice ariſing from the greater faintneſs, and other imperfections of the image.

171.

171. From the preceding obſervations, I think, it is very manifeſt that with both our eyes we can diſtinguiſh, pretty accurately, the places of objects that are not above five or ſix foot diſtance from us. And indeed it ſeems very neceſſary, that we ſhould have ſomething within our ſelves, or ſome means that never forſake us, whereby we might unerringly judge of diſtances ſo very near us: Otherwiſe we might be frequently in danger of our lives, without perceiving it; as well as ſubject to perpetual miſtakes, concerning objects ſo near as within the reach of our arm.

Limits within which, by the motions only of our eyes, we can eſtimate diſtances to a tolerable exactneſs.

But as the diſtance becomes greater than $1\frac{1}{2}$ or 2 yards, we begin to be leſs certain in our eſtimations of it; and beyond 3 or 4 yards, the means hitherto conſidered, ſeem to be of little or no help for aſcertaining the places of objects. For beyond that extent, the differences of the optic angles on the account of different diſtances, are ſo very ſmall, as to become in a manner inſenſible.

III. *The length of the ground-plane, or the number of intervening parts perceived in it, is another mean, by which we perceive diſtance.*

172. When the floor or ground on which we ſtand, is uniformly extended before us; in a line produced directly from us on this plane, we can diſtinguiſh that ſuch ſucceſſive parts, as form ſenſible angles at the eye, are ſucceſſive or one behind the other; and the greater number of viſible parts the line contains, the greater conſequently is the viſible extent of the whole. Let AB be the height of the eye above the ſtrait line AC: Now if the viſual angles EBD, DBC are each of them about 2 *min.* the parts ED, DC will appear as two contiguous points, each being[a] a *minimum viſibile:* But if the ſaid angles are each ſenſibly greater than 2 *min.* then three objects placed at E, D, C, may be diſtinguiſhed as being ſituated one behind the other. Fig. 27.

[a] 83. II.

173. An extent of ground lying before us, is itſelf properly an object, the viſible length or magnitude of which, is the viſible diſtance of an object placed at the farther end of it: The ſame obſervation is alſo juſt, with reſpect to the ſide of a ſtrait wall, or hedge, *&c.* In this caſe therefore diſtance is properly viſible, ſince the intervening chain that meaſures it, is itſelf viſible: And it ſeems very probable, that from the experience we have had in meaſuring ſuch a viſible extent of floor or ground lying before us, by walking it over or otherwiſe, it is that we firſt have the idea of diſtance. This idea is ſo fixed, that all objects have their viſible diſtance, although nothing is perceived to intervene betwixt them and the ſight. A near object directly facing us, is perceived to be within or without arm's length of us, or within the length of

Idea of viſible diſtance, how acquired.

of a ruler, containing a certain number of parts of a ſtated length: This judgment being made by the preceding methods of eſtimating diſtance. A crow flying in the air has its viſible diſtance, how widely ſoever this may differ from the real diſtance; that is, in all caſes, the viſible diſtance ſeems to be a line having a definite length.

Viſible diſtance a fixed idea, and much the ſame to all people.

174. We have the ideas of viſible diſtances ſo early, that we remember nothing of our having been ever without them: And theſe ideas are ſo firmly ſettled in our infancy, that our future experience does not afterwards ſenſibly alter them. Since I learned that the moon is many thouſand miles off, her apparent diſtance is to me, not a whit greater than it was before; nor did that new-acquired knowledge cauſe any alteration in her viſible magnitude. The viſible magnitude of the ſky, and viſible diſtances in general of given objects, I imagine are much the ſame to all common eyes; although for want of practice in meaſuring, and their being apt to confound the idea of viſible diſtance with preconceived notions of the real diſtance, people will differ in their eſtimate as to the number of real feet or yards, *&c.* contained in ſuch a viſible extent. Thus, if a given viſible extent be conceived as being divided into a few number of equal parts, which may be called degrees; I take it, that moſt people will judge much alike of the places of the dividing points, and that perhaps nearly as well in proportion, as they would judge of the places of points, that divide a ruler into a like number of equal parts: And if this be the caſe, it ſhews that viſible diſtance is much the ſame to different people, how widely ſoever they may differ in their notions of the real diſtance.

Viſion a faculty, and independent of the ſenſe called feeling.

175. *Schol.* It ſeems undoubtedly true, that all our ideas are acquired. But it ſeems as true, that ſeeing is learnt ſo perfectly in our infancy, that but little addition is made to it afterwards: Or, that viſible ideas, properly ſo called, or the appearances of objects during the act of viſion, are much the ſame that they always have been *, ſo far back as our memory reaches. Viſion therefore cannot, with any propriety, be reckoned an art, as ſome would ſeem to make it; ſince the appearances of objects are the ſame to the illiterate and the men of ſcience; and alſo continue always much the ſame, unleſs it may be in ſome few uncommon caſes, which had not earlier occurred to us. The judgments we form concerning objects are ſubject to great variations, according to the greater

* It ſeems as if our own ſtature is a kind of ſcale, to which we compare the magnitudes of objects: For it is certain, that in our infancy, the ſame things look bigger than they do afterwards; and diminiſh as we grow up, ſeemingly in proportion as we have enlarged our ſcale; and that having done growing, things have much the ſame apparent magnitudes ever after. But this affects not what is above ſuggeſted; for the proportions of apparent magnitudes of different objects, are not ſubject to theſe variations: However, it ſeems to create a doubt, not eaſy to be cleared, whether the ſame things do not appear larger to ſmall-ſized people than they do to others: But this queſtion affects not the preſent argument.

greater advances we make in knowledge: But it is not these rational ideas that are here considered. Vision then is a faculty, which conveys an image to the mind, analogous to that upon the retina; nor does it seem to be always founded upon prænotions from tangible ideas, or notions arising from feeling. How else could distorted images, as those in cylindrical speculums; objects seen in microscopes; diagrams drawn upon paper, the like of which we had never felt with our fingers; how else, I say, could these, and a thousand other objects that are not tangible, be so like in their appearances to different people? These instances seem to shew, that a person who never had had the use of his hands, would have the same visible ideas with another; so far does it seem from being true, that visible ideas have that near connexion or dependance upon tangible ideas, which some have imagined. And indeed, are not the ideas proceeding from each of our senses, independent one of the other; however one idea may bring to memory, another idea peculiar to a different sense?

There is a certain extent within which, visible distance is adequate to the real.

176. If an indefinite line, extended from the foot of a spectator directly before him, be divided into equal parts, as yards, &c. the visual angles subtended by these parts continually diminish, as the said parts are at a greater distance. Yet there is a certain extent, within which, visible distance differs not sensibly from real distance: That is, if a line of the said extent be divided into a small number of equal parts, the visible or seeming length of the remotest part, will not be much less than that of the nearest. This extent, in most cases, I do not take to be much less or much more, than about ten yards; and if a line of that length, drawn upon an even floor, be divided into 10 equal parts, distinguished by little stones or bits of paper, and a spectator stands at one end of the line; the first yard or part, will not appear much bigger than the remotest; although it is very certain, that to a person of a common stature, standing upright at the end of an horizontal line, the two first yards of it subtend a larger angle, than the rest of the line would, if it was infinitely extended.

The nearness that we can distinguish degrees of distances in given cases, estimated.

177. An indefinite line being extended obliquely before the eye, and divided into equal parts, as above; the visible extent of those parts that are beyond the limit above-considered, continually diminish, and that faster and faster the more remote they are; so as beyond a certain limit, all objects have the same visible distance from the eye, how great soever the differences may be of their real distances. There are several causes which, as they occur, tend to lengthen or shorten the apparent extent, of the same real extent. But in order to make some estimate of the nearness, that we can distinguish degrees of different distances; let us suppose this nearness at a medium, to be as the lengths of the least visible

visible parts, taken at given distances in an horizontal line uniformly extended before a spectator of a given stature. Let AC be that indefinite horizontal line, AB the height of the eye, suppose $5\frac{1}{2}$ foot; and suppose the *minimum visibile*, or the angle subtended by the least visible part, to be 2 minutes. Now, if the $\angle$ E B e = 2 *min.* and the distance AE be 10, 20, or 50 yards; the least visible part E e will be accordingly about $1\frac{1}{6}$ inch, $4\frac{1}{2}$ inches, or 29 inches; and therefore in the respective cases, these lengths are the least differences of real distances, that can be perceived by the eye. If the $\angle$GBC = 2 *min.* and AG be supposed 100 yards, the least visible part GC will be near 10 foot; and so beyond 100 yards, the visible falls very short of the real distance. If AG be $\frac{1}{4}$ or $\frac{1}{2}$ of a mile, GC will be accordingly 71 yards, or 347 yards; which last number is above $1\frac{1}{2}$ furlong. And therefore two objects, whereof one is at $\frac{1}{2}$ a mile, and the other at $\frac{3}{4}$ mile distance from a spectator, will seem very near together. When AG is $\frac{1}{4}$ mile, the $\angle$ AGB = HBG is less than 15′; and when AG is $\frac{1}{2}$ mile, the said $\angle$ is only about 7′: So that if the line AG be infinitely extended, all beyond G in the one case, would not contain above 8 visible points; and in the other, it would not contain 4 visible points. If the part AG be supposed 3151 yards, which is about $1\frac{3}{4}$ mile, all the line beyond G, if it be infinitely extended, will subtend an $\angle$ at B = 2′, and therefore will be a *minimum visibile*.

Fig. 27.

Visible extent of a given line, as the height of the eye.

178. *Schol.* 1, If in a plane passing through AB, a given line DF be seen very obliquely by an eye at B; the visual angle DBF, will be as AB, the height of the eye above the line DF produced to A, nearly.

Draw Fb parallel to AB: Now the triangles DFb, DAB being similar, and their bases DF, DA being invariable; the ratio of Fb to AB will be also invariable: But the $\angle$ DBF will be as Fb, nearly. Therefore, *&c. Q. E. D.*

Fig. 27.

Visible distance variable, for various causes.

179. *Schol.* 2. It is manifest from observations, as well as from the reason of the thing, that a given extent appears longer, according as it contains a greater number of visible parts: And hence two remote and equal distances may appear very unequal, according to the different circumstances of the intervening parts, and as the spectator is differently elevated. Thus, by the preceding, a given part of an extended plane contains a greater number of visible parts, and therefore the whole made of the collective parts, appears longer, according to the greater elevation of the eye: But without regarding the differences upon this account, let the spectator have a given station, or stand upon even ground, and there will be many circumstances, which may vary the apparent or visible distances. Thus, a hedge having in it several grown trees, generally looks longer than a clipt hedge, or the same extent of ground

ground in the open field; becauſe the ſeveral parts of the uneven hedge are more remarkable and perceptible, than the like ſpaces upon the even ſurface of the ground. For the ſame reaſon, a row of houſes, columns, or trees regularly planted, appear longer than a plain wall of the ſame extent: For, here we have not only more viſible and remarkable parts in the one caſe, but our præ-knowledge of the ſeveral intervening objects being equidiſtant, tends to protract the apparent length of the whole chain ſtill longer. A river at firſt, looks not ſo broad, as after we have had a ſide-view of the bridge acroſs it: And indeed, a given extent of water, does not appear ſo long as the ſame extent of land; it being more difficult to diſtinguiſh parts in the ſurface of the one, than it is in the ſurface of the other. In like manner, a village ſeen from an eminence, appears farther, as a greater number of fields, *&c.* are perceived to intervene: For the hedges being remarkable objects, and the preciſe breadths of the ſeveral fields unknown; their greater number affects the idea of viſible diſtance. And univerſally, a greater number of intervening objects, all other circumſtances being the ſame, render the whole apparent diſtance always greater; but not greater in proportion to that number.

180. Again, when part of the intervening chain is inviſible, or obſcurely ſeen; the apparent diſtances of objects that are beyond that part, will be accordingly leſs. Thus, a certain extent of rough uneven ground, appears ſhorter than the ſame extent of plain ground; the prominent parts hiding the cavities behind them, the whole apparent diſtance is ſo much the leſs, by the loſs of thoſe inviſible parts. For the ſame reaſon, the brow of an hill, ſeen over the top of another, frequently looks nearer, than it does after the vale between preſents itſelf to view. And the banks of a river at ſome diſtance, will ſeem contiguous, if none of the ſurface of the river is viſible. Theſe obſervations regard only the apparent diſtances of objects that are beyond a certain limit; [a] for there is a limit, within which the apparent diſtance will not be ſenſibly affected, by the interpoſition of a viſible chain.

A loſs of part of the intervening chain, leſſens the viſible diſtance.

[a] 171. II.

181. An aſcent, as at AT or DT, ſeen from B, contains more viſible parts than an equal extent, AC or DC, upon a level; ſuppoſing each to be alike even and ſmooth ſurfaces. For the aſcending plane being leſs oblique to the eye at B, than the horizontal plane AC; each ſucceſſive part of the one, ſubtends greater viſual angles, than the correſponding parts of the other. And hence, an aſcent appears longer, than the ſame extent of level ground; and eſpecially, an aſcent DT at a conſiderable diſtance from the eye, appears much longer than the ſame extent of level DC, at a like diſtance. And, within a certain limit, the greater

An aſcending plane appears longer than a level plane of the ſame extent.

Fig. 27.

is the acclivity, or the ∠CDT, the greater will DT appear. For the ſame reaſon, a deſcending plane appears ſhorter, than an horizontal plane of the ſame extent. However, the apparent lengths of thoſe correſponding parts, will not be exactly in the ratio's of the viſual angles; for we generally make ſome allowances, though ſeldom or never enough, on account of the different inclinations of thoſe planes with reſpect to the horizon.

Before we proceed any farther on the ſubject of apparent diſtance, it will not be amiſs to make a few obſervations on the appearances of planes in given poſitions.

An horizontal plane ſeems to aſcend.

182. An horizontal plane AC ſeems to aſcend, for ſome diſtance off, from the ſpectator. For the viſual rays cut a perpendicular, EI, to the horizon, in points that are higher and higher, or nearer to the horizontal line BH, according as they belong to points in the line AC, that are more remote from A; and the images of the more and more remote parts, having the like ſituations upon the retina, as the images of objects having greater and greater elevations, we form an idea of a gradual aſcent accordingly. And becauſe the greater is AB, or the higher is the point B, the [a]greater will be the viſual angles formed by the ſucceſſive parts of AC; ſo alſo, the greater will be the apparent aſcent of AC. The apparent aſcent of the ſea, to a perſon ſtanding on the ſhore is in ſome caſes ſo remarkable, that one might expect to ſee it come rolling towards him, and overflow its banks.

Fig. 27.

[a] 178. II.

Phenomena of the cieling and ſides of a long gallery, &c.

183. It is for a like reaſon as the above, that a cieling, as ML, appears to deſcend gradually towards the horizontal line BH; and that faſter than the floor aſcends, in proportion as BM is greater than BA. In like manner the ſides of a long room, or two parallel rows of trees, as AC, ML, ſeem to a ſpectator ſtanding between them, to converge towards the farther end; and that ſide which is fartheſt, will ſeem to converge the moſt.

But as the viſual angles EBF, FBD, &c. ſubtended by the equal parts EF, FD, &c. continually diminiſh, the more remote thoſe parts are from AB; ſo the two parallel lines ML, AC, ſeem leſs and leſs inclined to the line BH, the farther they are extended; and inſtead of appearing ſtrait lines tending towards a certain remote point, they ſeem more like the legs of two hyperbola's converging towards the line BH. And hence, a long horizontal floor or plane, and the cieling over it, ſeem convex towards the eye; or to aſcend and deſcend reſpectively leſs and leſs, the farther they are extended.

184.

184. Again, a tall upright pillar, or the wall of a high building, as AC, will seem to lean forward to an eye at B, towards the vertical line BH. But this phenomenon may be partly owing to another cause: The centers of the images of objects, that we directly look at, are always in the axis of the eye, or upon the same point of the retina; and we judge of the situations of objects in oblique directions, according to the situations of their images, with respect to that fixed point in the eye. In looking upwards, after inclining our heads all we can, we are apt to imagine that we have brought the axes of our eyes to point towards the zenith; but this not being the case, our eyes looking forwards many degrees from it; whatever is in the zenith, seems to be as many degrees behind us. And this accounts for the mistakes, that people generally are apt to be under, concerning the altitudes of stars having very high elevations. Why an upright building seems to lean forward.

IV. *Different degrees of apparent distances are suggested, by the different appearances of known objects, or by the known magnitudes of their least visible parts.*

185. As objects become more and more remote, we continually lose sight of parts that are in themselves still bigger and bigger; and these losses perpetually vary the countenances of objects, so as, within a certain limit, to afford us means of distinguishing different degrees of distances. Thus, when I am at the nearest distance of distinct vision from a given object, its smallest visible parts, and the little wholes made up of these, are then most distinct of all: But, as I retreat farther back, the smaller parts become first indistinct, then vanish; afterwards these are succeeded by others that are larger, which after the same manner dwindle away, and become absorbed in greater ones. And as thus I am continually losing sight of more and more parts, till at length they are all absorbed in one confused and ill-defined whole; so from the experience I have had in like cases, those different appearances of the object, which may be called different degrees of distinctness, suggest to me the respective degrees of real distances.

186. *Schol.* 1. Observations of the above kind, are the most universal means we have, and in some cases the surest, for discovering degrees of real distances: And the judgments we form of the real distances of remote objects, seem in some measure to affect the idea of visible or apparent distance, properly so called. It is very probable, that generally, the greatest apparent distance is not many score yards; yet we cannot help fancying, that a building, none of whose parts are discernible, appears much farther than another, whose windows and doors are vi-

ſible; and this other likewiſe appears farther than a houſe, having viſible parts which are known to be ſtill ſmaller; as the bricks in the wall, tiles on the roof, *&c.* In like manner, the different appearances of trees, animals, *&c.* as they are means which ſuggeſt different degrees of real diſtance; ſo they alſo cauſe ſome difference in the apparent diſtances.

187. *Schol.* 2. The occurrence of objects of unuſual magnitudes, and detached as it were from others, by their remarkable heights, *&c.* ſometimes miſlead us in our judgment of diſtances; the greater magnitude ſuggeſting the nearer diſtance. Thus, of two unequal ſteeples or towers, wide of one another, but at about the ſame diſtance from a ſpectator who is a ſtranger in the country, the larger tower may appear neareſt; and in conſequence of this miſtake, houſes or cattle that are near the larger, will appear ſmaller than others of the ſame ſize, near the other tower. In like manner, whenever we ſee a remarkable object afar off, whoſe real magnitude is much greater than common, or than we imagine it to be; we are apt from thence to imagine it nigher than it really is. But theſe are particular caſes, which ſeldom happen; and when they do, we are rarely left without ſome other helps, whereby theſe miſtakes might be corrected.

V. *All other things being the ſame, different colours and degrees of brightneſs of objects, cauſe a difference of apparent diſtances.*

188. As objects become more and more remote, the light proceeding from their leaſt viſible parts [a] is continually diminiſhed by the heterogeneity of the intervening medium; ſo they appear gradually more faint, languid, and obſcure: Alſo their colours not only loſe of their luſtre, but likewiſe degenerate from their native hue, and participate more of the blueiſh colour of the ſky, as the rays have paſſed through a greater body of air, or as the images upon the retina are tinged with a greater proportion of ſky-light. Theſe different appearances are of help to us in judging of degrees of real diſtances, of known objects; and ſo they affect the ideas of apparent diſtances; thoſe objects that are brighteſt, and whoſe colours are moſt vivid, appearing neareſt.

[a] 30, 31. I.

189. *Schol.* 1. By the above method of eſtimating diſtances, we are alſo ſubjected to frequent miſtakes; ſince objects at a given diſtance will appear farther, the darker and more obſcure they are: And alſo, the ſame objects, and at the ſame diſtances, will ſeem nearer or farther, as the weather is more or leſs ſerene. Thus, in foggy weather, all objects ſeem farther than ordinary: The diminution of light in that caſe, anſwering to that on account of greater diſtance, we are miſ-led accordingly.

dingly. A weſterly landſcape in a clear morning, with the ſun upon it, ſeems nearer than it does, when the ſun is higher up, or more weſterly: That aſpect of the ſun being the moſt advantageous for illuminating the objects, they appear brighter, and therefore nearer. To which may be added, that the air being leſs illuminated in the morning, than when the ſun is higher up, is another cauſe that favours the phenomenon. The diſpoſitions of the clouds, alſo affect the apparent brightneſs of objects; ſo that the different caſes that may affect the apparency of the diſtances of remote objects, are innumerable. And the miſtakes we fall into, on account of different circumſtances of brightneſs, are undeniable proofs, that the degrees of brightneſs of objects, have their ſhare in forming our ideas of remote diſtances.

190. *Schol.* 2. From the ſeveral preceding obſervations, it appears, that after joining together all the helps we can have, our eſtimations of diſtances, beyond a certain limit, are groſs and uncertain; and this limit alſo varies in different circumſtances. And the more certain eſtimates we always make of near diſtances, ſeems, as has been before obſerved, to prove that in theſe caſes, the ideas are principally formed from the motion of our eyes; otherwiſe the different colours and brightneſs of objects, and alſo the magnitudes of ſuch as we had not ſeen before, would lead us into perpetual miſtakes: And the leaſt viſible parts of unknown objects, can furniſh nothing to conjecture from.

191. *Schol.* 3. As the apparent ſituation, in reſpect of the optic axis, of a broad object, or of a row of ſeveral objects, is according to the apparent diſtances of the extremes from the eye; ſo by miſtaking the neareſt extreme or object to be the remoteſt, which may frequently happen, we alſo miſtake the ſituation, with reſpect to right and left, of the ſide next towards us; and the more is the real obliquity, the more liable we are to commit theſe miſtakes. Let A, B, C, D, be a row of lamps, trees, columns, the arches of a bridge, &c. Now if a ſpectator at O, happens to imagine the neareſt object A, to be the remoteſt, as at *a*; he will conſequently imagine all the reſt to be at *b*, *c*, *d*, in a line extended on his right hand, contrary to the ſituation of the line AD, which here is to his left †: And ſo whilſt he is under the above miſtake, if he ſees a body moving from A towards D, it will appear to him to take a quite different courſe, as from *a* towards *d*. The poſition of the line *a d* will be variable, according to the imaginary proportion of O*a* to O*d*. For the viſual angles AOB, BOC, COD will be the ſame, wherever in the lines OA, OB, OC, OD, the points *a*, *b*, *c*, *d*, be taken.

The ſituations of objects frequently miſtook, and why.

Fig. 28.

192.

† The learned and worthy preſident of the *Royal Society*, MARTIN FOLKES Eſq; is the author of the above ſolution.

192. When the object is but of small extent, in proportion to its distance from the spectator, it is so much the more difficult to determine its true position. And hence, beyond a certain distance, we are very liable to mistake the position of a weather-cock, the fly or sails of a windmill, *&c.* and consequently to mistake the course of their circular motion. Thus, if a spectator at O, situated nearly in the plane of the sails, AE, CF, imagines the nearest end A, of the sail AE, to be the remotest; and the real motion of the sails be according to the order of the letters ABCDE: Now when A is got to B, it will seem in D, as if it had moved from E to D; and so the sail seems to go round according to the order of the letters EDCBA, contrary to the true course of its motion. This deception is so strong, that it is frequently an even wager, that one mistakes the true course of the above circular motion. We are also liable to the same mistakes, and that for the same cause, as to the circular course of a branch or hoop of lighted candles, turned round at some distance. Fig. 29.

Greatest apparent distance: or, visible extent of the horizon.

193. Before we dismiss this subject, it will not be amiss to make an enquiry concerning the limit or extent at a medium, of the greatest apparent distance. It is very certain, that no objects appear farther than the visible horizon; and also, that the horizon itself appears farthest, when the whole intermediate tract of land is visible. But as that visible extent will be different, [a]according to the height of the spectator; [a]the evenness or roughness of the ground; [a]as the ground lies either level, ascending, or descending: Let us suppose a spectator of a common stature, to stand in the midst of an indefinite horizontal plane, and the visible distance of the horizon will be [b]less than two miles; supposing the least visible angle to be 2 *min.* and in the present case, we may rather suppose that angle to be more instead of less. But as the appearance of the distance, upon which the above calculus is founded, is itself contracted; the apparent distance of the horizon must be still lessen'd; and, from [b]what has been already observed, it cannot sensibly exceed a quarter of a mile. Let AC be an indefinite horizontal plane, AB the height of the eye $5\frac{1}{2}$ foot, the least visible angle $= 2'$, AG 100 yards, CG 1 yard; and suppose GC appears from B, equal to a foot, at a near distance: Now the $\angle AGB = GBH$ is only $63'$; and therefore upon the present supposition, the greatest distance beyond G, would be visibly not above $31\frac{1}{2}$ feet. If we begin our calculation from a lesser distance, the whole apparent distance will be still less, so as to fall far short of 100 yards, and even so as not to exceed half that length. This at first sight, will seem surprising to most people; and, as[c] before observed, it is not unlikely, but that a pre-conceived notion of a greater distance, may in some measure affect the idea of apparent distance. But

[a] 179, 180, 181. II.

[b] 177. II.

Fig. 27.

[c] 186. II.

But to throw aside all doubt that might arise, from the supposition of the spectator's height, the extended plane, or the least visible angle being too great or too small; let us make a calculus from the apparent magnitude of an object, seen in the horizon. Suppose the moon, for instance, when near the horizon, appears a yard in diameter, or equal to a round table just at hand, whose diameter is a yard: The visual angle subtended by the moon is about $\frac{1}{2}$ degree; and therefore the apparent distance of the moon is the radius of a circle, whose circumference is twice 360 yards. For it is manifest, that twice 360 apparent moons, or yards, would go round the periphery of a circle of the above magnitude. Upon the above supposition, the apparent distance of the moon will be about 115 yards. In like manner, by having the visual angle, and apparent distance of any object, we may find its visible or apparent distance. But as the moon seldom looks near so big, as above supposed, I am apt to imagine upon the whole, that the greatest apparent distance, is generally much less than an hundred * yards; and sometimes not above fifty: But in some cases, it may probably amount to a quarter, or may be half a mile; as when we stand upon an eminence, viewing a large vale or a gradual ascent, on which are a great number of objects, whose different distances are very perceptible.

194. *Schol.* One main purpose of vision being to discover, as near as may be, the true places of objects; we do not rest satisfied with mere apparent distance, but from thence, and from such other helps as occur, we always make some judgment of the real distances of terrestrial objects. And hence it is, that most people are apt to think the simple apparent distance of a remote object, to be much more extensive than it really is. It is very certain that the apparent distances of two steeples, one at a furlong, and the other at 10 miles distance, are much the same, if no objects are perceived to intervene betwixt them; and yet we are seldom without helps, by which we can distinguish that one is much more remote than the other. The principal aids for estimating real distances, are those which have been already considered; and for very remote distances, the [a] two last are of most use. By these helps, those who have had frequent experience, as country-men and people who often travel, will be able to guess at the real distance of a remote object, within, may be, $\frac{1}{3}$ or $\frac{1}{4}$ of the whole distance; from the observations they often had made, that certain objects, at such and such distances, have such and such appearances. Thus, seamen will not err many leagues, in their estimations of the distance of an object ten

Our estimations of real distances.

[a] 185, 188, II.

* It is scarce necessary, I think, to observe, that by a *yard* here is meant, the apparent extent of a ruler of that length, when it is pretty near us.

ten leagues off; and as the object becomes nigher, they will be more exact in proportion.

Of apparent Magnitudes.

When the apparent and real distances are alike, the apparent are also as the real magnitudes.

195. When objects are so near us, that their apparent distances are as their real distances; their apparent magnitudes are also as their real magnitudes. This, I think, is universally true, excepting a difference with respect to two objects, which in some cases proceeds from a difference of colour and brightness. Thus, a man at a yard from me, or at six yards from me, appears in both cases of the same magnitude. And the ideas we form of the magnitudes of different objects placed in the same room, are always pretty accurate.

Apparent distances being the same, the apparent magnitudes are as the visual angles.

196. Again, the apparent distances of objects, having nearly the same colour and brightness, being the same; their apparent magnitudes will be as the visual angles. Thus, the apparent distance OA, of two objects AB, AC being the same; their apparent magnitudes will be as the visual angles AOB, AOC. That is, in this case, the two objects being really at the same distance from the eye, their apparent magnitudes are as their real magnitudes. In like manner, if the distances OA, OD are so great as to appear equal, the apparent magnitudes of the two equal objects AC, DE, will be as the visual angles subtended by them, or as AC and AB. But if, as in the preceding, the objects AC, DE are so near, that their apparent distances, are as the real ones OA, OD; then will their apparent magnitudes be as the real ones, or as AC and DE.

Fig. 30.

Greater brightness suggests greater magnitude.

197. *Schol.* The apparent distance of an object being fixed in the mind, the degree of brightness affects a little the idea of apparent magnitude; the greater brightness, for instance, suggesting a greater magnitude. Thus, if two pieces of paper, of the same figure and bigness, one whereof is white and the other black, be stuck to a wainscot of a different colour from either; the white paper will seem to be bigger than the black. As the eye receives more light from an object of a given brightness, the bigger it is; so on the other hand, its receiving more light from a brighter object, seems to be the cause of that object's appearing bigger. The phenomena above, are not very sensible, but when two adjacent objects of opposite colours, are compared together.

Apparent magnitude, in the compound ratio of the visual angle and apparent distance.

198. It is very difficult to ascertain precisely, how far, or in what proportion, apparent distance affects the idea of apparent magnitude: But it seems probable, that the latter is always either exactly or very nearly, as the visual angle multiplied by the apparent distance. If that be the case, and the apparent distance was always as the true distance, then the apparent magnitude would be always as the true magnitude; or

or the ſame object would always appear of the ſame magnitude, at all diſtances; as the caſe really is, within a few yards of us, or ſo far as apparent diſtance is adequate to the real. But objects placed beyond a certain diſtance, generally appear leſs, the more remote they are: For two parallel rows of trees, *&c.* ſeem to converge towards the farther end. But it is certain, that the diminution of apparent magnitudes, is no ways proportionable to that of the viſual angles, or to the increaſe of real diſtance: A pane of glaſs appears nothing nigh ſo big, as the front of a houſe, or ſtreet, ſeen through it: And it is certain alſo, that in moſt caſes the apparent diſtances of remote objects, fall ſo much the ſhorter in proportion of the real diſtances, as the objects are more remote, which ſerves to account for the decreaſe that is obſervable in the apparent magnitudes. So that upon the whole, it ſeems likely, that *apparent magnitudes are either exactly, or very nearly, in the compound ratio of the viſual angles and apparent diſtances.*

199. *Schol.* 1. We are as frequently deceived in our notions of magnitude, as we are in thoſe of diſtance; and theſe deceptions are farther confirmations of the preceding theory. A fly ſkipping before an unattentive ſpectator, will ſometimes excite the idea of a crow flying afar off; but as ſoon as the miſtake in the diſtance is found, the large object or the crow, will dwindle into the little fly; and one may even perceive the diſtance and magnitude, as it were, contracting both at the ſame time. In like manner have I miſtook a crow, for a fly hopping betwixt me and the window.

200. *Schol.* 2. It is obſervable, that in foggy weather, and in the duſk, objects generally appear farther than they commonly do: So, for the reaſons above aſſigned, in theſe caſes they alſo appear proportionably larger; and ſometimes they appear of a very extraordinary ſize, according to the greater miſtake in the diſtance. A ſmall heap of ſtones, has been miſtook for the ruins of a large building, *&c.* And it has been obſerved, that a chamber appears leſs, when its walls are whitened; and that fields and mountains appear leſs, when covered with ſnow: The reaſons of which muſt be, the white walls and fields appearing ſo much nearer. Why objects appear larger in foggy weather.

201. *Schol.* 3. In forming our ideas of magnitudes, it is not enough that we know, either from former experience of our own, or from the teſtimony of perſons on whom we can entirely rely, how far ſuch or ſuch an object is from us: But we form our idea, or calculus, if it may be ſo called, of the apparent diſtance, as it were *extempore*; from ſuch circumſtances as then preſent themſelves to the ſight, and by which we uſually judge of diſtances when we are left to our ſelves. And

hence it is, that we are not able to correct ſome miſtakes, as to apparent magnitudes. An horizontal walk, terminated by two parallel ſides, will not appear ſo narrow at the farther end, as a walk upon a deſcent of the ſame length and breadth: And the parallel ſides of an aſcending walk, will yet appear to converge leſs; and in ſome caſes, it may appear as wide at the farther end, as it does at bottom, and even wider. An inſtance of this latter ſort, I have frequently before me: The parallel ſides of a ſett of ſteps facing my window, and going up to a mount on the Tower wall, ſeem manifeſtly wider at top than they do at bottom. All the above different appearances, ariſe undoubtedly from the miſtaken judgments we make of diſtances; and theſe miſtakes alſo proceed from the reaſons that have been [a] before aſſigned.

[a] 181. II.

Apparent diſtances independent of the viſual angles.

202. *Schol.* 4. The diminution of apparent magnitudes, is ſo very ſmall, in compariſon of the greater increaſe of diſtance; that in the general, it ſeems that the viſual angles ſubtended by objects, have little or no ſhare in directing our judgments of their diſtances. And indeed, if the caſe was otherwiſe, it would be almoſt impoſſible for us ever to gueſs aright, either as to diſtance or magnitude. For inſtance, if we did not judge of diſtance independently of the viſual angle, how could we know a child from a grown perſon; or even a pin from a maypole: For the largeſt object, by being removed to a greater diſtance, may ſubtend a leſs angle than any aſſignable one.

Why objects at unuſual heights appear ſmaller thanordinary.

203. As objects generally [a] appear nigher, when the intervening chain is not perceived; ſo in theſe caſes, we have a reaſon why they ſhould alſo [b] appear proportionably ſmaller: As the caſe is, when we look from below at objects on the top of a ſteeple; or from above, at objects at the bottom. In looking upwards or downwards, the oblique and unuſual view we have of objects, adds ſtill to our deception; as from above, we look on the crowns of hats; tops of houſes, coaches; backs of cattle, *&c.* But after frequent experience in theſe caſes, things do not look ſo ſurpriſingly ſmall, as they did formerly.

[a] 180. II.
[b] 198. II.

A tree upon the ground, appears longer than it did ſtanding; and a ſteeple which is viſible all the way from the bottom upwards to the top, does yet not appear ſo high, as it really is; or not ſo long, as the ſame extent upon level ground. This phenomenon ſeems to be entirely owing to our want of experience, in meaſuring heights; and its being ſuch, accounts why even a globe on the ſpire of a ſteeple, appears much ſmaller, than a globe of the ſame diameter on the ground, and at the ſame diſtance from a ſpectator. When we are deprived of that help, which a view of the whole extent of the ſteeple affords us, as in looking

ing down, &c. it is a reason why objects should appear still smaller, as I believe, all other circumstances being alike, they always do.

204. *Schol.* To prevent mistakes, let it be here again observed, that, according to the [a] *defin.* by *distance* in this §, was every where meant, the extent of an imaginary line joining the eye and the object; and not the distance betwixt two objects, unless it was so declared. Also, by *apparent magnitude of an object*, was meant its apparent diameter, breadth, or distance betwixt two of its opposite sides; and therefore all that has been said upon that head, is directly applicable to the apparent distance between two objects; and that, whether the line joining the two objects be at right angles to the optic axis, or obliquely to it: For the apparent distance betwixt two sides of an object, and betwixt two objects or points in them, are cases that are exactly coincident. [a] 161. II.

205. Our faculty of distinguishing degrees of distances, is also of the last importance to us, in distinguishing things. For an object is not a mere coloured extension, contained within a certain out-line; but a figure, wherein are comprehended, besides their colour and out-lines, the particular figures, *relievo's* or swellings, hollowings, &c. of every visible part, and member of the whole; or a figure, wherein is considered the distances, from an imaginary plane, of every visible point. Thus, a globe placed before me does not appear a circle, but a convex surface, swelling all the way from its outward or visible verge, towards the vertex or point nearest to me. The inside of an empty bowl, is visibly a spherical concave: In a face, the nose is seen projecting before the rest, and the eyes are plainly sunk lower than the forehead, &c. Of the visible or apparent figures of objects.

We have other helps for distinguishing the prominences of small parts, besides those by which we distinguish distances in general; as their degrees of light and shades, and the prospect we have round them: Both these, I think, contribute their share to the visibility of such small objects as the pile upon cloth, &c. And by the parallax on account of the distance betwixt our eyes, we can distinguish besides the front, part of the two sides of a near object, not thicker than the said distance; and this gives a visible *relievo* to such objects, which helps greatly to raise or detach them from the plane, on which they lie: Thus, the nose on a face, is the more remarkably raised, by our seeing each side of it at once. These observations, I say, are of use to us in distinguishing the figures of small and near objects; and when the breaks, prominencies and projections are more considerable, we do not want them. The distances betwixt the legs of a chair, are visible many yards off; and the projection of a building is visible still farther. But as the distance is increased, different degrees of eminencies, cavities, &c.

disappear one after another; so that at length the corners, projections, &c. of buildings become imperceptible; and both square and round towers, at great distances, appear alike dark and flat surfaces, terminated by blunt and obscure lines. It is for the same reason, that the moon appears a flat surface, and not globular; the difference betwixt the distances of the limb and middle part from us, being much too small to be perceived.

Things that we are daily conversant with, we survey in different positions; and from this familiarity, a transient glance only oftentimes suffices, to give us an idea of even the back part which is not seen: And we learn also to make allowances, on account of the different views we have of things. A chair appears a chair, and seemingly much the same object, in various positions. And when we look attentively, we have frequently great helps for discovering the figures of objects, that we are not acquainted with. Suppose a person, who had never seen one before, looks obliquely at a chair: The seat he sees is parallel to the floor, and supported in the front by two legs perpendicular to it; one of the back rails he sees from top to bottom, and that it is joined to one corner of the seat; he likewise sees the whole upper part, and underneath, a little of the foot of the other rail; and by considering the uses and similarity of the visible parts, he will go near to form a good idea of the whole figure. The figure of that part of the cavity of a bowl viewed obliquely, which is not seen, will be suggested by the convexity of the outer part. But there must be always something visible to help us; without this, prænotions will not do: The moon appears alike flat to an astronomer and to a shepherd.

§. 5. *Of the apparent distances, magnitudes, and figures of the cœlestial objects.*

Of the sky.

206. We seem to be inclosed by a kind of a vaulted roof, extending every way round us, and within which all visible objects seem to be included: For all remote objects, detached from the earth, seem to float, or as being placed in this concave surface, which we call the *sky*. We are certain nevertheless, that instead of being confined within a concave shell of a small diameter, as the sky seems to be; the medium round us is transparent, to a distance far greater than even our imagination can reach; for such is the immensity of the distance of the fixed stars. We are certain also, that light is a mediate cause of vision; and therefore, that the above sky-like appearance must be owing to the inflection of light by the air, or by some particles floating in it. For,

For, if light was not inflected by the atmosphere, it is pretty manifest, that the appearance which we call the sky, would vanish; and in its room would succeed a dark void be-sprinkled with stars. And as we are undoubtedly surrounded every way with an atmosphere, so we receive light from every part of it; as indeed both are proved by the phenomenon itself.

207. Every pencil of light that enters the eye, excites the idea of colour, and of a visible point: But the particles of air themselves, are much too small to be seen; and as we undoubtedly receive light, by the reflection of airy particles at innumerable different distances; it remains still to be accounted for, why the sky should appear at a particular distance, or as having a definite magnitude and figure. Its apparent figure and magnitude.

The air being transparent, the sky cannot appear nearer than the remotest object in view; nor do I see any reason why it should appear farther, as nothing occurs here to the sight, whereby we usually distinguish one object to be more remote than another. The sky therefore seems to touch the remotest land-object in view: And light being reflected to us from all parts whatever, and that much in the same proportion as to sort and quantity; seems to be the reason, why the serene sky should be a concave surface, of pretty much the same colour. And different parts of the atmosphere being at different times charged with heterogeneous particles, suited for inflecting different sorts of light, seems to be the reason why we see clouds, or parts in the sky of different colours. But to define the magnitude and figure of the sky, we must have recourse to observations.

The apparent distance of the visible horizon [a] has been already considered: And by comparing the apparent distance between two given stars, or the apparent diameter of the moon, at different altitudes, we may pretty nearly determine the apparent figure of the sky. Or, by guessing at the places, whereabouts perpendiculars dropt from several points in the sky, at different altitudes, would fall, and having the angles of elevations of these points; we may from thence also make an estimate, of both the apparent figure and apparent magnitude of the sky; due allowances being made betwixt the apparent and real distances of the places, where the perpendiculars are supposed to fall. As if a cloud 40 degrees high, be supposed to hang over a place 50 yards off; then supposing the apparent distance of that place to be 40 yards, the apparent distance of the cloud will be about 52 yards; or the hypothenuse of a triangle, whose base is 40 yards, and angle at the base 40 degrees. And thus by finding the apparent distances of several points, we shall have the apparent figure and magnitude of the whole sky. [a] 193. II.

The

The two sorts of observations above, are to be considered as being but gross ones; for people will differ in their notions, both of the apparent magnitude of the moon, and also of the distance of that point, over which she seems suspended: However, we may be certain from them, that the figure of the sky is not, at least is not always a hemisphere; but rather a kind of spheroid; the apparent distance of the horizon being generally much greater, than that of the zenith. And this is very agreeable to the theory of the apparent distances of terrestrial objects.

Variations in the visible magnitude and figure of the sky, from different situations of the spectator.

208. To those who have very extensive prospects, the visible horizon appears somewhat farther, and the surface of the sky appears somewhat flatter, than to those whose prospects are more confined. But these variations are not so considerable, as might be expected from the preceding theory; and that partly, because the apparent distances of objects fall so very short of their real distances. And also, because the sky is an object very familiar to us, and by our moving about, frequently seen under various circumstances, as to extensiveness of land-prospects; we retain much the same idea of its figure and magnitude, so long as its colour and brightness is the same also; even though a wall is before us, beyond which nothing is visible but the sky itself.

Difference of serenity, a cause of difference in the apparent figure and magnitude of the sky.

209. The different brightness of the sky, at different times, is a reason why its apparent figure and magnitude are neither of them immutable, but variable in different weather: A greater brightness, as in other cases, suggesting a nearer distance; as objects seen through a fog, generally seem farther than usual; and nearer, when the air is clear and the sun shines upon them. The atmosphere is generally more loaded with vapours, the lower down; and so the sky commonly appears duller that way, than it does higher up: And this also is another reason, why the sky commonly appears farthest towards the horizon. The atmosphere is also subject to greater changes, the lower down; and this accounts why the apparent distances of the lower parts of the sky, should be subject to greater variations, than those of the parts which are more elevated.

Phenomenon of the horizontal moon.

210. From the preceding, the reason is manifest, why the moon when near the horizon, generally appears bigger than when she is elevated higher up; and also, why sometimes in the same situation, she appears much bigger than ordinary. The sun also frequently appears of an extraordinary bigness, when near the horizon; but the greatness of his splendour, prevents our observing so particularly the sun's magnitude, as we do that of the moon. It is observable likewise, that the distance betwixt two stars, appears different according to their elevations.

By

By obſerving whether the apparent diſtance of two ſtars, in the neighbourhood of the moon, keeps the ſame proportion at different altitudes, with the apparent diameter of the moon at theſe altitudes; it may be determined, whether a greater faintneſs of the moon, abſtractedly conſidered, or without regarding the ſky, may of itſelf in ſome meaſure excite the idea of a greater magnitude. But as the moon in an eclipſe, appears no bigger than ſhe ordinarily does at the ſame altitude; it ſeems very probable, that the variations in her apparent magnitude, at different elevations, are moſtly, if not wholly, to be imputed to the apparent figure of the ſky.

How the aſtronomers expreſs the apparent magnitudes of the planets.

211. *Schol.* As the apparent magnitudes of remote objects, will be different, according to the judgment every one forms of their apparent diſtances; (thus, one deſcribes the moon as appearing about the bigneſs of the crown of his hat; another thinks ſhe looks bigger, and ſo on; but none think her ſo ſmall as a pea, though a pea at a ſmall diſtance will ſubtend a larger angle, than the moon:) So, to prevent miſtakes, aſtronomers expreſs the apparent magnitudes of the planets, by their viſual angles, as they find them by actual meaſuring with inſtruments; and not by comparing them to common meaſures, as inches, feet, *&c.* Thus, the moon's diameter is ſaid to be about half a degree; becauſe thereabouts is the viſual angle ſubtended by the moon, at the diſtance we are from her. The apparent diameter of the ſun, is about the ſame bigneſs with that of the moon; although in reality, the ſun's diameter is much bigger than the diſtance between the earth and moon. The angles ſubtended by the fixed ſtars are altogether imperceptible, or too ſmall to be diſtinguiſhed; although theſe bodies may, very likely, be each as big as the ſun; ſo the ſtars are ſaid to have no ſenſible diameters.

Why the ſtars appear of different magnitudes.

212. The ſtars appearing of different magnitudes, is probably owing entirely to their different ſplendor; thoſe ſtars appearing bigger, as they excel in brightneſs. And indeed, as the fixed ſtars have no definite figures, or ſenſible diameters; what is commonly expreſſed by degrees of magnitudes, can be properly conſidered as nothing elſe, than degrees of brightneſs. With reſpect to the naked eye, all the planets, excepting the ſun and moon, may be alſo claſſed among the fixed ſtars; the appearances of them all being much alike.

Phenom. of new moon. Fig. 31.

213. The enlightned part of the moon's diſk, a few days from the change, ſeems to be a ſegment of a larger circle, than the remaining obſcure part, which is then alſo viſible; ſo that the *anſæ*, or horns, project ſenſibly farther from the center, than the limb of the ſaid obſcure part, as repreſented in the annexed figure. This phenomenon, I think, can be no ways eaſier or more naturally accounted for, than by

by ſuppoſing the exceſs of apparent magnitude of the enlightned part, to be owing to its greater brightneſs. For, here the apparent, as well as the real diſtance of both parts being the ſame, both appearing contiguous and in the ſky; the greater quantity of light received from the one part, excites the idea of a greater magnitude. I have looked at the new moon through concave glaſſes of different figures; * but could not perceive any ſenſible difference, as to the proportions of the diameters of the two afore-mentioned parts. And indeed it is difficult to conceive, how the aberrations of the rays coming from the ſame object, could cauſe ſo remarkable a difference in the apparent magnitudes of two contiguous parts.

Radiations of the ſtars.

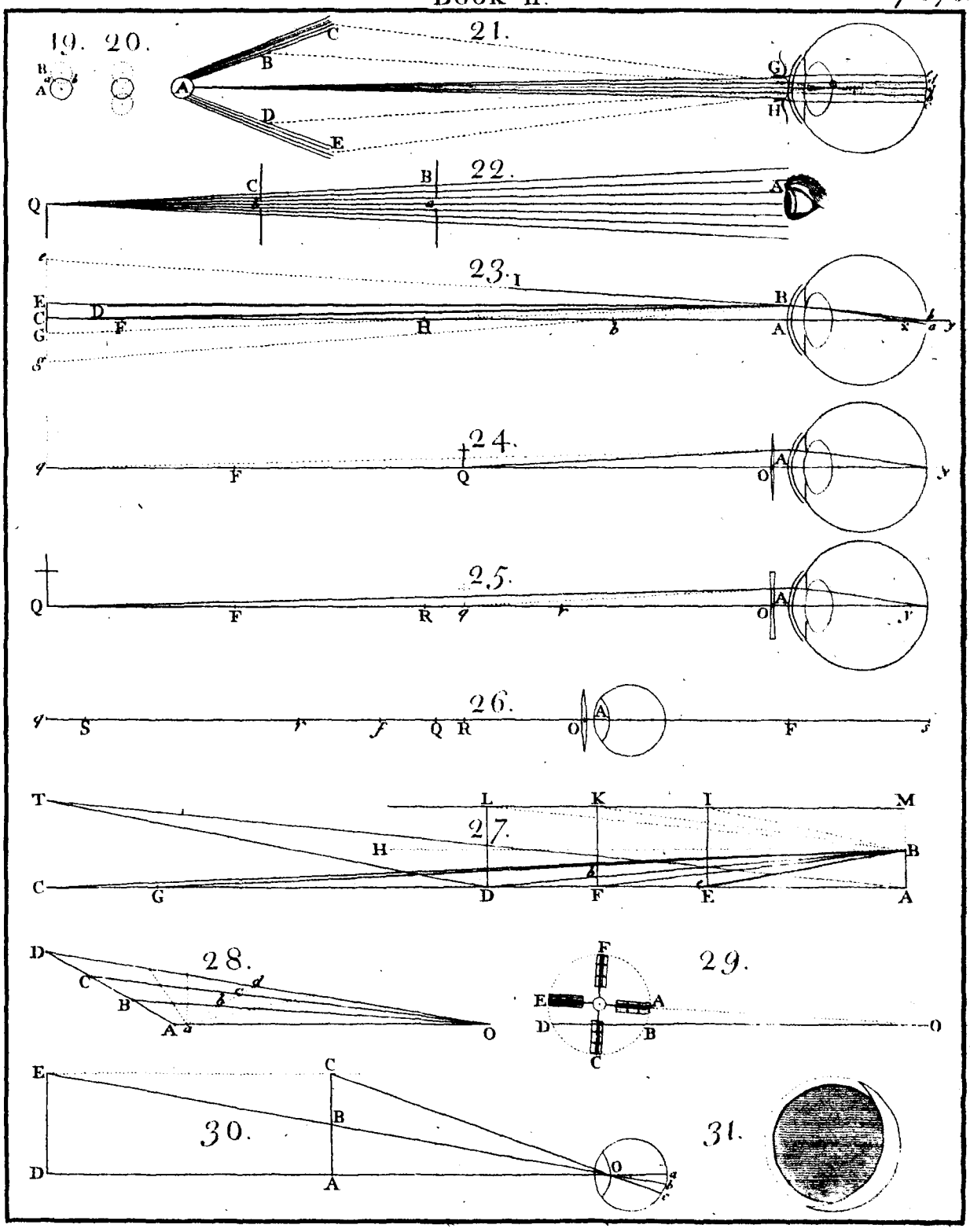
214. The apparent figures of the ſtars, is a phenomenon that preſſes hard upon the hypotheſis of aberrations and falſe images of remote objects: For by that hypotheſis, the ſtars and planets, and all bright objects ſubtending inſenſible angles, ought to have definite and circular figures, gradually decreaſing in luſtre from the middle towards the circumference. But the ſtars, and alſo lamps or candles, ſeen at a diſtance in the night, inſtead of appearing whitiſh circles, ſeem only indefinite glares of light, ſhooting out, as it were ſpontaneouſly, all round as from a center into an indefinite number of ſlender and ſharp-pointed ſpokes, or *brachia*, of different lengths. The ſtar-like appearances of ſmall ſtones, bits of glaſs, *&c.* ſeen at ſome diſtance in the ſun-ſhine, as alſo the radiations from the edges of very bright objects, having ſenſible magnitudes, muſt all probably be owing to a like cauſe.

The hypotheſis for explaining the phenomenon of two beams ſhooting from a candle, will not do here; becauſe the beams are obſerved to ſhoot round every way alike: Nor can theſe phenomena be ſolved directly, by any of the foregoing principles. But it ſeems probable, that they are owing to ſomething in the conſtitution of the eye itſelf; and perhaps one of the following may be the cauſe of them.

If it be ſuppoſed, that an impreſſion made upon any point of the retina, is, by ſome vibrations of the fibres in that part, communicated and as it were ſpread further, like the undulation of water when a ſtone is thrown into it; but that, inſtead of ſpreading itſelf over one continued circular ſurface, it branches out into a ſtar-like image: And the light of a ſtar impinging but upon a ſmall point in the retina, and there being no impreſſions made round that point, at the ſame time, at leaſt but ſuch as are very faint and languid, compared to the impulſe of the ſtar's light; the undulating motion above ſuppoſed, has time to ſpread itſelf undiſturbed, and ſo to excite the ſame idea, as would be cauſed by the direct impulſe of light upon thoſe parts where the motion reaches.

* See 128. II,

Or,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Or, if it be supposed, that some of the light proceeding from a visible point into the pupil, instead of being collected with the rest into a point on the retina, is, by an hetereogeneity in the humours of the eye, irregularly scattered round the center into a star-like image. Either of these hypotheses, will account for the above appearances of small bright objects; and also shew why bright objects having sensible magnitudes, seem to branch out from the limb into pyramidal *spicula*, whilst the whole space within is all over bright: But they are hypotheses only, and here offered as such.

215. *Schol.* 1. The reason, why none but bright objects have these branches shooting from them, seems to be, that the sky-light, which intrudes into the eye at the same time, overpowers and obliterates those feeble impressions which are made by the few rays scattered from the true image: So that these impressions are never sensible, but when the rest of the retina is very dark, in comparison of the image of the radiating object. After the same manner may be accounted, why the stars do not sparkle or radiate so much in the twilight or moon-shiny night, as they do when the sky is less luminous; and as they sparkle less, they appear less, being more like points: Also, why a star near the moon, appears less than ordinary. The flame of a candle also from a remote distance, will have less apparent magnitude, as the medium between is more illuminated. Why the stars appear less by twilight.

216. *Schol.* 2. When the flame of a candle is at such a distance, as to be a *minimum visibile*, its pyramidical figure cannot then be discerned, and the candle has only a star-like appearance, for the reasons already assigned. It is for the same cause, that *Venus* never appears horned to the naked eye: For her whole diameter not subtending a sensible angle, it is impossible to distinguish her moon-like figure; and the whole disk being as a point, Venus and all the rest of the planets excepting the moon, have star-like appearances. Why Venus never appears horned like the ☽.

217. Those who are accustomed to look through long telescopes, find that the air is generally affected with a kind of tremor or undulating motion, which is in different degrees in different weather. This tremor of the atmosphere, some have thought to be the cause of the *scintillation* or *twinkling* of the fixed stars *. The stars do not appear as Twinkling of the stars.

* Another opinion concerning the scintillation of the stars, as it has been often printed, should not be here passed unobserved: *viz.* "The stars being but points, their twinkling is "said to be caused by small opaque particles "spontaneously floating in the atmosphere." But although the stars are points, yet since they propagate light all manner of ways; it is pretty manifest, that an opaque body of a less breadth than that of the pupil, cannot totally hide one of them, in whatever part of the atmosphere such a body be supposed to float. And particles of that size in the air, are much too large to be admitted.

as if they were ſhaken out of their places, but only as if they were either nimbly covered and uncovered; or as if their light was impaired and renewed, by fits; and to an attentive eye, the brighter ſtars either do not twinkle at all, or very ſeldom in compariſon of the leſſer ones. The tremor of the air therefore, if it has any effect, does not ſeem of itſelf ſufficient to account for the phenomena: It is very likely that ſomething which paſſes in the eye itſelf, may here have the principal ſhare. We are continually ſhifting and cloſing our eyes, even when we are not conſcious of ſo doing; and a ſtar being but a point, the ſmalleſt motion of the eye is ſufficient to make us loſe it, or very ſmall opaque particles floating in the vitreous humour pretty near the retina, are capable of preventing the ſtars light from proceeding any farther. Perhaps, without any ſtoppage of the rays, the extream feebleneſs of their impreſſions, are not capable of keeping the viſive faculty conſtantly awake, but only as it were by fits. Theſe are only conjectures.

§. 6. *Of Viſion by images.*

The particular phenomena of viſion, in given caſes, by reflected and refracted light, are the chief ſubjects of the two enſuing ſections; but becauſe of their univerſality, it will not be amiſs to cloſe this with the following few obſervations.

218. Viſion of real objects ſeen directly, or naked viſion, and viſion by images, are both founded upon the ſame principles: That is, the like impreſſions, or the ſame kinds of images upon the retina, excite like ideas in both caſes. Whence,

219. Objects ſeen by reflected or refracted rays, are ſeen in the places of their laſt images, if theſe images are at moderate diſtances * before the eye; the ſeveral circumſtances by which we form the ideas of the apparent diſtances of objects ſeen by naked viſion, being here alſo taken into the account. And univerſally, every viſible point of any object appear ſomewhere in the directions of the axes of the pencils of rays proceeding from them to the eye, after their laſt reflection or refraction.

220. *Schol.* In viſion by images, we are generally deprived of many circumſtances by which we uſually judge of diſtances; and this makes it

* In naked viſion, the pencils of light always enter the pupil of the eye diverging, though in many caſes their divergencies are ſo ſmall, that they may be reckoned parallel: But in viſion by reflected or refracted light, the rays ſometimes fall on the eye converging, and yet the object, in ſome caſes, is ſeen tollerably diſtinct; but here we regard only ſuch caſes as are analogous to thoſe we meet with in naked viſion.

it difficult in most cases, to determine the place of an image, if it be farther off than two or three yards [a], though we see it with both eyes; [a] 171. II. and with one eye alone, we cannot determine distance [b] beyond a foot [b] 167. II. or two. These difficulties are again frequently increased, by some peculiarities appertaining to the images, which we are not accustomed to observe; and for which therefore we are at a loss how to make the proper allowances. But when the distance of the image is within the above limits, we can determine its place, in most cases, pretty accurately; and, as in naked vision, the nearer we can determine the place from whence the rays diverge to the eye, the more distinct will the image appear.

221. The apparent magnitudes of objects seen by reflection or refraction, are, either accurately or very nearly [c], *as the rectangles under the visual angles and apparent distances, of their last images:* Also [c] 198 and 218. II. in all cases, the apparent place, position and figure of an object seen by reflected or refracted light, are as those of its last image: For the rays * diverging to the eye, when the image is before it, from a succession of physical points, after the same manner as if they came directly from a real object [d], excite an idea of that image, or of an object equal [d] 218. II. and similar to it.

222. *Corol.* 1. A given speculum or lens magnifies the most or least, or diminishes the least or most; when the rectangle under the visual angle and apparent distance of the image, is the greatest or least possible.

223. *Corol.* 2. In vision by images, we are liable to many deceptions, some of which are entertaining as well as surprizing. For not only the place of an image, but also very often its position, magnitude and even figure, some one or all of these, shall be quite different from the original object.

224. If in the place of an image, formed by rays converging either by reflection or refraction, be substituted a white plane, as a sheet of paper; upon this white surface there will be formed a picture, exactly equal and similar to the image †. And if the room be dark, so that the eye may be pretty free from extraneous light, this picture will be visible to a spectator placed in almost any situation before it: For here the rays are reflected from every point of the white body all manner of ways, after the same manner as they are reflected at the surface of common objects. But if the white obstacle that stopped them be taken away, the rays will proceed directly onwards; and therefore the image cannot be seen unless the eye be placed directly behind it: And even then the visible field will, in most cases, be much narrower than

* The cases of vision by parallel or converging rays, are not considered here.

† See 244. I.

than the picture; as the rays proceeding from parts of the image that are remote from the middle, do necessarily escape wide of the eye.

SECTION III.

Of vision by light reflected from plane and spherical speculums.

THE abstract theories of images, formed both by the reflection and refraction of plane and spherical surfaces, having been already delivered in the first book; in this and the next section, what we have chiefly to consider are the phenomena themselves, of vision by such images.

§. 1. *Of vision by* mirrors *or plane speculums.*

225. *Phenom.* Objects seen by the reflection of plane speculums, generally appear so perfect and natural, that if the speculum itself is not perceived, one is liable to mistake the images for the real objects of which they are types. A man appears alive, corporeal, and not a mere surface; and any series of objects placed before the speculum, exhibit alike series on the other side, all appearing in their due places, agreeable to the [a] theory of images, and that of vision in like cases. The principal difference observable here is, that the images appear somewhat darker than the objects, on account of the loss of light by reflection; and this, when the images are remote, may [b] affect their apparent distances; and more especially if, as the case often is, part of the floor sustaining the real objects [c] is not seen, or if the speculum [d] is not vertical. But when the images are near, we find, as in vision of real objects, that the above causes have no sensible effect.

[a] 37 to 49. I.
[b] 188. II.
[c] 180. II.
[d] 181. II.

Fig. 32. Let PQR be an object, whose image formed by the plane speculum ST, is *pqr*. Now an eye placed before the speculum ST, sees every point of the object PQR, by the very same rays that originally diverged from those points, without having undergone any other change, excepting that they are bent at the speculum, as if they came directly from the corresponding points in the image *pqr*; and therefore it is, that the said image appears equal, similar and of the same colour with the real object. Thus, an eye placed at A, B, C, D or E; will accordingly

cordingly see the radiating points P, R, by the pencils of rays whose axes are S*a*, 5*b*; 1*a*, 6*b*; 2*a*, 7*b*; 3*a*, 8*b*; or 4*a*, T*b*, proceeding directly as from the respective points *p*, *r*. The rays that enter the pupil suppose at C, from any point P, have the very same divergencies as if they came from a distance[a] $= P2 + 2C^2 = Cp$; and so the eye[a] 43. I. receiving the same impressions, as it would if the object P was placed in *p*, we form an idea accordingly.

226. *Schol.* If in any speculum a person sees his own image, or that of an object that is behind him: If the image appears erect, it will appear inverted as to right and left; and the reason is, because the object and its image face each other, or look contrary ways. The case is not unlike when the object is between us and the speculum, though we are apt to make it different, for want of considering that with respect to the image seen in the speculum, it is the back of the object that is now towards us; and so we now call that the right side of the object, which we would call the left if we were on the other side of it. Thus, if a person sees the image *pr* of PR; the image *p* appears to the right wherever he stands; if he is behind the object, he will reckon the point P also to the right; but if he is between the object and the speculum, he must turn his back to the speculum to see PR, and then P is to his left. If PR is a printed page, the letters in the speculum will be equally inverted as to right and left, whether the spectator be before or behind the paper. The phenomena are contrarily, when a person sees an inverted image, or one that is turned upside down; that is, if the object is behind him, the image will seem in its due situation as to right and left; but in that respect it will seem also inverted, if the object is between the spectator and the speculum. But we seldom attend to the positions of images as to right and left, excepting in particular cases. Apparent position of images seen in speculums. Fig. 32.

The phenomena of vision in plane speculums agree, in most if not in all cases, with the theories of the images; and what remains to be added under this head, is to explain the phenomena when two or more speculums are combined together, or when one is placed in an inclined position.

PROP. XII.

227. *If two reflecting planes* BC, DC, *be placed at right angles to one another: A body* A *moving along the line* AB *perpendicular to the one speculum, and consequently parallel to the other, will have its images* a, b, *move the contrary ways; that is, while the body* A *is in motion, the* Fig. 33.

the image b *goes onwards the same way, whilst* a *moves directly towards* A; *and the motion of* A, b *and* a, *are all exactly at the same rate.*

It is manifest that the lines *a*B, *bc*, described by the images *a*, *b*, are the images of the line AB in the respective speculums BC, DC; therefore *a*B = [a] AB, and *ab* is [a] perpendicular to BC; also *b*D = [a] AD, and *bc* is parallel to DC. Therefore &c. *Q. E. D.*

[a] 40. I.

Fig. 34. 228. *Schol.* If the line AB be inclined to the speculums BC, CD; the lines *a*B, *bc* described by the images *a*, *b*, may be found by drawing the perpendicular A*a* to BC, and the perpendiculars A*b*, *bc* to DC; then taking *da* = *d*A, D*b* = DA, and C*c* = C*b*: Or, the lines *a*B, *bc*, may be found by making the ∠ *a*B*d* = ∠ AB*d*, and the ∠ *be*D = ∠ A*e*D.

PROP. XIII.

Fig. 35. 229. *If a plane speculum,* RS, *be inclined to the horizon in an angle of* 45 *deg. with its face downwards: An upright object* CD *will have its image* cd *in an horizontal position; and the image* ab, *of an object* AB *lying horizontally, will be vertical.*

For each object and its image forming [a] equal angles with the speculum, and those which the objects form being each [b] half a right angle; the ∠ [a] CR*c*, BS*b* are right angles. Therefore, &c. *Q. E. D.*

[a] 40. I.
[b] Hypoth.

230. *Corol.* If RS be a plane speculum, with its face upwards, AB will be the image of an object *ab*; and the image of an object *cd*, will be CD. And so a person standing upright before such a speculum, will see his image extended horizontally, as it were on the floor, with his face upwards.

231. *Schol.* 1. A person looking in a plane speculum inclined to the horizon in an angle of 45 *deg.* will see himself, as it were, in a flying posture: For he will seem suspended horizontally in the air, with his face downwards. And if the speculum is sufficiently long, he will seem to fly downwards or upwards, as he walks towards or from the speculum.

Fig. 35. 232. *Schol.* 2. If the speculum SR be inclined to the horizon in any other angle than 45 deg. the images *cd*, *ba*, of the vertical and horizontal objects CD, BA, will be accordingly inclined from the position they have now in the figure. And particularly, the objects CD, AB retaining their vertical and horizontal position; if the ∠ RSB be greater than 45 deg. the image *cd* will recline upwards, the point *d* being higher than *c*; and the image *ab* will recline backwards from the speculum: And the contrary will happen, when the ∠ RSB is less than 45 deg.

233. *Schol.* 3. Upon a flat and smooth board FI, let there be made a grove or channel *a b c d e*, by means of thin ledges A, B, C, D, E glewed to the board FI; and let the edges of the boards A, B, C, &c. that are next the channel, form the same angles with the sides of the rectangular board FI *. The board being thus prepared, and ledges fixed to the sides of it, and another at *a e*; if the end FG be a little elevated, a marble or ivory ball placed at *a*, will roll with the same velocity, along the channel, till it gets to the bottom at *e*. Now if the board be placed at the bottom of a box, in which is a looking-glass so inclined that the board FI appears vertically, the end HI appearing uppermost; the ball will seem to roll upwards, till it gets to the top at *e*. Fig. 36.

A ball seeming to roll upwards.

This little deceipt is surprizing enough till the cause is known, and to prevent its being immediately detected, the box should be placed a little above the eye, and the aperture in the front, and the size of the glass should be so limited, that, from a certain station, nothing shall be visible but the image of the board FI. In the side *a e* of the box, and below the end *e* of the channel, may be a narrow board, whose end *e* may be lifted up for throwing the ball back again to *a*; and then it will begin its journey as before. The end *e* may be lifted up by a string guided by pullies to any particular place, and which might be concealed from the spectators various ways. Note, there should be a provision made to prevent the ball from getting under the lifting board *a e*.

If the side FH of the board be placed to the front, the ball will seem to go almost directly up and down. Or, if HI be to the front, and made the uppermost side, it will be surprizing enough to see the ball roll downwards from *e*, so very gradually. The channel *a b c d e* may be made in a serpentine form; but then the board must have a greater declivity, and the motion of the ball will not be so uniform.

PROP. XIV.

234. *In any number of plane speculums, all lying in the same plane, there can be seen from the same place, only one image of the same object.*

For in this case, there can be but [a] one image; and the object is seen by rays proceeding directly to the eye, as [b] from the image. Therefore &c. *Q. E. D.*

[a] 37. I.
[b] 41. I. and 219. II.

235.

* The breadth of the channel *a b c d e*, may be less than half the diameter of the ball; and the ball will run upon a less declivity, if the edges of the boards against which it bears, be champered or sloped considerably.

235. *Schol.* If two ſpeculums be inclined backwards, ſo that perpendiculars to each, meet ſomewhere beyond them; it is manifeſt that rays diverging from any object cannot be reflected by both ſpeculums to an eye having any given poſition; and therefore they can exhibit only one image of the ſame object. See *fig.* 34. where *a* may be ſuppoſed the object, *e* the eye, BC, CD the two ſpeculums.

PROP. XV.

Fig. 37. 236. *If two plane ſpeculums* AB, AC, *be joined together ſo as to form any* ∠ BAC; *there will be a certain diſtance* AF, *within which a ſpectator will ſee his image in each ſpeculum, at the ſame time: And if to theſe be joined two other ſpeculums* BD, CE, *forming* ∠[a] ABD, ACE; *a ſpectator placed any where within a certain diſtance* HG, *will ſee his image in each of the ſeveral ſpeculums. And univerſally, if ever ſo many plane ſpeculums are joined together in an arch of a circle, not too wide to be comprehended at one view; a ſmall object placed near the center of that circle, will be multiplied as many times as there are reflecting planes.*

Upon AB, AC, raiſe the perpendiculars BF, CF; now an object placed any where between F and A, will have its images in lines[a] parallel to FB, FC, till it arrives at A where both images will be united. Again upon BD and CE, raiſe the perpendiculars DG, BH, and EG, CH; and by the ſame way of reaſoning as before, a ſpectator placed any where between G and H, will ſee his image or part of it, alſo in BD and CE. And univerſally, if O be the center of an arch ED, any number of ſpeculums joined together within this arch, will each of them exhibit the image of an eye placed at or near O. Therefore &c. *Q. E. D.*

[a] 37. I.

237. *Schol.* As a ſpectator walks from F towards A, his images in the two ſpeculums AB, AC continually approach nearer together; and at a certain ſtation he will ſee two heads and but one body.

If the place of an object and of the eye be given, the phenomena may be eaſily[b] deduced: As if H be the place of an object, its image will be viſible in the ſpeculums AB, AC, at all diſtances; becauſe the angle of incidence HBF or HCF, is not leſs than the angle HFB or HFC. And in the ſpeculums BD, CE the image will be alſo viſible to a conſiderable diſtance beyond H; becauſe the angle of incidence HDG is not much leſs than the angle HGD. For, it is manifeſt that when the angle of incidence, is equal to the angle formed by the perpendicular to the reflecting plane at the point of incidence and the line AO, the reflected ray is[c] parallel to AO.

[b] 40 & 44. I.

[c] 11. I. & 27. I. *Euclid.*

PROP.

PROP. XVI.

238. *The images of any sector* SRT, *contained between two reflecting planes* RS, RT, *inclined to one another in the given angle* SRT, *will be all concentric; and the number of these images will be such, as will exactly compleat the whole circle.* Fig. 38.

The images of the sector SRT, formed by the speculums RS, RT, are [a] respectively the sectors SRG, TRM: But it is manifest that the [a] 40. I. rays forming the image SRG fall on the speculum RT, after the same manner as if SRG was a real object; and so the said rays [a] form a second image or sector MRL. In like manner the rays forming the image TRM, do, by the reflection of RS, form the image GRH; and the images MRL, GRH, form the sectors HRI, LRK, by the reflections of RS, RT respectively: Again, by the reflection of the rays forming either of the two last images, at the respective speculums, the sector IRK will be exhibited; and so on till the whole circle be compleated. But it is manifest, that none of the rays forming the image IRK, or which exhibit the space IRK contained between the backs of the reflecting planes SR, TR produced, can be again reflected or form another image. Therefore &c. *Q. E. D.*

239. *Schol.* 1. When the sector SRT is not an aliquot part of a circle, the last image, as IRK, will be less than SRT, if the two last perfect images HRI, LRK do not incroach one upon another; but when they do so incroach, they will both frequently appear only as one image, greater than SRT; but the said image can never be twice as great as SRT. For it is manifest, that each speculum forms the same number of images of the sector SRT; and therefore the remaining sector IRK, may be of any magnitude not so great as the double of the sector SRT. Or, the two last images may perfectly join, so as to leave no space between them.

240. *Schol.* 2. The images A, B, C, D, *a*, *b*, *c*, *d* of an object Q, placed between two speculums RS, RT inclined to one another in any angle SRT, will be all in the periphery of the circle whose center is R, and radius = RQ: There will be one image, and no more, in each image of the sector SRT; and the arches contained between the images will be alternately = 2QS and = 2QT. This is sufficiently manifest from the *Prop.* but it will not be amiss to illustrate it farther. Upon the reflecting planes RS, RT, let fall alternately the perpendiculars Q*m*A, Q*na*; A*o*B, *apb*; B*q*C, *brc*; C*s*D, and *ctd*: Then take *m*A = *m*Q, *na* = *n*Q; *o*B = *o*A, *pb* = *pa*; *q*C = *q*B, *rc* = *rb*; *s*D = *s*C, and *td* = *tc*. Now A, B, C, D; *a*, *b*, *c*, *d*, are the images formed

 by

by rays diverging from Q upon the ſpeculums RS, RT reſpectively, after one, two, three, four reflections. For A, *a* are[a] firſt images of the object Q; and the rays proceeding from theſe images, fall upon the adverſe ſpeculums ſo as to form the ſecond images B, *b*; for the ſame reaſon theſe again form the third images C, *c*; and laſtly are formed the images D, *d*. But in this example, becauſe the ∠ SRT is an aliquot and alſo an even part of a circle, the two images D, *d* coincide, and make but one image. Hence it is ſufficiently manifeſt, that 1. All the images are in the periphery of the circle whoſe center is R, and radius is RQ; 2. there is one image and no more, in every image of the ſpace SRT; 3. the arch A*b* = B*c* = 2QT; *b*C = *a*B = 2QS; C*d* = Q*a* = 2QT; and D*c* = QA = 2QS. *Q. E. D.*

37. I.

241. *Corol.* When the ſector SRT is $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{10}$, $\frac{1}{20}$, &c. of a circle; the number of images of the ſaid ſector, and of an object Q placed between the two reflecting planes will be accordingly 2, 3, 4, 5, 9, 19, &c. But when the angle SRT becomes pretty acute, one or more of the laſt images will become inviſible.

Fig. 39.

242. *Schol.* 3. If two plane ſpeculums SR, TR, be placed parallel to one another; there will be a ſeries of images of the floor or ſpace between them, indefinitely extended both ways; there being no other limitation of their number, than what is cauſed on account of the decreaſe of light, by the continual reflections: Alſo an object Q placed between the ſaid ſpeculums will have a like ſeries of images, all ſituated in the indefinite line BQ*b*, perpendicular to the reflecting planes; and the diſtances between the ſaid images will be = 2QT and = 2QS, alternately, all which is evident from the preceding *Schol.* A perſon ſtanding between two ſuch ſpeculums, will ſee in that he fronts, the images of both his fore and back part, repeated ſeveral times in a long row; but continually fainter the farther off.

PROP. XVII.

Fig. 40.

243. *Having the angle of inclination* SRT *of two plane ſpeculums* RS, RT, *and the ſituations of an object* Q, *and of an eye* O, *placed between them; to trace the whole progreſs of any ray diverging from* Q, *which after any number of reflections comes to the point* O.

To prevent the confuſion of too many lines, let us trace only thoſe rays which diverge from Q upon RS.

1. By drawing the perpendiculars QA, AB, BC, CD, upon the reſpective ſpeculums, and by taking *m*A = *m*Q, *o*B = *o*A, *q*C = *q*B, and *s*D = *s*C; we[a] have the places of the images A, B, C, D.

[a] 240. II.

2. Draw

2. Draw a line from O to any given image (A, B, C, or D, &c.); and from the point where this line intersects the speculum, draw a line to the next preceding image; and from the point where this line meets the speculum, draw a line to the next preceding image, and so on, till you get to the object itself Q. This ray traced backwards from Q, will be that which exhibits the given image to the eye at O.

Thus, to find the ray that exhibits the image D; draw the line O*k*D, and from the point of intersection *k*, to the next preceding image C, drawn the line *ki*C; then draw the lines *ih*B, *hg*A, and lastly *g*Q. I say Q*g*, *gh*, *hi*, *ik*, *k*O, is the ray that exhibits the image D to the eye at O.

For the ∠ O*k*T = [b] R*k*D = [c] R*k*C; and therefore the ray *ik* is reflected from *k* [d] to O. Again the ∠ *ki*S = [b] C*i*K = [c] K*i*B; and the ∠ *ih*R = [b] B*h*T = [c] T*h*A; also the ∠ *hg*R = [b] A*g*S = [c] S*g*Q. Therefore the ray diverging from Q to *g*, is reflected from *g* to *h*, from *h* to *i*, from *i* to *k*, and from *k* to O. *Q. E. D.*

[b] 15. I. *Eucl.*
[c] Const. and 4. I. *Eucl.*
[d] 11. I.

In like manner the image C, is seen by the ray Q*d*, *de*, *ef*, *f*O; the image B by the ray Q*b*, *bc*, *c*O; and the image A, by the ray Q*a*, *a*O.

After the same manner, we may trace the rays diverging from Q upon the speculum RT, till they reach the eye at O.

244. *Schol.* 1. It is worth observing, that the pencil of rays which exhibits any image to the eye, is reflected orderly as from each of the preceding images; beginning with the first image, and from no other parts of the speculums: And so the whole number of reflections, is always equal to the whole number of images; as the images A, B, C, or D, are seen respectively by rays having undergone 1, 2, 3, or 4 reflections.

245. *Schol.* 2. The distance OD, of any image D, from the eye at O, is equal to the whole ray proceeding from Q, till after all its reflections it arrives at O; that is OD = Q*g* + *gh* + *hi* + *ik* + *k*O.

It is manifest that Q*g* + *gh* = *h*A = *h*B, and *ih* + *h*B = *i*C; also *i*C + *ik* = *k*D. Therefore &c. *Q. E. D.*

In like manner OC = Q*d* + *de* + *ef* + *f*O; OB = Q*b* + *bc* + *c*O; and OA = Q*a* + *a*O.

246. *Schol.* 3. The last image, as D, of a small object Q, will be invisible, whenever the eye O is so situated that the strait line OD passes through the center R; or so as not to touch either of the speculums when they do not meet in R; and so in that case the next preceding image C, is to be reckoned the last image. Wherefore in such cases, if the arch ST be $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{10}$, or $\frac{1}{20}$, &c. of a circle; the number of

visible images of a small object Q, will be accordingly 1, 2, 3, 4, 8, or 18, &c.

Fig. 39. 247. *Schol.* 4. In like manner as in *Art.* 243, may be traced the rays exhibiting any image of an object Q to an eye at O, placed between two parallel speculums SR, TR. Thus the image A, is seen by the rays Q*r*, *r*O; the image B, by the rays Q*s*, *st*, *t*O; the image *a*, by Q*v*, *v*O; and *b*, by the rays Q*x*, *xy*, *y*O. But the images on one side, will be visible to a much greater distance, if the eye be placed behind one of the speculums, looking through a hole in it, or over its edge, at the other speculum.

Fig. 41. 248. *Schol.* 5. By placing two plane speculums AB, CD, at the end of a box parallel to one another; and looking over or by the edge of one of them, the images of the bottom of the box will [a] appear continued to a considerable distance. Or if two speculums AC, BD be placed upright in a box, but inclined to one another; the trapesium CB [b] would be repeated on each side, till the images met; the images of the lines AB, CD being the chords of two concentric circles, whose center is the point where AC, BD produced, would meet.

[a] 242. II.

[b] 238. II.

A shew-box by plane speculums.

By joining the above four speculums together, as is expressed in the figure, the bottom of the box will be multiplied surprizingly; there being here no other limitation to the number of images, but what is owing to their continual loss of light. The top of the box may be covered with thin canvas, which will let sufficient light within, where may be placed some toy, as two people playing cards, or soldiers standing century, &c. If these be set in motion, by wiers coming from their feet through the bottom or side of the box; it will afford a spectacle still more entertaining. Or, three plane speculums joined together, will have a like effect: For the polygon formed by two of the reflecting planes, will be repeated by the other, &c. The bottom of the box may be covered with artificial moss, shining pebles, flowers, trees, &c. but whatever they are, the upright figures between the speculums, should be slender and not too many in number; otherwise they will too much obstruct the reflected rays from coming to the eye.

Fig. 42. 249. *Schol.* 6. If the sides of a little room, whose floor or base is a polygon as expressed in the figure, be all lined or covered with looking-glasses; the room being enlightened either through the top, or by a lustre of candles suspended in the middle; objects in the room will be surprizingly multiplied, and that for the same reasons as were given in the preceding. The shows lately exhibited, called *luminous amphitheatres* are of this sort. The room should not be very large; for

for the ſmaller it is, the oftner will the images be repeated before they grow too faint to be perceived.

The preceding propoſitions are, I think, ſufficient for explaining any phenomena, that may ariſe from the combinations of plane ſpeculums; and ſome of theſe phenomena are ſo very entertaining, that I believe thoſe who have a reliſh for theſe ſorts of amuſements, will not think I have dwelt too long upon them. But before we cloſe this head, it will not be amiſs to explain the theory of looking-glaſſes, conſidered as they are mediums terminated by a refracting and a reflecting ſide.

PROP. XVIII.

250. *Having the focus of incidence, of rays diverging upon a plane looking-glaſs, whoſe thickneſs is given; to find the focus of the emergent rays, after their ſecond refraction at the fore ſide of the glaſs.*

Let CF be a plane looking-glaſs whoſe back, reflecting or quick-ſilvered ſide is EF, and thickneſs is AB; and let Q be the focus of incident rays. Having through the point Q, drawn the indefinite line NP perpendicular to the ſides of the glaſs; in this line take the point G ſuch that AQ : AG :: n : * m; [a] then will G be the focus of the rays after their refraction at the firſt ſurface of the glaſs CD. 2. Take BH = BG, and H [b] will be the focus of the rays after their reflection at the ſurface EF. 3. Find the point R, by ſaying m : n :: AH : AR; and [a] R will be the focus required. That is, an incident ray, as Qa will be firſt refracted into ab, ſo as to belong to the focus G; then at b it will be reflected into bc, proceeding from the focus H; and laſtly at c, it will be refracted into ck, ſo as to belong to the focus R. *Q. E. I.* Fig. 43. [a] 50. I. [b] 37. I.

251. *Schol.* 1. Let Ar = AQ; that is, let r be the focus of rays reflected by the ſurface CD, before they enter the glaſs: And it will be Rr = $\frac{2n}{m}$AB; that is, in a looking-glaſs, Rr = $\frac{4}{3}$ its thickneſs.

For AH = [c] AG + 2AB; Ar (= AQ) = $\frac{n}{m}$AG; and AR (= [c] $\frac{n}{m}$AH) = $\frac{n}{m}$AG + $\frac{2n}{m}$AB; whence Rr = $\frac{2n}{m}$AB = $\frac{4}{3}$AB. *Q. E. D.* [c] Conſt.

252. *Schol.* 2. Some of the rays diverging upon the looking-glaſs ED, may be ſeveral times reflected betwixt its ſurfaces, before they totally emerge; whilſt ſome others of the pencil emerge each time at the ſurface CD. The emergent rays will have different focuſes s, t, v, &c. all lying orderly one behind the other in the perpendicular BP; and the

* See Art. 35. I.

the distances Rs, st, tv, &c. between the focus's s, t, v, will be each $= Rr = \frac{2n}{m}AB$.

Thus, of a slender pencil of rays supposed to flow along the physical line Qa, some will be immediately reflected at a into ap, belonging to the focus r; but the greatest quantity will enter the glass, along the line ab, and being reflected at b, along bc, will emerge into ck belonging to the focus R. Again, some of the rays proceeding along bc, will be reflected into cd, de, as if they proceeded directly from I, K respectively; and then some of them will emerge along cl, proceeding directly from s. In like manner, some of the rays may be reflected back at e into ef, then into fg, and at g, some will be refracted into gm; whilst some will be returned back along gh, hi, in, &c. the focus's of the reflected rays ef, fg, gh, hi, will be respectively L, M, N, P; and of the refracted rays gm, in, will be t, v; so that GI, IL, LN, &c. and HK, KM, MP, &c. are each $= 2AB$; and Rs, st, tv, &c. are each $= Rr$.

By making the angles of incidence and reflection equal; it is manifest that $AI = {}^{d}AH = {}^{d}AG + 2AB$; $BK = {}^{d}BI = {}^{d}BH + 2AB$ or $= BG + GI$; and therefore $GI = 2AB = KH$. In like manner it is proved that IL, KM, LN, MP, &c. are each $= 2AB$. Again $As = {}^{e}\frac{n}{m}AK = \frac{n}{m}AH + \frac{n}{m}HK = {}^{d}AR + \frac{n}{m}2AB$; and therefore $Rs = \frac{2n}{m}AB = {}^{e}Rr$. In like manner it may be proved that $st = tv = Rr$. Therefore, &c. *Q. E. D.*

[d] 250. II.

[e] 251. II.

Light considerably obstructed by glass.

253. That some of the rays of light, are reflected several times betwixt the surfaces of a looking-glass, and that some of them are transmitted each time they come to the fore surface; appears very plainly from the flame of a candle seen obliquely in it, exhibiting a series of several images: In this case, all the images beyond R, will continually decrease in brightness, till at last the rays become too faint to exhibit a visible image. The flame of a candle will exhibit more visible images than an opaque object, because of its great brightness: And the several images are more distinguishable, the more obliquely they are viewed; because then the angles formed by the spaces rR, Rs, &c. become so much the larger *. In other cases, the images will partly cover one the other; and the more so, the larger is the object, or the thinner is the glass. From the above phenomenon we have a proof, that light suffers a considerable diminution in its passage through glass; that is, a

* It frequently happens that a looking-glass exhibits several images, at much greater distances one from another than according to the above theory; but these are owing to the irregularities of the surfaces of the glass.

a great part of the light which comes to the ſurface of glaſs, is prevented from going the way it ſhould take, according to the theory of refraction.

254. *Schol.* 3. The diſtance beyond the fore ſurface CD of a *looking-glaſs*, of the image of an object ſticking cloſe to that ſurface, is $= \frac{4}{3}$ the thickneſs of the glaſs. For in this caſe, the points Q, A, r, do coincide; and ſo $RA = Rr =$ [a] $\frac{4}{3}AB$. [a] 251. II.

Hence we have a ready way of eſtimating the thickneſs of a looking-glaſs, by obſerving the diſtance betwixt a wier or a ſlender object laid cloſe to the glaſs, and its image, when both are ſeen pretty nearly in the ſame viſual line: For the thickneſs of the glaſs in this caſe will be $= \frac{3}{4}$ of that diſtance, nearly; and the ſaid diſtance may be eſtimated, by comparing it with the diſtance betwixt two parallel lines ſeen at the ſame time. How to eſtimate the thickneſs of a looking-glaſs ſet in a frame.

255. *Schol.* 4. In the annexed figure, $Qabc$O is the courſe that a ray would take in coming from Q to an eye at O; if a ſlender pencil of rays impinging obliquely on the looking-glaſs CF, had its *foci* after refraction at the ſurface CD in the points G, R, found as in the preceding figure. But becauſe of the [a] aberrations of refractions, at the ſurface CD; the ray Qa will be refracted into ad, whoſe focus is ſomewhere in the line AG, [a] farther from Q than G. Again the ray ad will be reflected into de, making the $\angle Bdh =$ [b] $Aed =$ [c] $Aag =$ [b] Dep; and laſtly the ray at e will emerge into eo, whoſe focus is x lying ſomewhere between R and r; and the greater is the obliquity of the incident ray, the greater will be the aberration Rx. For, when the point of incidence a, is neareſt of all to A, then $Ax =$ [d] AR: And it will always be $Aa : ae ::$ [e] $Ar : rx$: For the $\angle Aag$ being $= Dep$, the $\angle Deo = $ [f] $QaA =$ [g] Aar; and ſo ex is parallel to ar. Suppoſe the line ag to be produced, till it meets the perpendicular AG produced in a point y; then it will be $yA : AB ::$ [e] $Aa : \frac{1}{2}ae$. But as the point a recedes from A, the ratio of yA to AB, is continually increaſing; and therefore the ratio of ae to Aa is conſtantly decreaſing; and ſo the point x is conſtantly approaching nearer to r. But x can never coincide with r, till Aa becomes infinite; for till then the ratio of Aa to ae, and conſequently of Ar to rx, will be finite. Therefore the point x is always betwixt R and r; and as the point a is nearer to or farther from A, the point x will be nearer to R or r. *Q. E. D.* Fig. 44. [a] 56. I. [b] 29. I. *Eucl.* [c] 11. I. [d] 50. I. [e] 2. VI. *Euc.* [f] 13. and 16. I. [g] Conſtruct.

In like manner the diſtances between the focus's R, s, t, v, &c. which here correſpond to thoſe in fig. 43, will, as a recedes from A, be alſo leſſened, ſo as to be always about $= rx$. But the focus of a pencil whoſe axis is $Qadeo$, will be in the line oex, nearer to e than x.

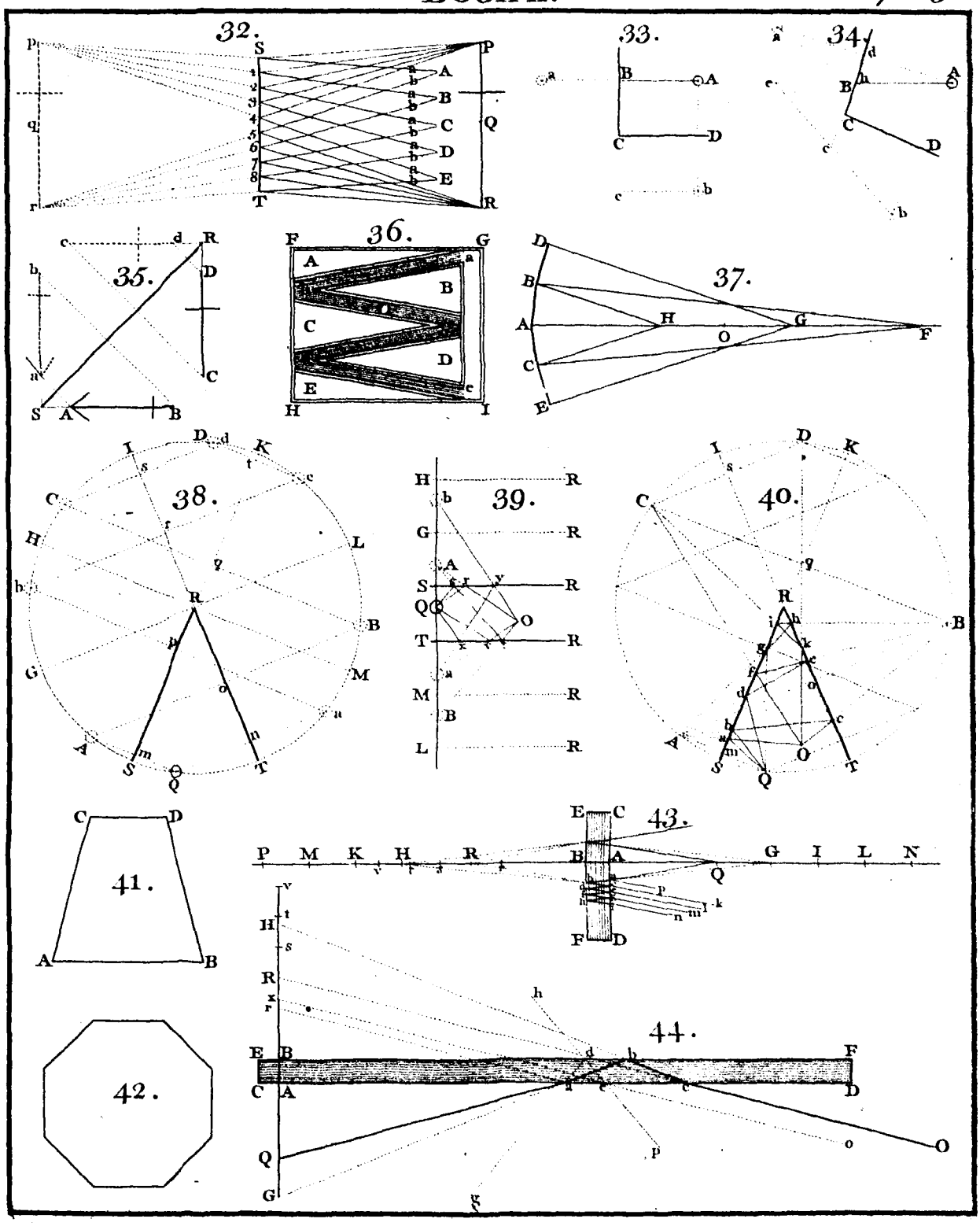
3. For

For a ray diverging from Q upon a point of the ſpeculum which is
56. I. nearer to A than *a*, will have its focus after emerſion, [a] betwixt *x* and R; and a ray falling on the other ſide of *a*, will emerge in a line proceeding in a point [a] betwixt *x* and *r*; and ſo theſe two rays will interſect the line *ex*, in a point between *x* and *e*. So that a line connecting the ſeveral ſucceſſive images made by the reflections within the glaſs, or thoſe correſponding to R, *s*, *t*, *v*, &c. will be inclined from BR towards *d*, and alſo a little curved. When the ∠ AQ*a* is 60 deg. *rx* will be only about = $\frac{2}{3}$AB.

Some obſervations whereby to judge of the goodneſs of plane ſpeculums.

256. All ſpeculums ſhould be exactly figured, and well poliſhed: The goodneſs of figure of a plane ſpeculum is eaſily known, by obſerving if images, ſeen in all poſitions, eſpecially in very oblique ones, and from all parts of the ſpeculum, appear exactly equal and ſimilar to the objects; that is, if the images, eſpecially of the remoteſt objects in the room, appear natural without having any part of them diſtorted, the ſpeculum has a good figure: The ſtrait edges of the rails of wainſcot, are good objects for this experiment. A plane muſt be exceeding bad, that will diſtort a face looking into it; becauſe the rays being returned almoſt directly back to the eye, ſmall aberrations cannot be ſenſible: But let two perſons look at each other's image as obliquely as they can, and they will ſooner perceive if the figure of the ſpeculum is faulty.

It is deemed a very difficult thing to grind a true plane; and the difficulty of making a good looking-glaſs is ſtill greater, becauſe the two ſides ſhould be exactly parallel, as well as flat. If the images of a candle, ſeen very obliquely, as in *fig.* 44, and in different obliquities and from all parts of the glaſs, do not always keep pretty nearly, at equal diſtances one from another, it is a proof that the ſides of the glaſs are neither flat nor parallel. Another method, analogous to the preceding, of trying the goodneſs of a looking-glaſs, is thus: Stick a pin or ſlender wier by the point, in the upright jaum of a window frame, or in a bar of the ſaſh, ſo that the pin may be nearly horizontal and in the plane of the window; then hold the looking-glaſs, and turn it about ſo, as to ſee the images of the pin very obliquely, and from all parts of the glaſs: In this caſe, there will be ſeldom viſible but two images, correſponding reſpectively to R, *r*, *fig.* 43, 44; and if theſe images keep always exactly ſtrait, parallel, and at regular diſtances one from another, according to the theory in the preceding, the glaſs is well figured; and the phenomena, if the glaſs is good, will be alſo a confirmation of that theory. The phenomena of the different diſtances of the images from each other, will be the more conſpicuous, if two pins be ſtuck parallel to one another, and at a ſmall diſtance aſunder:


32.
33.
34.
35.
36.
37.
38.
39.
40.
41.
42.
43.
44.

By this method alſo, the correctneſs of figure of a metalline plane ſpeculum, may be alſo examined. By making ſuch trials, one ſhall obſerve a prodigious difference in the goodneſs of looking-glaſſes; and it is very rare to meet with a common glaſs, as will bear the teſt in all its parts *.

In all ſpeculums, *cæteris paribus*, the better they are poliſhed, the brighter will be the images; that is, the more light the eye will receive from given objects: And this affords us a method of examining the goodneſs of ſpeculums as to their poliſh, whenever we have an opportunity of comparing ſeveral of the ſame ſort, and in the ſame light, together. The darker is the colour of a glaſs ſpeculum, it is generally a ſign of a higher poliſh: For the glaſs itſelf can be no otherwiſe ſeen, than by the reflections of ſcabrous particles, not lying even and ſmooth with the reſt of the ſurface. But different glaſſes, though equally well poliſhed, will not always appear equally dark; yet generally the rule above takes place, and the darkeſt is to be preferred.

§. 2. *Of viſion by the reflection of ſpherical ſpeculums.*

PROP. XIX.

257. *To explain the phenomena of viſion by light reflected at a convex ſpeculum.*

In a convex ſpeculum, the images of objects are always ſeen erect, a little convexed towards the eye, and diminiſhed, yet pretty nigh to the ſpeculum: And the greater is the convexity of the ſpeculum, or the ſmaller is its radius, the nearer, the ſmaller, and the more convex will be the images of given objects.

Theſe phenomena agree exactly with the theories of [a] images, and of [b] viſion. In the annexed figure, BAC is a convex ſpeculum, whoſe center is O; and the principal *foci* in the axes OP, OQ, OR, are E, F, G; the curved line *pqr*, is the geometrical image of the ſtrait object PQR; and *svt* is the image of the object SVT. The lines ſpreading from BA towards PR, repreſent rays that originally diverged from P, and are now reflected from the focus *p*; and the lines proceeding from the part CA of the ſpeculum, repreſent rays belonging to the phyſical point R, and now diverging from the focus *r*. In like manner, as rays diverge upon the ſpeculum from every phyſical point in the object PQR;

[a] 111 to 116. I.
[b] 169 to 171. II.
Fig. 45.

* I have a ſmall looking-glaſs by me, ground in a peculiar manner by a ſkilful hand, that anſwers much better than ordinary; though by examining it very critically, one may diſcover that even this is not quite perfect.

PQR; so after reflection, they diverge from every correspondent point in the image *pqr*. And as it is manifest, that some rays belonging to *p* and *r*, will after reflexion from some part or other of the speculum, be mixed throughout the whole space BCKH; so wherever the eye is placed within that space, it will see the whole image *pqr*: Thus, if K*a* be the breadth of the pupil; an eye placed at K*a*, will see the point *r*, by the pencil of rays whose base is K*a*, and vertex is *r*; and the point *p*, by the pencil of rays K*pa*. And as this is agreeable to the manner of naked vision, so *pqr* appears as a real object, of the same colour with PQR; because it is seen by rays propagated from PR, without having undergone any other change, than that of being made to diverge more, which is the cause of the diminution and convexity of the image.

If the speculum is a pretty large segment, it will exhibit the images of objects that are pretty wide asunder; so that part of the ceiling, floor, and two sides of the room, may be seen at the same time; the whole making a kind of * picture, very agreeable in its effect: And the nearer the eye is to the speculum, the [c] larger will be the field of visible images. If while the objects and speculum remain fixed, a spectator moves towards or from the speculum; the apparent places and magnitudes of the images, will remain invariable: But if an object moves towards or from the speculum, its image will also appear to move the contrary way; and as it approaches nigher or recedes farther, it will accordingly appear more and more enlarged or diminished. All which is entirely agreeable to the [d] theories of apparent distances and magnitudes of near objects; and a notable confirmation of that theory.

[c] 122. I.

[d] 169 and 195. II.

258. *Schol.* 1. If a convex speculum be placed against a window having an extensive prospect, or facing the end of a street; the great multiplicity of objects that are seen, one behind the other, and the diminution of their images; will, sometimes after poring into the speculum, make us fancy that these images are a great way off, and perhaps farther than the objects are: But if we look more attentively, this mistake will soon be corrected; and the apparent places of the images, will not differ sensibly from their real places, or those places from whence the rays diverge to the eye.

259. *Schol.* 2. When the eye sees the image very obliquely, the rays in that case diverging from focus's [e] that are nearer to the speculum than *pqr*; the place and figure of the apparent image will be different from those of *pqr*: But the image will not be sensibly the less distinct; because

[e] 105. I.

* The appearance here is a kind of mean betwixt that of the objects themselves, and that of a good picture of them on a flat surface; and upon this account therefore, and also for grouping the objects, a convex speculum may be very useful to a landskip painter.

because, though the rays that enter the eye have great aberrations from the axes of the pencils, yet as they make but a slender pencil, they have but small aberrations from one another, and therefore cause no confusion.

Thus, to an eye at *bc*, the apparent image of the object PQR is *stv*, lying in a different position from the geometrical image *pqr*, and betwixt it and the speculum. And as the pencil of light coming to the eye from P, falls more obliquely on the speculum, than that which comes from R, so *sp* is greater than *vr*; and for a like cause the curvity of *stv*, is different from that of *pqr*; and it will still vary according to the different situations of the eye and the object. Fig. 46.

260. *Schol.* 3. As the rays diverge much more after reflection by a convex speculum, than they did before; so this is another reason, besides that on account of the usual loss of light by reflection, why images seen in such speculums, appear much darker than the objects. Thus, the eye at *bc*, receives no more light from P, than what is contained in the slender pencil P*d*; the two extream rays of the pencil P*d*, being reflected respectively to *b* and *c*, the edge of the pupil. In like manner, the two extream rays in R*g*, are reflected respectively to *b* and *c*. But the pencil R*g* is a little broader than P*d*, because it falls less oblique on the speculum; and therefore the point *v* of the image, will be a little brighter than the point *s*. Why images in convex speculums appear very dark. Fig. 46.

PROP. XX.

To explain the phenomena of vision in spherical-concave speculums.

This proposition admits of three cases, according as the object is between the speculum and its principal focus; between its center and that focus; or farther off than the center. In the annexed figures, BAC is a concave speculum, whose center is O, and principal foci are in the arch EFG: PQR is a rectilineal object, placed at right angles to the axis AOQ, of the speculum; and *pqr* is its geometrical image. The strait lines issuing from the speculum, represent rays which diverged upon it from the respective points P, Q, R; and which now diverge from, or converge to, the respective points *p*, *q*, *r*; the aberrations of the more remote rays not being here considered. The rays issuing immediately from the object to the speculum, are not here represented, to prevent the confusion of too many lines. Fig. 47, 49, 50, 51.

261. CASE 1. *When the object is between the speculum and its principal focus.*

In this case, the images of objects are always [a] erect, and [b] magnified; and the nearer is the object to the principal focus F, the greater will be Fig. 47. [a] 112. I. [b] 111. I.

be its image. If the object is a ſtrait line or a plane, placed at right angles to the principal axis AO, its geometrical image will be always
116. I. [c] concave; and the nearer the point Q is to F, the greater will be the concavity of the image *pqr*. In the figure, the rays diverging from the part BA of the ſpeculum, are the rays belonging to P; and thoſe diverging from the part CA, are thoſe which came from R. If the rays belonging to P and R, had been alſo drawn from the other halves of the ſpeculum, they would have interſected throughout the whole ſpace BCKH; and therefore to an eye placed any where within that ſpace, the images of the points P, R, and conſequently the images of every other point of the object PQR, will be viſible. As if K*a* be the place and breadth of the pupil; the eye will ſee the image of the point P, by the pencil of light *ab*AK; and the image of the point R, by the pencil *ad*CK. In like manner, the eye will ſee every point of the image *pqr*, by pencils of rays, reflected from the ſpeculum between *b* and C.

Phenom. In the caſe above, the image always appears erect, and larger than the object; and, if while the object and ſpeculum retain the ſame poſition, the eye moves a little either way in the axis AO; the apparent place, figure and magnitude of the viſible image, will continue much the ſame. Alſo, if the geometrical image is pretty near the ſpeculum, ſo that the rays in the pencils entring the eye, have ſenſible divergencies; the viſible image will appear pretty near in the ſame place, and of the ſame figure and magnitude with the true image. But when the geometrical image is remote from the ſpeculum, its apparent place will be in a manner indeterminate; and variable according to different circumſtances of light, and particularly as the eyes happen to have a different conformation for ſeeing at different diſtances. But the viſible image never appears very remote; and in ſome caſes, it will ſeem to approach the ſpeculum, when the geometrical image recedes from it; and ſometimes it will appear ſtationary, when the object is in motion.

Thus, let PR repreſent the length of the face of a perſon viewing his own image in the ſpeculum BAC, from different ſtations in the axis AF. When PR is near to the ſpeculum, its viſible image on the other ſide will be pretty nearly in the ſame place, and of the ſame figure and magnitude with the geometrical image *pqr*, as has been obſerved above; the rays in this caſe having a ſenſible divergency, and the geometric image *pqr* being not very concave. As PR recedes from A, its image for a little while will alſo appear to recede from A, but not ſo faſt as the geometric image *pqr*, till at length the viſible image

will become ſtationary; or it may even ſeem to approach towards the ſpeculum, more eſpecially if PR is an object whoſe image is viewed by a ſpectator placed near the center O. For, as the object advances towards F, the divergencies of the reflected rays become leſs and leſs ſenſible, whilſt at the ſame time the images of the ſmaller parts of the object, appear, pretty nearly, equally bright and diſtinct; alſo as our ſpectator recedes from A, his image fills a larger part of the ſpeculum; this part * being a ſegment, whoſe breadth is the meaſure of about half the angle POR. So from all theſe conſiderations, we ſee why, as the ſpectator retreats from A towards F; his apparent image at firſt retreats from the ſpeculum pretty quick; then ſlower and ſlower; at length becomes ſtationary; and at laſt may even ſeem to advance forwards †. If the ſpectator be at O, the viſual angle of an object PR will be conſtantly increaſing, as Q retreats from A towards F; that angle in this caſe being always $= \angle$ POR.

262. Again, When the apparent diſtance of the image is leſs than the geometrical diſtance; its apparent magnitude [d] will be leſs than the geometrical magnitude. And while the apparent diſtance increaſes but ſlowly, or remains the ſame; the apparent magnitude will increaſe but ſlowly, or it may remain for a little while invariable; for the viſual angle ‡ decreaſes, though ſlowly, whilſt the ſpectator ſees his image reflected from a larger part of the ſpeculum; and for this laſt cauſe, the apparent magnitude ſcarce ever ſeems to decreaſe by retreating from the ſpeculum. Apparent magnitude of the image. [d] 198. II.

For the above reaſons, the image of a flat and blank ſurface, will not appear ſo concave as the geometrical image: For not being able to determine the true diſtances of the different parts of this image, we ſometimes fancy it to be pretty near flat, like the object. But when the geometrical image is not very remote, our prejudice will be overcome; and the image will appear more concave, as we view it more attentively. In ſome caſes, we have farther aſſiſtance for judging more accurately of the place of the image; and then the phenomenon is ‖ ſurprizing.

263.

* If P be the eye, and PQ = QR; the image of the point R will be ſeen by a ray reflected from a point near to A, and that of P, by a ray OP after reflection.

† As the rays have but a little way in coming from the object to the eye, and therefore as many as come ſtrike as vigorouſly upon it, as they uſually do when we ſee near objects; this may be thought a cauſe why the image ſhould not appear very remote: The pencils of light are alſo denſer, than when we ſee an object at the ſame diſtance by naked viſion; and the *impetus's* upon the retina being probably chiefly in proportion to the quantities of light in each pencil, this may be yet another and more powerful cauſe of the above phenomena.

‡ If P be the eye, and PQ = QR; the viſual $\angle$ of the image will be $= AP'p = PAO + POA$; and the $\angle AP'p$ is leaſt when Q is in F.

‖ See Prop. XXI.

Phenomena of a hollow cone or ſtick held in the axis of the ſpeculum.

263. *Schol.* 1. If a hollow cone be held with its baſe towards the ſpeculum, and at ſuch a diſtance that its vertex may be near the principal focus F; its image will appear projected into a prodigious length, according to the theory of the geometrical images of different parts of the cone. The reaſon of which, is, our having in this caſe, a ſeries of theſe images, lying orderly one behind the other, we are enabled thereby to judge of the places from whence the rays diverge to the eye, more truly; and the whole viſible length is protracted [a], becauſe of its really containing a great number of viſible parts. It is for the ſame reaſon, that if a ſtrait ruler or ſtick be held before a concave ſpeculum, ſo as to lye pretty near in the axis, as AF, and ſo that the fartheſt end from the ſpeculum is pretty near to F; the image of this end, will appear much farther, than will the image of the ſtick if it be held croſs-wiſe at the ſame diſtance from F. The image of the ſtick will alſo appear conical; the remoteſt end of the image appearing the biggeſt. And for the ſame reaſon, the image of the above cone, will be nearly cillindrical, or much more acute than the cone itſelf. In like manner, and for the ſame reaſon; the noſe, and ſide of a face, will be protracted much more, and the hollows ſunk deeper, than in the proportion that the length and breadth of the face are magnified.

[a] 172. II.

Fig. 48.

264. *Schol.* 2. The pencils of light coming to the eye from an object PQR, placed before a concave ſpeculum, diverge from [a] a greater diſtance than the geometrical image *pqr*, as from *stv*. The image *stv* will be alſo bigger, and have a different figure than *pqr*; and if the eye is out of the principal axis, as in the annexed figure, the poſitions of *pqr* and *stv*, will be alſo different. But the phenomena will be as in *Art.* 261, and that for the reaſons there aſſigned; excepting that when the eye ſees the image obliquely, as from *bc*, its apparent poſition will be nearly ſimilar to the real poſition of *stv*.

[a] 105. I.

265. Case 2. *When the object is between the principal focus, and the center of the ſpeculum.*

We have before brought a ſpectator viewing his own image, as far from the ſpeculum, as its principal focus; and for the reaſons [a] that have been given, his image ſeen from that focus will appear erect, magnified, and pretty near the ſpeculum. As he retreats from hence towards the center, the geometrical image of his face [b] will be behind him, [b] advancing ſtill towards him, and at the center [b] the image and object will both unite. During this retreat, the image appears ſtill erect, becauſe the rays belonging to each viſible point, do not interſect before they reach the eye: The image becomes leſs and leſs diſtinct,

[a] 261. II.

[b] 97. I.

and at the ſame time its viſual angle is increaſing, ſo that at the center or rather before, the whole will be loſt in confuſion.

In this caſe, the picture is made before the retina; and yet the indiſtinctneſs is not like that of remote objects, but on the contrary, like that of objects viewed too near. For the ſmall parts of the object, inſtead of becoming inviſible, for want of ſubtending angles ſufficiently large, do appear larger than common; the increaſe of magnitude being manifeſt, by the image filling a larger part of the ſpeculum. But yet in the preſent caſe, I think, the image never appears before the ſpeculum, but rather as it were ſticking to it: And that perhaps may be owing to a prejudice ariſing from our frequent looking into plane ſpeculums, where the images are always on the other ſide: Or, having here no aſſiſtance from the different conformation of our eyes for viſion at different diſtances, we naturally look directly at the ſpeculum, and nothing being perceived betwixt us and it, we cannot fancy the image nearer than the ſpeculum we look at; nor can we fancy it much farther, becauſe of the manifeſt increaſe of its apparent magnitude ‡. So that here, the apparent places of the image continue pretty near the ſpeculum; and therefore its apparent magnitudes, are nearly as the viſual angles.

266. To illuſtrate this caſe farther: Let an object PQR be fixed between the principal focus F and the center O, whilſt the eye looks for its image in the ſpeculum, from different parts of the axis A*q*. If the pupil of the eye be *bc*; the images of the points P, Q, R, will be ſeen by the pencils of rays * *bd*A*c*, *bef c*, *b*A*gc*, whoſe reſpective focus's are *p*, *q*, *r*: Now if the conſtitution of the eye be ſuch, that the rays contained in the above pencils are brought to unite before they reach the retina, the repreſentation of PR will be † confuſed; as remote objects appear to ſhort-ſighted perſons. But although the rays come to the eye converging, which is a caſe that never happens in naked viſion; yet as they are orderly reflected according as they come from different parts of the object, they convey an idea of that object, though an indiſtinct one, both as to its ſhape and colour. And in the preſent Fig. 49.

‡ See *Art.* 261. II. and its *note* †.

* Some have thought the confuſion in the preſent caſe, to be owing to the mixture of the pencils upon the ſpeculum: But although the largeneſs of the ſpaces upon the ſpeculum taken up by the pencils entring the pupil, are proportionable to the degree of convergencies of the extream rays; yet, from the obſervations lower down in the above paragraph, it is manifeſt enough, that this is not the immediate cauſe of the indiſtinctneſs. The pencils belonging to the adjacent viſible points of an object, are mixt upon a plane ſpeculum; and the nearer the eye is to the ſpeculum, looking at the image of an object that is behind him, the larger will be the ſections made at the ſpeculum of the ſeveral pencils: But yet no indiſtinctneſs ariſes upon that account.

† The confuſion occaſioned by the convergencies of the rays, may be remedied by a concave lens of a proper concavity; and the nearer the eye is to the image, the more concave the lens muſt be. If the eye be at *q*, the confuſion will be irremediable by any lens held cloſe to it; but it may be remedied by a concave lens held at a proper diſtance before the eye, which diſtance muſt be the greater, the leſs concave is the lens.

present case, an aged person might see distinct enough, and perhaps distincter too, than he is ever able by his naked sight: For whatever be the disposition of the rays coming from every visible point of an object, before they enter the eye, as diverging or converging; vision will be still distinct, if the several pencils are not mixed upon the retina; and the apparent places and magnitudes will be according to some general laws of vision in other cases; that is, as some of the ordinary helps are wanting, others will preponderate.

As the eye recedes farther back from the speculum, the confusion of the image necessarily increases: As when the pupil *xy* is at the center O, it receives from the points P, Q, R, the pencils B*xyh*, *dxyg*, *kxy*C, whose extream rays converge about twice as much as those of the former pencils. And before the eye is got quite so far as *q*, the whole image will be lost in confusion. Again, all the way from A to *q*, the image necessarily appears erect; and also its visual angle is constantly increasing.

Fig. 49. 267. *Schol.* 1. As the eye recedes from A towards *q*, the field of vision in the given speculum, of an indefinite object PR, is continually diminishing; so that beyond K, none of the rays belonging to the points P, R, can enter the pupil; and at *q*, the said field will be diminished to nothing. After the eye has past *q*, the field of vision of the indefinite object PR, will again begin to open, and the farther back the eye is from *q*, the wider will be the said field. The cause of all which is manifest, from an inspection of the figure: *See also* 121. I.

After the eye has past *q*, the image will appear inverted, and for some time continue indistinct, as in the case of objects viewed too near, till the eye is got so far, that the rays entring it have a proper divergency for distinct vision. But as the image itself is so much the rarer, in proportion as it is magnified, and the pencils of light entring the pupil continually decrease in density, as the eye retreats farther back; so beyond a certain distance, it will be difficult to find the image, unless it be that of a bright object, as a candle; and when it is found, for the above reasons, the spectator will be apt to fancy it farther from him, than it really is; and conforming his eyes accordingly, this also will cause an indistinctness.

268. *Schol.* 2. If the object be a strait stick or a plane at right angles to the axis of the speculum, its image, to a spectator at a proper distance behind it, instead of appearing concave, will be convex, and that pretty much too. This phenomenon, as at first sight it may seem to involve some difficulty, deserves to be farther explained.

Let,

Let, as before, *pqr* be the geometrical image of the object PQR, after reflection by the speculum BAC; and let *bc* be the breadth and place of the pupil. Now the extream rays *dtc*, *gtb* of the pencil belonging to Q, being not remote from the axis AQ*a*, will diverge to the eye from a point *t*[a] that is very near to *q*. But the nearest ray *ksc* to the axis POD of the pencil belonging to P, that can enter the pupil, being very oblique to that axis, will intersect it at a point *s*, [a] much nearer to the speculum than *p*; and the rest of the rays belonging to this pencil will intersect the axis a little farther from *p*, than *s*; the extream ray C*yb* intersecting the ray *ksc* in *y*. In like manner, the nearest ray *hvb*, to the axis ROH of the pencil belonging to R, intersects that axis at the point *v*; and the extream ray B*xc* intersects the other extream ray *hxvb*, at the point *x*. And so the apparent image is a convex curve *ystvx*, agreeable to the theory. Fig. 50. [a] 105. I.

269. CASE 3. *When the object is farther from the speculum than its center.*

In this case the geometrical image is always [a] inverted, a little [b] concave if the object is a plane, and [c] between the center and principal focus of the speculum. The farther the object is from the center, the [d] less will be its image, and the [c] farther from the said center, or the nearer to the focus; and the converse. If the object be in the center, its image will coincide with it. [a] 112. I. [b] 116. I. [c] 97. I. [d] 111. I.

Phenom. Let PR, represent a spectator viewing his own image, at a considerable distance beyond the center O of the speculum: In this case, his image will appear inverted, small, faint and indistinct: It is faint, because but few rays are contained in the pencils that enter the pupil; and this faintness, [e] causes the apparent distance to be so much the greater, so that the image generally appears beyond the speculum; and this mistake adds to the indistinctness. As our spectator advances forward, his image will appear gradually larger and brighter, and also to approach nearer to him; but if he does not see it betwixt him and the speculum, it will continue still indistinct. Fig. 51. [e] 188. II.

270. In the case above, if the observer holds out a stick with its end towards the speculum, he will immediately perceive its image come out to meet him; the length of the image now assisting him to judge more accurately of its distance: The image of the stick will appear shorter than the stick itself; and if the object be a cillinder or a ruler with parallel sides, the image will appear taper; that end being smallest which is held in the hand, or which appears fartheſt off: Both which phenomena, of the shortening and tapering of the image, are exactly agreeable to the theory of the geometrical image. If the stick be Phenom. of a stick held before the speculum.

 stretched

ſtretched onwards nearer to the ſpeculum, its image will be ſeen to advance farther out; and when the end of the ſtick is in O, its image will be ſeen there alſo. If the ſpectator advances farther, he will ſee the ſtick and its image paſs by one another. After this, ſtretching out his arm without the ſtick, he will ſee its image come out, and he may graſp, as it were, his own fiſt: And if he now minds the image of his face, he will ſee it ſuſpended in the air, at the proper place, betwixt him and the ſpeculum.

When the inverted image appears diſtincteſt.

271. As ſoon as the image is ſeen in its true place, it will become a determinate and fixed object, whereas before it ſhifted its place as it were caſually; and then, and not till then, it will appear perfectly diſtinct; and ſmall ſpecks or pits in the ſkin, though diminiſhed, will yet be well defined. This phenomenon is ſo ſtrong and lively, that I have known people ſtartled at their own image, the firſt time of their obſerving it.

After making a few trials, a perſon will find the true place of his image pretty readily, if he is not too far off or too near it; and it will continue diſtinct, as he moves to or fro, for ſome diſtance. When he has got within a few inches of it, the image will begin to be indiſtinct; and this indiſtinctneſs will increaſe, till he has got to the center O, where the image will be loſt in confuſion. If he advances beyond the center, he'll begin to have ſome glimmering of an erect image, which will grow diſtincter as he advances forwarder, as has been already explained.

272. If PQR be an object remaining fixed, while the eye is in motion along the axis AOQ; the image will appear inverted all the while that the eye is behind *pqr*; and when it is at ſuch a diſtance, that the image appears at the place *pqr*, it then alſo appears diſtinct. If the eye is far behind the image, it will appear faint for want of ſufficient light; and if it be too near it, it will be indiſtinct becauſe of the two great divergencies of the rays: If the eye gets before the image, it will appear erect and confuſed, which confuſion will leſſen as the eye gets nearer the ſpeculum, becauſe the convergencies of the rays will be leſſened.

If the eye be fixed, while the object PQR moves towards the center of the ſpeculum; as the real magnitude of the image will be thereby all the way increaſing, ſo alſo will its apparent magnitude. If the eye is pretty near the center O, the above motion of the object will cauſe an indiſtinctneſs, which will increaſe as the image approaches nearer the eye. If the eye had been placed between the image and the ſpeculum, the indiſtinctneſs of the image would have leſſened, as

the object advanced nearer towards the center. And in both cases, the contrary would happen, if the object retreated farther back.

273. *Schol.* 1. In the two last cases, if the eye be out of the principal axis, the figure and position of the image formed by the pencils entring the pupil, will be different from those of *p q r*; as has been before [a] exemplified in other cases. But as generally here, vision is indistinct, and the apparent image necessarily differs in some circumstances from that formed by the concourse of the rays entring the pupil, it would be needless to dwell upon this any farther. [a] 259 and 264. II.

When the eye is in the axis Q*q*, at a small distance behind *p r*; the apparent image will be less concave than *pqr*, and may be even convex; because the pencils entring the pupil from the extream parts, diverge from focus's that are [b] nearer the speculum than the points *p*, *r*, as in *Art.* 268. Fig. 51. [b] 105. I.

274. *Schol.* 2. When the eye is in or near the principal axis, and between the image *p q r*, of a plane object PQR, and the speculum; the image generally appears concave or hollow. In this case the object is somewhat less indistinct about the middle, than towards the extreams; because the rays belonging to the middle of the object converge less than the others, at their entrance into the pupil. And this seems to be the reason why the middle appears farther, or at a properer distance for distinct vision; the greater indistinctness in this case, suggesting a nearer distance of the object, as when we approach one too nigh, though the causes are quite opposite. See 265. II. Fig. 49, 50, 51.

275. *Schol.* 3. A small image *q* of an object Q, seen with both eyes in a concave speculum, will appear double, as at *m* and *n*; if the optic axes *a v* F, *b v* F are not directed to *q*, but to some point F beyond it. This phenomenon frequently happens, as we are apt to look towards or beyond the speculum, for the image; and the [a] cause of it is, the points *r*, *s*, where the visual rays Q*m q v r*, Q*n q v s* meet the retina, falling on different sides of the points *a*, *b* in the optic axes. In like manner, if *p q r*, fig. 49, be the image of an upright and slender object, as a wier, &c. this image to a spectator beyond it, will also frequently appear double, and that for the same cause. Fig. 52. [a] 59. II.

Again, when the image *q* is behind the spectator; there will appear a double image, if the speculum be broad enough, to throw rays diverging upon it from Q, into both eyes. For in this case also, the points *r*, *s*, where the visual rays B*v r*, C*v s*, meet the retina as they are tending towards *q*, are on different sides of the optic axes; and therefore vision is double: The case here is the same, as when the object seen, is farther than the point we look at; which is the reverse of the preceding. Fig. 53.

Fig. 51, 45. 276. *Schol.* 4. The inverted image of a perſon ſtanding a little way behind it, appears to him equally bright and diſtinct, with his image in a convex ſpeculum of the ſame radius with the concave one, and when he is at ſuch a diſtance that his image is of the ſame ſize in each. For, it is manifeſt that the denſities and divergencies of the rays in the ſeveral correſponding pencils, at their entrance into the eye, are the ſame in both caſes.

I have been the more particular in endeavouring to illuſtrate the phenomena of viſion in ſpherical ſpeculums, not only becauſe ſeveral of theſe phenomena are curious and entertaining, but becauſe they alſo greatly illuſtrate and confirm the theory of viſion in general. In the concave ſpeculum, we have a quick tranſition, by a ſmall motion of the object, through all the caſes that can happen, as to the different degrees of the divergencies and convergencies of the rays at their entrance into the eye: And the ſeveral phenomena do all concur to prove, that our viſible Ideas are always conformable to certain general laws, notwithſtanding any prejudices we may have in oppoſition to them; ſo far are we from being left to the uncertain guidance of mere fancy. The opportunity we have of viewing the images here with both eyes, is an advantage we ſeldom have when we look through lenſes; and ſo the phenomena are generally more ſtriking, conſtant and regular in the former, than they are in the parallel caſes in the latter. Before we diſmiſs this ſubject, it will not be amiſs to ſubjoin the following obſervations.

Fig. 54. 277. The image of an object PR, placed in the principal focus of a concave ſpeculum, will appear under the ſame viſual angle to an eye D, wherever in the axis the point D is taken. For, neglecting the aberrations, the viſual rays pD, rD, [a] will be always parallel to the reſpective axes OPB, ORC; and ſo the $\angle$[s] pDr will be always $=\angle$ POR ſubtended by the object from the center. But the apparent magnitude of the image will be greater or leſs, as it appears farther or nearer.

[a] 85. I.

Fig. 55. Alſo, if the eye be placed at the principal focus F of the ſpeculum; the viſual angle ſubtended by the image of an object PQR, will be invariably the ſame, wherever in the axes the object be placed. For the rays PB, RC will be reflected to F, wherever in the indefinite lines BP, CR the points P, R be taken; and ſo the viſual angle will be always $=$ BFC. In this caſe the image will appear always erect; but its diſtinctneſs, apparent place and magnitude, will be variable, according to the different places of PQR.

Fig. 45, 47, 49, 50, 51. 278. *In all caſes, the object and its geometrical image, form each the ſame angle at the vertex of the ſpeculum*; that is, the $\angle$ PAR $=\angle$ pAr.

At the points Q, q, draw perpendiculars to the axis QO, till they meet the axes or lines POp, ROr, produced if need be; and, to avoid having new references, let these points of intersections be called respectively P, p; R, r. Now it will be, QO : qO[a] :: QP : qp; but because QF : AF (=OF)[b] :: OF : Fq; it will also be AQ (QF ± AF) : OF :: Aq (AF ± Fq) : Fq. But OF : Fq[c] :: QO : qO; and therefore QO : qO :: AQ : Aq :: QP : qp. But because the ∠PQA = [d] pqA, the ∠QAP = [e] qAp. After the same manner is proved that the ∠QAR = qAr; and therefore the whole ∠PAR = pAr. *Q. E. D.*

[a] 4. VI. *Euc.*
[b] 94. I.
[c] [94]. I.
[d] Constr.
[e] 6. VI. *Euc.*

279. *Corol.* Hence the ∠pAr is always increasing, whilst the object is approaching nearer towards the speculum; and in a convex speculum, the magnitude of the image is also increasing, whilst the object is advancing forwards. When the object is between a concave speculum and its principal focus, the image is least of all when the object is close to the speculum; and the magnitude of the image continually increases, as the object advances towards the focus; after this, the inverted image will constantly decrease, as it approaches nigher to that principal focus, or as the object retreats farther from that focus. But the apparent magnitude of the image, will not be always as its true magnitude; because the apparent distance is always finite, and frequently much less than the true distance. It may be observed farther, that the image in a convex speculum is always between the speculum and its principal focus: But in a concave speculum, the distance betwixt it and its principal focus, is the only part of the axis, produced both ways to infinity, that the image is excluded from.

280. In all cases, if the eye be in or near the principal axis, and the breadth of the pupil be neglected, the visual angle of the apparent image, is equal to that angle which the real image subtends at the center of the pupil. When the geometrical image is before the eye, it affects us then as if it was a real object, and any mistake we may be under as to its true distance, causes no difference in the visual angle. Fig. 49. When the image is behind the eye; suppose the point b or x to be the center of the pupil; then is the visual angle dbA or Bxk, formed by the rays reflected towards p and r, equal to the ∠pbr or pxr. But in this case, if the breadth of the pupil be considered, the above angle will be somewhat greater, and that because the rays enter it converging. Thus, if bc be the breadth of the pupil, the visual angle of the whole image will be $dog = por$, formed by the intersections of the rays db, gc tending towards p, r. And since, when the image is behind the eye, the [a] apparent distance remains much the same, the apparent magnitude will increase as the eye recedes from the speculum towards q.

[a] 265. II.

281.

281. The limits of the diminishing powers of convex speculums, and of the magnifying powers of concave speculums, are reciprocally as their focal lengths. For if two equal objects PR, be placed each at the greatest distances before two convex speculums, or in the focus's of two concave speculums; the image *pr* in the one case, will be directly as OF; and in the other, it will be reciprocally as OF. This is manifest, because OQ is supposed to be infinite in the one case, and O*q* in the other; whence *pr* in the one case is as OF; and *pr* in the other is as the ∠ *p*O*r*. But yet these powers, as to their effects, are contained within bounds not very extensive. Fig. 45, 47.

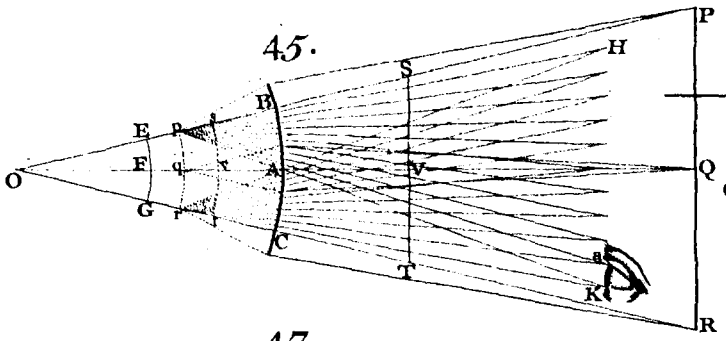
An object seen in a convex speculum, unless it be the sun or moon, must be at a finite distance, and the more convex is the speculum, the nearer the object must be to it, to be sufficiently distinct. If the distances of the objects from the focus's of two convex speculums, be respectively as their focal lengths; the distances of the object and its image from the center of each, will be also in the same proportion; that is, QF : OF :: OQ : O*q*; and OQ : O*q* :: QP : *qp*. And therefore in this case, the diminishing powers of the two speculums are equal. Fig. 45.

An object in a concave speculum appears distinctest, when it is at such a distance from the focus, that the geometrical image is at a moderate distance off, not exceeding a foot or two from the speculum; and generally the more concave is the speculum, the nearer the image, *cæteris paribus*, will appear to it. And if the apparent distances of the images seen in two speculums, be reciprocally as their visual angles; the apparent magnitudes of the images, will be as the objects. That is, if the objects before each speculum be equal, their images will appear also equal.

282. Hence it is, that one does not usually perceive any great difference in the bigness of his face, when seen in two speculums of different concavities. And the effects of concave speculums in * magnifying a large object, as a face, are not so great as at first sight one might imagine. The image generally appears biggest when the spectator is near the principal focus, and then the visual angle of the image is about double of what it would have been, if the speculum had been plane, the spectator standing at equal distances from each speculum: And admitting the image appears three times the focal length of the speculum beyond it; the whole apparent breadth of the face, will be four times greater than in a common looking-glass. But the image seldom appears so far, and therefore is seldom magnified so much, as four times in breadth. But the apparent magnitude of the whole face, being as the Fig. 47.

* Their effects in magnifying small objects, will be considered in the next book.

45.



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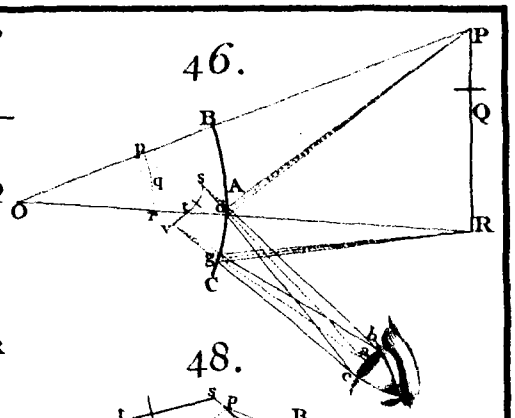

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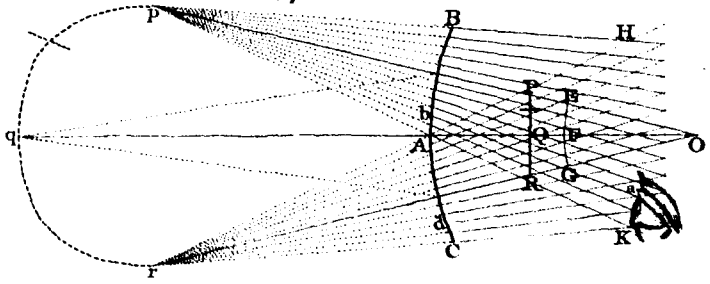

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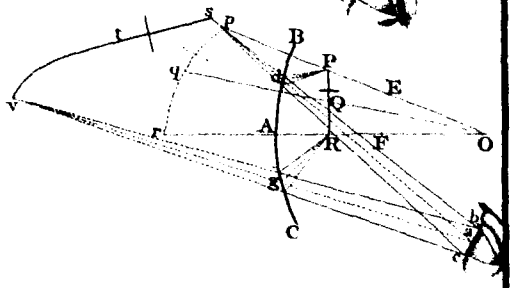

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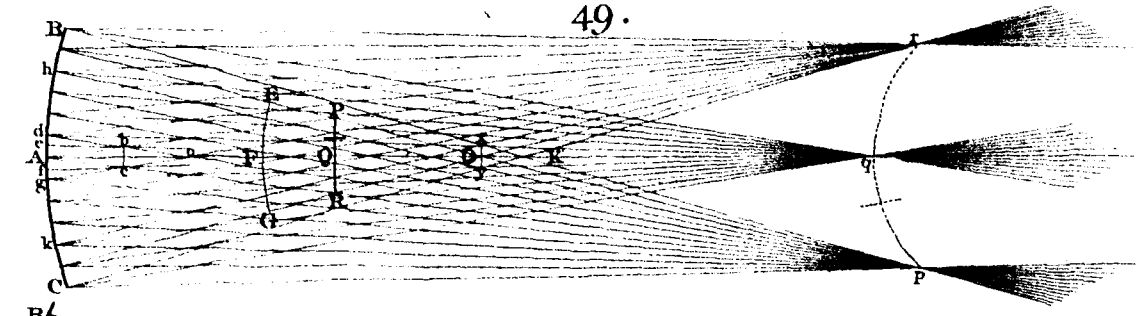

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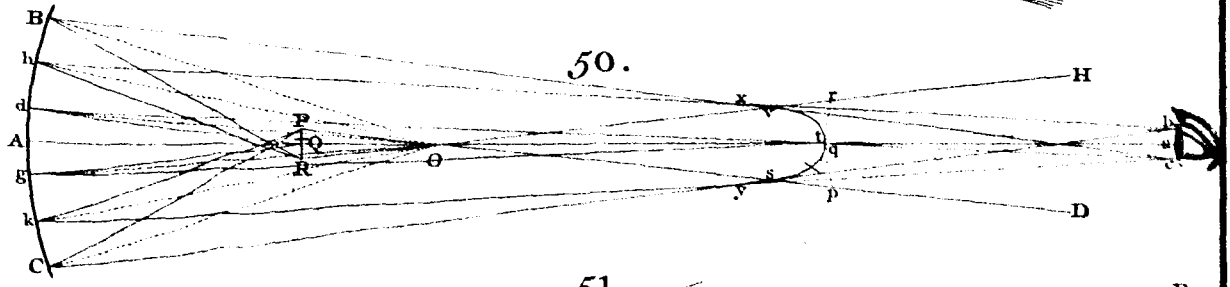

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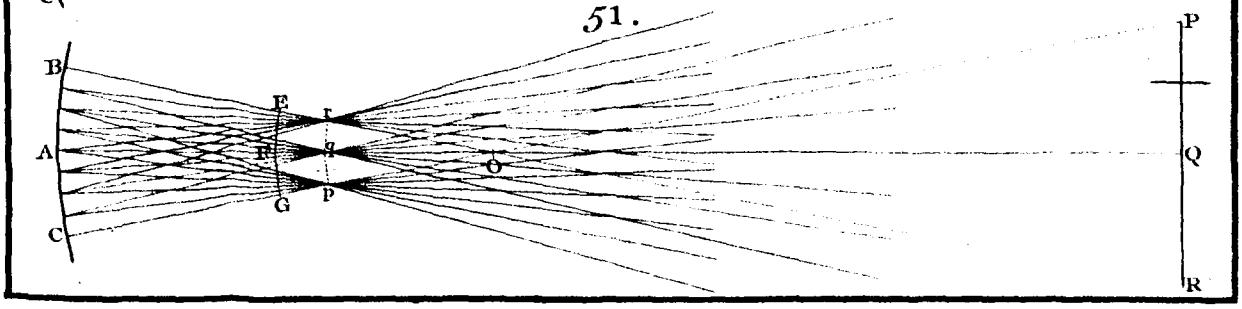


the ſquare of that of its breadth, is the cauſe of our ſurprize; the whole face being really magnified nine times, when it appears only three times broader than uſual.

PROP. XXI.

283. *To explain the phenomena of pictures ſeen in concave ſpeculums.*

Phenom. If a picture drawn according to the ſtrict rules of perſpective, and wherein are repreſented objects having different real diſtances, be placed before a concave ſpeculum, a little nearer to it than its principal focus; the image of the picture will appear extremely natural, the whole ſcene appearing very like the real one from whence it was taken: For not only the objects are greatly magnified, ſo as to approach nearer to the natural ſize, but they have alſo different apparent diſtances; inſomuch that a view of the inſide of a church, appears very like a real church: And landſkip pictures, &c. appear very like as the real objects would do, to a ſpectator at the place from whence the view was taken. Theſe curious and very entertaining phenomena, may, I think, be accounted for in the following manner.

In the annexed figure, the curve *pqr* is the geometrical image of the ſtrait object PQR; or it is that curve in which are contained the foci of all the pencils of light that diverged from PR, and whoſe axes paſs through the center O of the ſpeculum BAC, after their reflection by that ſpeculum. Now the geometrical image of a circle facing the ſpeculum, and whoſe center is at Q, and diameter is PR; will be a hollow figure, formed by the rotation of the curve *pqr* round the axis O*q*. If this hollow figure be ſuppoſed a real thing, whoſe inſide ſurface is variouſly diſtinguiſhed into parts by different colours, and a picture of it be drawn in perſpective upon the circle PR, the point of ſight being at O: A ſpectator placed at O, would be affected much alike by the rays coming to him from the picture, after reflection by the ſpeculum, as he would by rays coming to him directly from the hollow figure, the ſpeculum and the picture being removed; for the hollow figure, and the geometrical image of the picture upon the circle, are, in this caſe, both coincident. Fig. 56.

Again, the geometrical image of a rectangular parallelogram AB, fig. 57, placed where PQR is fig. 56, will be alſo a hollow figure; but more like a pyramid with four ſides, than to the figure deſcribed by the rotation of the curve *pqr*. For the geometrical images of each of the ſides of the parallelogram, lye all in planes paſſing through the center of the ſpeculum, and theſe ſides reſpectively: Theſe images, it is true, will be curvilineal; but theſe curves lying in the planes above mentioned,

mentioned, the ſtrait lines joining their extreams, will be the ſides of a parallelogram, like unto the original. And in like manner, a leſſer parallelogram *ab* within the former, will have the images of its ſides like thoſe of the former, but at a greater diſtance. And ſo likewiſe the ſides of the ſeveral parallelograms *cd*, *ef*, &c. will have their images in a ſeries one behind the other; the middlemoſt being fartheſt of all: So that the geometrical image of the whole figure, does ſomewhat reſemble the *fruſtum* of a hollow pyramid with four ſides, and which becauſe of the greater apparent diſtance of the ſmaller or middle parts, appears nearly like a hollow priſm, a ſection whereof is *pstr*.

Now if *ps* or *rt* be the length, and *st* or *pr* the breadth of the inſide of a church, a perſpective view of which from Q, is drawn upon the plane PR; the geometrical image in this caſe will not be very unlike the church itſelf. For the picture upon the plane PR, is a figure properly conſiſting of ſeveral parallelograms, diminiſhing towards the middle, after the manner of thoſe above deſcribed; and if the picture is not too large in proportion to the ſize of the ſpeculum, the curvities above noted of the ſides of the ſeveral included parallelograms, will not be very conſiderable, and theſe are ſtill leſſened in the appearance, by the viſible concurrence of the images of certain lines, inclining to one another in the picture. But as moſt of the pencils of light entring the eye, diverge from points that are at great diſtances, their different divergencies are not alone ſufficient for determining the true places of their foci: And the apparent image of a blank ſurface placed at PR, will not appear near ſo concave as the geometrical image. We muſt therefore have recourſe beſides to ſome other cauſes; and indeed we have here many concurring together.

The contiguous parts of the picture of the floor, for inſtance, eſpecially if it be checquered or otherwiſe diſtinguiſhed, form a long ſeries of viſible images contiguous likewiſe to one another: The images of the remoter parts appear alſo at the ſame time fainter, and ſmaller; they appear fainter, becauſe they are drawn ſo in the picture, which faintneſs contributes much to enlarge the apparent diſtance; and they appear ſmaller, becauſe the pictures of the remoter parts of the real floor, diminiſh faſter in proportion, than the apparent diſtances of the images of theſe pictures increaſe: So that from all theſe cauſes conſpiring together, the eye receives much the ſame impreſſion as if it looked at a real floor; and ſo in both caſes the appearance is much the ſame; a long extended ſurface, a little diminiſhed in breadth, and that gradually, towards the farther end. In like manner, the walls appear erect and extended on each ſide, and the roof above facing the pavement; and all gradually inclining, after the ſame manner as a large

large room appears to the naked ſight when viewed from one end. Beſides the above helps, the window-lights in the ſides, the ſhades of upright objects thrown on the pavement, &c. in the picture, do alſo contribute their ſhare; and all theſe conſpiring together, do ſufficiently out-weigh the imperfections of the geometrical image of the whole: So that inſtead of a diſtorted picture, we ſee in a manner a real church; the great magnitude of the whole, its viſible concavity, and proportionable lengths and diſtances of the parts, contributing very powerfully to excite in us that idea.

In like manner, landſkip pictures, different views of buildings, &c. are alſo ſurprizingly improved by the concave ſpeculum; moſt or all of the cauſes above aſſigned, conſpiring together to make the images appear natural, as in naked viſion. Theſe phenomena appear rather more perfect to both eyes, than to one alone; and the appearance will be the ſame, to one who does not know what the picture on the cloth or paper repreſents; both which * ſhew that the ſaid phenomena are not founded upon mere prejudices, but upon the general laws of viſion in like caſes.

The pictures that receive the greateſt improvements by a concave ſpeculum.

284. Thoſe pictures appear to the beſt advantage in a concave ſpeculum, which ſtand moſtly in need of improvements; that is, thoſe which repreſent objects having conſiderable differences of apparent diſtances, from the point whence the view was taken: For the geometrical images of different parts of the picture, will be really at different diſtances. Alſo moſt pictures in general that are drawn leſs than the life, as landſkips and all ſorts of views of remote objects, are generally improved by the ſpeculum becauſe they are magnified. But portraits, and pictures of objects that lye nearly in a plane at right angles to the optic

* I have known a young child, who had never ſeen any thing like what was repreſented, ſhew great marks of joy and ſurprize, upon looking at the image of a print in a concave ſpeculum. And if it could be ſaid with propriety that nature is any where improved upon, I think, this is the place: For, if the print or picture is finely done, the opportunity we have of viewing its image without extraneous light, intruding at the ſame time into the eye, is an advantage we cannot have when we look at remote objects, and has a very pleaſant effect. For the eye being leſs tired, ſees the parts ſo much the diſtinctier, and examines them more at leiſure; and I do not know but that in ſome caſes, the image of a fine picture, wherein perſpective has been accurately kept, may convey as good an idea of the ſcene repreſented, as if we were carried to the very ſpot from whence the artiſt took his view. How uſeful and entertaining are good prints when thus improved? And what ſtrong inſtances have we here, of the excellencies and defects of painting, both at the ſame time: How lively and natural is the image of a good picture drawn in perſpective; and how faint is the reſemblance, or how little there is of likeneſs, in the picture itſelf. But though pictures for this purpoſe, ſhould be drawn according to the ſtrict rules of perſpective; it does not follow from hence, that thoſe rules are to be cloſely kept in pictures that are to be viewed only by the naked ſight.

optic axes, receive no great improvement by ſuch ſpeculums. The glaſs ſpeculums commonly ſold in the ſhops, are ſegments of ſpheres of proper ſizes enough for this purpoſe; but the pictures to be viewed in them, ſhould not be ſo broad as they are; and they generally appear beſt, when they are not much above half that breadth: For when the picture is pretty wide, the images of the extream parts generally appear too near, and ſomewhat diſtorted; which is perfectly agreeable to theory. And hence, a given picture generally appears beſt, the larger is the ſpeculum, or the longer is its focal length. The picture, as it ſtands in its place, ſhould alſo be flat; otherwiſe it is manifeſt that its image cannot appear natural. Objects that are intended to ſtand upright, as perſons talking, &c. ſhould not be drawn in a part of the picture that is intended to appear level, as in the midſt of the pavement of a church, &c. For in this caſe, as the geometrical image will be in a manner thrown flat upon the floor; ſo the apparent image, inſtead of being erect, will have an uncouth poſture, and generally be protracted into an unnatural length. To avoid theſe inconveniences, all erect figures or objects ſhould be placed near the ſides in the picture; and then the defects of the geometrical images are the eaſier overcome, by the other aids that are preſent. Theſe few hints may ſuffice as to this head.

The propereſt poſition of a picture before the ſpeculum.

285. The picture ſhould be placed nigher to or farther from the principal focus, according as the image ſhould appear more or leſs concave. That is, if the objects repreſented, have conſiderable differences of apparent diſtances from the point of ſight; the image in the ſpeculum will appear moſt natural, when the picture is placed a * little within the principal focus, as in *fig.* 56. If the picture be moved much nearer to the ſpeculum, the apparent image will be flatter or leſs concave, and ſmaller, which is exactly agreeable to theory. And ſo, as the difference of the apparent diſtances of the neareſt and remoteſt objects, is greater or leſs; by moving the picture nearer to or farther from the principal focus of the ſpeculum, we ſhall obtain that poſition wherein it will exhibit the moſt natural image.

Again, the image is beſt, and appears beſt, when that point in the picture which repreſents the remoteſt part of the exhibited ſcene, is in or near the principal axis of the ſpeculum, and the eye is placed behind a hole made round the ſaid point. But as ſuch a hole would damage

* If the picture be placed in the focus or without it, the image to a common eye, will not be ſo diſtinct, and the indiſtinctneſs will increaſe, as the picture is moved farther out; ſo that before the picture has got half-way between the focus and the center, the parts are generally loſt in confuſion.

damage the picture; if the horizontal line is near the bottom of it, place the picture higher against the upper part of the speculum, so that the image may be seen underneath it. In this case, if the speculum is vertical, the ground plane will appear ascending; the reason of which is manifest from an inspection of *fig.* 56. To remedy this inconvenience, let the speculum be inclined a little backwards, and also the picture, but not quite so much; for the picture ought always to be nearly at right angles to the axis passing through the principal point in it. But after all, if the breadth and depth of the picture be respectively above $\frac{2}{3}$ and $\frac{1}{3}$ of the breadth of the speculum, some part of the image, in most cases, will appear unnatural: For instance, the geometrical image of the roof of a church, in a picture placed and viewed as above, will be shorter than that of the pavement; and so will also the apparent image. If the picture be placed lower, and the image viewed over it, the phenomena will be the reverse, but not so natural as before.

In all cases, the horizontal line in the picture, should have an horizontal situation; and when the said line is near the top, it will be best to place the picture against the lower part of the speculum, and look over it; and so incline the speculum and picture a little forwards, for the reasons above given. Again, when the picture represents an oblique view, as of a long range of buildings, &c. it should be then placed a little sideways, and a little obliquely, and also its image viewed on that side next the middle of the speculum. From these hints, it will be easy by trials to find the propereſt positions of the pictures, in the several cases that offer. And it is also to be observed, that the picture should be no more enlightened, than what is just necessary: For the faintness of the image, contributes also its share, to enlarge its apparent distance.

286. A proper apparatus for supporting the speculum, and the pictures, in their due position, will be very advantageous; as the image will remain steady, and may be viewed at leisure without trouble or fatigue to the spectator: This may be effected by two different methods; both which I shall describe, regarding at the same time, as much as may be, the principles explained in the preceding. A box for holding the speculum and the pictures.

In the annexed figure DEGHIKLM, represents a vertical section of a box through the middle, the fore part of which ML is open. The glass * speculum BAC, whose principal axis is AF, and focus F, is represented with its wooden frame, and placed in the narrow chamber GHIK fitted for it; the top part of the space HK is left open for admitting the speculum in and out, and is shut close occasionally by the Fig. 58.

 narrow

* Speculums for this purpose are commonly made of glass, as metal ones would be very cumbersome and expensive. One that I have is a segment of about 26½ degrees.

narrow board HG. The board GK has a round hole *a b* cut in the middle of it, of the bigness of the speculum: This board serves to strengthen the box, otherwise the speculum might have been confined by narrow slips of boards. For supporting the prints or pictures; on each side of the box is a dove-tail slider, carrying each a wooden bar, as *r s*, which turn stiffly upon a pin *t* going through each of the said sliders. If the prints are stuck on pasteboards, a slit in each of the above bars will be sufficient to hold them. But as this method of stiffening the prints is inconvenient; a thin board whose width does not exceed half that of the speculum, and whose ends slide in grooves made in the bars *r s*, will answer better. The board should slide so stiff as to stand at any height, and this will be easily effected by a spring fixed at each end of it. The grooves should be also deep enough to hold the board a little slanting; that is, when one of the bars *r s*, is a little forwarder than the other. The board may remain fixed while the prints are shifted; and the prints may be kept flat to it by a narrow frame turning upon hinges, or by sliding ledges binding their ends, &c. The front ML is closed by a pair of folding doors, having at their edges about the middle a deep notch, making a transverse slit or opening long and deep enough, for looking into the box with both eyes at once. This slit is closed occasionally by a sliding board; and which, by means of the grooves on each door, serves to fasten them together. In the top ED is a square hole, which is shut occasionally by the loose board *e d*. The pictures are enlightened through the slit or † opening GE, which extends the whole width of the box, and may be made wider or narrower by two sliders drawing to each side, and with which the slit may be shut quite close. The prints may be put in and taken out, either through the front ML, or the top ED, or through the slit GE, or through a slit made for the purpose in one of the sides of the box. If the print or picture has the point of sight near the bottom of it, as is commonly the case, place it opposite to the upper half of the speculum; then ‡ incline the box backwards, till the image appears vertical, which may be easily done by placing a book under the front L, giving the bottom an inclination, as I*i*: This being done move the sliders to or fro, and if needful incline the bars *r s*, till the image appears to the best advantage. If the print requires to be placed at bottom, the

† This opening may be closed with thin canvas; but this is needless unless it be for concealing the print within, as it is easy to proportion the degree of light upon the print by a different position of the box with respect to the window.

‡ If the chamber HGIK be made about three times as wide, the proper inclination may be given the speculum without tilting the box. But this would be attended with more trouble, besides enlarging the box, and also endangering the speculum, from its unnecessarily having much room to play in.

the box muſt be inclined the contrary way, ſo that its bottom may have the poſition L *l*.

The box above deſcribed is deſignedly contrived with a view to other optical purpoſes, as will be ſeen in the next ſection; and I think it ſerves each of theſe purpoſes as conveniently, as if it was deſigned for one only. But by the above method it is troubleſome to ſhift the prints, and to keep them flat; and alſo the image is inverted as to right and left. Both theſe inconveniencies might at once be remedied, by a plane ſpeculum ſo placed as to reflect the rays diverging upon it from a print placed at the bottom of the box, upon the concave ſpeculum; and ſo as to be reflected from thence again to the eye. But in this caſe ſome part of the image will be much diſtorted, as the rays come to the concave ſpeculum very obliquely: This may in a great meaſure be remedied in the following manner.

287, The out-lines, in the annexed figure, repreſent a vertical ſection of the * ſame box, but at right angles to the former; that is, this ſection is parallel to the front ML in fig. 58. The concave ſpeculum BAC is placed near the top of the box, where it is ſupported with its face downwards, by the ledges *g*, *h*. The ſpeculum is placed a little inclining, that the rays coming to it from a picture at *a b*, may be the better reflected towards O; the principal axis being AF, and focus F. The ſpeculum is alſo placed higher than is neceſſary for a ſingle print, for the ſake of receiving a book; the print when furtheſt from it, being at about *a b*. *r s* is a plane ſpeculum; and *a c*, *c r*, *r* O; *b d*, *d s*, *s* O, are the principal rays in the pencils diverging from the points *a*, *b*, that come to the eye at O, after their reflection, by both ſpeculums. The ſpeculum *r s* need not be above 2 inches long, and $1\frac{1}{2}$ inch wide; and its frame or a little board for holding it, may turn upon a ſtiff hinge, one ſide of which may ſlip into a groove in the ſide of the box; and the proper inclination of *r s* is ſoon found by trial: The hole at O may be about 3 inches wide, and 1 inch deep. The prints are here enlightened through the front ML of the box, part of which is ſufficient for the purpoſe, or the whole might be ſcreened with thin canvas.

Another method of placing the ſpeculum and the pictures.

Fig. 59.

The ledges *g*, *h*, may have a vertical motion, which will ſerve both for giving the due inclination to the ſpeculum, and for adjuſting the diſtance betwixt it and the print at bottom: Or, the print may be raiſed to

* The top part of the box is now uſeleſs; and it might be reduced ſtill ſmaller by cutting the concave ſpeculum into a parallelogramic form. But as the box in the ſhape it is now in, not only ſerves for both the above methods, but alſo for other optical purpoſes; it ſeemed needleſs to alter it.

to the proper height by placing one or more books under it, the ſpeculum remaining fixed. The plane ſpeculum might be placed near the oppoſite ſide of the box to O; in that caſe, the picture muſt be placed near the ſide O; the plane ſpeculum muſt be wider, and the ſpeculum BAC muſt have a contrary inclination: But I think the other method is preferable. If the ſpeculum BAC be more inclined, the print *a b* may be brought nearer to the middle of the box; and the contrary.

Both the methods above deſcribed of placing the ſpeculum and the prints, may be uſed very readily in the ſame box; but I think the laſt method of placing the prints at the bottom, is upon the whole the moſt convenient. Thoſe who are fond of ſurprizing unſkilful ſpectators, might, by either method eaſily conceal the whole contrivance, and ſhift the pictures unobſerved *.

PROP. XXII.

To find the focal length of a ſpherical ſpeculum, by experiment.

There are different methods of ſolving this propoſition, which are as followeth.

By the chord and verſed ſine. Fig. 60.

288. Let O be the center, OA the principal axis, and F the principal focus of the ſpeculum BAC. If the ſpeculum is concave, lay the edge of a ſtrait ruler B*v*C over the middle of the ſpeculum, and meaſure exactly the length of the chord BC, and of the verſed ſine A*v*: But if the ſpeculum is convex, the ruler muſt be laid as at *a b*, ſo that the diſtances *a*B, *b*C, betwixt its edge and the verge of the ſpeculum, are equal. And in both caſes, it will be;

$vA\ (= Ba \text{ or } Cb) : vB\ (= vC \text{ or } \tfrac{1}{2}ab) :: vB : vO + AO$; to which adding *v*A, the fourth part of that ſum will be = AF the focal length

* Dr. *Graveſande* deſcribes a box for concealing a concave ſpeculum, in order to ſurprize a ſpectator ſtanding at a proper diſtance, and looking through a large hole in the front of it, by exhibiting his inverted image ſuſpended near that hole. Our box will alſo ſerve for the ſame purpoſe. But the doctor takes no notice of the phenomena of pictures ſeen in ſuch ſpeculums; nor have I met with any account of theſe phenomena any where, which made me the more particular in endeavouring to explain them. By the beſt information I could procure, the phenomena above-mentioned were not obſerved here till about the year 1730; and it might be about 6 or 7 years later before they were commonly known. Mr. Felix Clay at *Aldgate*, I am informed was the firſt here who made a box for the above purpoſe, the hint of which, he told me, he took from a ſmall kind of ſhow-box or toy brought from *Holland*; and this was in the year 1741. Soon after theſe things became known to the printſellers, and boxes were copied from Mr. *Clay*'s; theſe were different, tho' upon the ſame principles from that above deſcribed. Mr. *Clay* I believe was alſo the firſt who placed the ſpeculum at top, as in fig. 59. But theſe boxes were not calculated for ſerving any other purpoſe than that one; and large ſpeculums being expenſive, theſe ſhew-boxes have of late been laid aſide; to give room for another ſort with a convex lens, which ſhall be deſcribed in the next ſection.

length required. But this method will not admit of great exactness, because a small error in the length of A*v*, will produce a great one in AF.

289. Place the speculum so that its principal axis may point very near towards the center of the sun, or towards the moon when it shines bright: If the speculum is concave, find the burning point, or receive the image upon a white piece of paper; in either case, the distance between the focus so found, and the vertex of the speculum is the focal length required. Or, cover the speculum with a sheet of opaque paper, in which make two or more holes, as A, D; and by observing where the beams of light reflected from these holes unite, the focus may be found rather more accurately, than when the whole speculum is exposed. By exposing the speculum to the sun.

If the speculum is convex, it must be either covered, as above, leaving two holes as A, D; or leaving the speculum naked, stick on it two round opaque patches, A, D; then holding a white paper as GH perpendicular to the principal axis of the speculum, observe where the shades of the patches A, D fall upon it, as at G, H; measure exactly the distance AG, and the distances betwixt the centers of the shades G, H, and betwixt the centers of the patches A, D; and it will be, GH — AD (= *e*H) : AD :: AG (= D*e*) : AF. For D*e* is here parallel to FAG, and GH to DA, and so the $\triangle^s$ D*e*H, FAD are similar. In this case, it would be best to describe circles at G, H beforehand, whose diameters and distance betwixt their centers, shall be in some given ratio to the diameters and distance betwixt the centers of A, D: Suppose this ratio to be as 2 to 1, then AG = AF; if it be as 3 to 1; AG will = 2AF; for FG : FA :: GH : AD.

The focus of a concave speculum may be found with rather more ease and exactness, by the following method: Because here it will be troublesome to hold the speculum in its due position, unless it be furnished with an apparatus for the purpose; and the burning focus, or place of the distinct image of the sun or moon, admits of some latitude.

290. At the end of a long table, place the speculum in a vertical position, or so that its principal axis AOQ may be nearly parallel to the table; then place a candle towards the other end of the table, so that its flame Q may be opposite to the vertex of the speculum. This being done, take a clean piece of white paper, and having fixed it at the end of a stick, place this stick in the socket of another candlestick, so that the paper may be supported at about the same height with the candle; then move the paper or the candle to and fro, till the image *q* Focus of a concave speculum found by the image of a candle. Fig. 61.

of the candle appears moſt diſtinct upon the paper: If the candle and the paper are not nearly in the principal axis AqQ, they muſt be moved a little, as occaſion requires. And having meaſured exactly the diſtances Aq, AQ; becauſe,

[a] 278. II. [b] [94]. I. [c] 94. I.

$$AQ : Aq :: {}^{a}OQ : Oq \; (:: {}^{b}OF : Fq) :: {}^{c}QF : OF;$$ it will be,

$$AQ + Aq : Aq :: Qq\,(QO + Oq) : Oq :: AQ\,(QF + OF) : AF = OF.$$

And ſo either qO or AF gives the focus required. The ſame may be found without calculation, by moving the candle till its image hangs over it; or by obſerving where the end of a ſtick and its image touch one another; in both caſes, the point of coincidence is the center of the ſpeculum.

Fig. 60. The focal length of a convex ſpeculum may be alſo found by the light of a candle, thus: The ſpeculum, candle and paper being placed upright, as in the preceding; let the ſpeculum be covered with paper, having in it two round holes A, d; and let the candle and one of the upright ſides of the paper, be nearly in the principal axis AQ, the candle being placed hindmoſt, ſo that none of its direct light falls on that ſide of the paper which faces the ſpeculum. Then having found the places of the white circles g, h, upon the paper, and having meaſured the diſtances Ad, Ag, gh; the point of interſection q, of the lines gA, hd produced, may be found as in the preceding; and calling the flame of the candle Q, it will be in this caſe; $AQ - Aq : Aq :: Qq : Oq$.

A more exact method of finding the focal length of a concave ſpeculum.

Fig. 61. 291. If the greateſt nicety be required: The ſpeculum being placed in a vertical poſition, as in the preceding, find its principal axis, and alſo its center O, which may be done near enough by moving a candle till its image coincides with it. Before a convex lens a of about one or two inches focal length, and to a proper frame for ſupporting it, fix a thin opaque plate b with its edge upright, and a little ſide-ways of the axis of the lens, and alſo at ſuch a diſtance that the ſaid edge appears diſtinct through the lens. The edge of the plate being placed near to the center O, and the axis of the lens nearly in the principal axis of the ſpeculum; move the lens a little to or fro along this axis, till the edge of the plate and its image from the ſpeculum, are both ſeen diſtinctly through the lens, at the ſame time, and alſo very near to one another. Now the diſtance betwixt the ſaid edge and the vertex of the ſpeculum, is exactly twice the focal length required. Note, during this experiment, the ſpeculum muſt be in the dark; that is, it muſt be either ſcreened in a corner of the room by a tube or otherwiſe; or day-light muſt be excluded from the room altogether. In this laſt caſe,

the plate muſt be illuminated by a candle placed a little ſide-ways before it, with a ſcreen to prevent its light falling directly upon the ſpeculum.

The plate above might be perforated near the edge into ſeveral holes of different ſizes; ſo that the holes and their images might be ſeen at the ſame time: Or the objects might be ſmall wiers fixed to the frame of the lens.

292. *Schol.* 1. A ſpeculum free from ſpecks, fiſſures, *&c.* is *cæteris paribus* preferable to another not free from theſe blemiſhes: The goodneſs of the poliſh may be eſtimated from the luſtre of the ſpeculum, but with more certainty from the brightneſs of the images. But the principal thing to be ſought after is the goodneſs of the figure; and this may be judged of in the general, by the likeneſs of the images. But the laſt preceding method for finding the center of a ſpeculum, affords alſo a pretty critical one for examining its figure; and that by obſerving the degree of diſtinctneſs of the ſmall holes in the thin opaque plate, with the ruggedneſſes, and duſts, *&c.* ſticking to them, compared with theſe objects ſeen directly through the lens; both being ſo brought as to be ſeen at about the ſame diſtance from the middle of the lens. Let a ſmall round hole in the thin plate be placed near the center of the ſpeculum, and place the candle in the axis behind it: If the ſection of the rays received upon a white paper, placed a little before or behind their focus, does not appear circular, but oval, ſquariſh, *&c.* it is a ſign that the ſpeculum has not the ſame curvature throughout its ſurface. If the light received upon the paper, when placed before the focus, is brighter towards the circumference, and darker about the middle, than when the paper is placed at the ſame diſtance beyond the focus; it is a ſign that the ſpeculum is flatter about the middle, than towards the circumference: And if the phænomena be the contrary, the ſpeculum is flatteſt towards the circumference. Inſtead of receiving the rays upon the paper, an eye-glaſs might be uſed, moving it a little to and fro along the axis, the perforated plate and the candle remaining fixed *. By ſuch trials as theſe, which may be alſo a little varied, a perſon who has had experience will be able to judge pretty nearly what figure the ſpeculum is of: And it will be a farther help, if trials be made by different zones of the ſpeculum, the reſt of it being covered. If the ſpeculum is for a teleſcope, it muſt be

Goodneſs of ſpeculums, how to be examined.

* *John Hadley*, Eſq; who by his ſuperior ſkill and unwearied pains was the firſt who brought reflecting teleſcopes to any degree of perfection, is the author of the above method of examining the figure of a concave ſpeculum.

be farther examined by the effect, when joined with its proper appurtenances.

293. *Schol.* 2. If it be required to find a point in the axis of the speculum, at which an object being placed, its diameter shall be to that of its image in any given ratio, as that of a to b; a being always greater than b, if the speculum is convex. Make $b : a :: AF : FQ$; and the point Q will be that sought.

Fig. 45. to 51.

For $FQ : AF ::$ [a] $OQ : Oq ::$ [b] $a : b$. Therefore, *&c.* *Q. E. D.*

[a] 290. II.

[b] 111. I.

294. *Schol.* 3. Both convex and concave glass speculums, quick-silvered on one side, will have double images, as has been shewed of plane looking-glasses; and the places of each of these images may be found by some of the rules in the preceding book. But as these glasses are usually so thin, that these double images are scarce ever perceived, it would be needless to exemplify the manner of finding their places here. However, without regarding the reflection by the first surface, it will not be amiss to conclude this section, with some observations concerning the effects of glass lenses, when used as quick-silvered speculums.

Fig. 62. Let the plano-convex lens AB be quick-silvered on the plane side, and parallel rays will have a focus Q before the lens. Let O be the center of the spherical side AB: Now a ray DB parallel to the axis will be refracted towards f, so that $AO : Af ::$ [c] $m - n : m$; and at b the ray will be reflected towards q, so that $aq = af$. Again, at c the ray will be refracted at its emersion out of the glass towards Q, so that $fq : Of ::$ [d] $Aq : AQ$. If the thickness of the glass lens be neglected, its focal length in this case as a speculum, will be to its focal length as a lens, as 1 to 2. But because of the thickness aA, the focal length AQ as a speculum, will be shorter and aQ longer than in the above proportion.

[c] 126. I.

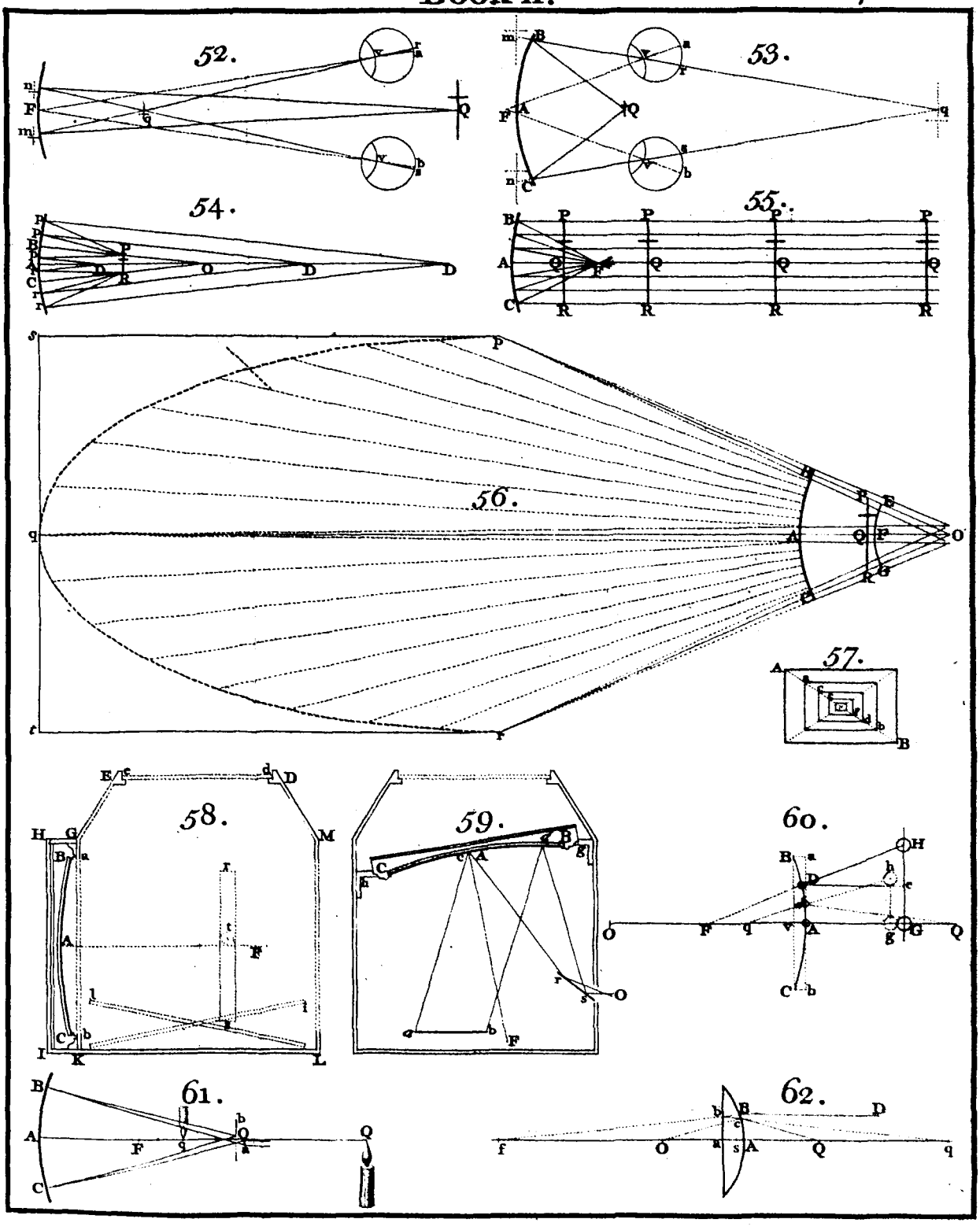
[d] 144. I.

It is manifest, that if the convex side of the lens be quick-silvered, it will have a real focus, and nearer to it than the former. Also all *double convex lenses*, quick-silvered on either side, will have a focus before them: And if the radii of the sides are equal, the focal length in that case will be $= \frac{1}{4}$ the focal length of the lens, nearly †.

A *plano concave lens* will have its focus behind it, which ever side is quick-silvered; and consequently so will likewise all *double concaves*, and *concavo-meniscus's*, when the concave side is quick-silvered. A *concave meniscus*, the radii of whose sides are as $m - n$ to m, when the rays are incident upon the concave side, has no focus: This is [e] manifest, because an incident ray parallel to the axis, will be refracted perpendicularly

[e] 126. I.

† Supposing the glass lens, fig. 62. to have the side ab of equal convexity with AB, and neglecting the thickness, the center of this side will be in Q; then the focal distance from A of the ray reflected at b will be $= \frac{1}{4}$ AQ (by the rule 94. I;) and the focus of the emergent ray will be $= \frac{1}{4}$ AQ, by 144 I.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cularly to the reflecting side. And when the said proportion of the radii of the sides is nearer to an equality, or more unequal, the lens will accordingly have either a focus on the concave or convex side.

But because of the great aberrations by the reflections and double refractions, lenses are very unfit for speculums, unless it be for illuminating objects, for which purposes those that have their focus's before them, may answer indifferently well.

SECTION IV.

Of vision through given mediums: of pictures seen through convex lenses; and a description of different sorts of camera-obscura's.

§. 1. *Of vision through mediums, terminated by plane sides; and some farther observations relating to the refraction of light.*

PROP. XXIII.

295. *An object seen through a medium, terminated by plane and parallel sides, appears pretty nearly in the same plane, and of the same magnitude, and exactly of the same figure, as it would do to the naked eye in the same position, the medium being removed.*

For the geometrical image *p q r*, is [a] erect, equal, and parallel to the object PQR; and the rays here undergoing but small refractions, the proper foci of the pencils of light that enter the eye will be very nearly in *p q r*. The image also is very near to the object; for the distance Q*q* between them is to the thickness of the medium, only in the ratio of $m - n^2$ to m. And although the pencils of light CO, DO, at their entrance into the eye diverge, as if they came from the focus's *p*, *r*, and consequently form a greater angle than if they proceeded directly from P, R; yet if the image appears in its true place, it will appear [b] equal to the object. Therefore, *&c. Q. E. D.* Fig. 63. [a] 79. I. [b] 195. II.

296. *Schol.* 1. We seldom see objects through mediums that are very thick: The thickness, for instance, of a coach-glass in the case before us, being very inconsiderable. But an object, by being rendered more obscure by the medium, may appear [c] farther than otherwise it would; [c] 188. II.

 and

[d] 198. II. and in that case it will also appear [d] larger: So that the phenomena of vision through such mediums vary, as the circumstances are different. It may be farther observed, that the nearer the object and the eye are to the medium, the greater, *cæteris paribus*, will be the effect; because the distance Q *q*, between the object and its image, is invariable.

Vision of objects in a dense medium.

297. *Schol.* 2. Objects placed in a dense medium, whose refracting surface is plane, appear erect, and nearer to that surface than their true place; but in most cases, they appear somewhat distorted. We have frequent opportunities of observing these phenomena, by looking into smooth and clear water; and the very near agreement of the phenomena with the [e] theories of the images in the present case, is another notable proof of the great use we make of the different conformations of our eyes, and the inclinations of their axes, according to the different places from whence the rays diverge, in our estimations of the said places, where those places are not very remote. For what else have we here to direct us? The greater obscurity of the objects, and our knowing perhaps from experience that the water is deeper than it appears, are causes that here tend to deceive us; and yet we find that they have little or no effect. The phenomena are also the same to persons who have no skill in optics.

[e] 62. to 66. I.

PROP. XXIV.

298. *To explain the phenomena of vision through a medium, whose refracting sides are planes inclined to one another.*

Phenom. An object seen through such a medium, appears in a new direction, and generally in a different position, somewhat indistinct, distorted, and coloured; and the greater is the refracting angle of the medium, the greater will be these alterations.

Fig. 64.

Let PQ be a rectilinear object, seen by the eye at O, through the triangular prism RST. Now the geometrical image of the object is *p q*; but because of the great obliquities of the rays to the refracting sides of the prism, the apparent image will not be *p q*; and here, in order to define the apparent image, it will be necessary to find the proper foci of the extream rays, at least, in each pencil. The dotted lines represent the [a] method of finding the proper foci of the rays Q *c*, Q *e*, after their refractions at each side of the prism; those rays being supposed homogeneal, and their sines of incidence and refraction as the numbers 3, 2. Thus *g*, *h*, are the proper foci of the rays *c d*, *e f*; and *m*, *n*, the proper focus's of the rays *d* O, *f* O; and so upon the above supposition, the point *b*, where the rays *d m*, *f n* intersect, would be the place of the visible image

[a] See 57. I.

image of the point Q. In like manner was alſo found the point *a*, the place of the viſible image of P. But as the pencils of light conſiſt of heterogeneal rays, which impinge the ſides of the priſm very obliquely; they will be here ſeparated, eſpecially at their emerſion out of it, into their different kinds: The image will therefore neceſſarily appear [b] coloured; it will be indiſtinct, becauſe the rays in the ſeveral pencils have different foci, as they differ in refrangibility, and have different inclinations to the refracting ſurfaces. The rays entering the eye being produced backwards, will form a kind of a compound image, which will be diſſimiliar to the object, and which may alſo appear in a different place and poſition from *a b*; and becauſe of the indiſtinctneſs, the apparent place and poſition may be different at different times of looking, the eye, priſm, and the object retaining their former places and poſitions. Therefore, *&c. Q. E. D.* [b] 283. I.

299. *Schol.* In like manner, if a medium has ſeveral plane ſides HK, E, D, A, B, C, inclined to one another, ſo as to form the triangular priſms BFK, CHK, DGH, EKH, and the light proceeding from a ſmall object Q, be tranſmitted through each of the ſaid priſms to the eye, as at O; an image will be ſeen through each priſm, and ſo there will be as many viſible images in the whole, as there are priſms, together with one more ſeen through the ſides A, HK, if theſe are parallel. And hence a glaſs having plane ſides cut in this manner, is called a MULTIPLYING-GLASS. Of multiplying glaſſes. Fig. 65.

The image *a* ſeen through the parallel ſides A, HK, will be diſtinct, equal, and ſimilar to the object Q, and alſo pretty near it, the diſtance betwixt their centers being $= \frac{1}{3}$ thickneſs of the glaſs AHK. The images ſeen through the common ſide HK, and each of the ſides B, C, D, E, will be reſpectively *b*, *c*, *d*, *e*. And theſe will be all of them ſomewhat indiſtinct, diſtorted and coloured; and the more ſo, the more remote they are from the middle image *a*, becauſe the priſms through which they are ſeen have larger refracting angles.

The diſtance OQ being given, the greater is OA, the leſs will be the angles Q O *b*, *b* O *c*, *&c.* but the more indiſtinct and coloured will be the extream images; theſe images appearing beſt, when the incident and emergent rays form equal angles with the reſpective refracting ſides: And that will happen, if the flat ſide be towards O, when the glaſs is much nearer to O than to Q. When OA is given, the greater is AQ, the greater alſo will be the angles Q O *b*, Q O *c*, *&c.* and the more remote from the middle of the glaſs, will be the incidences of the pencils coming to the eye. And hence the diſtance OQ being given, the diſtance OA, at which an object may be ſeen through each ſide of a given glaſs

glaſs, is limited; that is, OA muſt not exceed a certain given diſtance, or be leſs than another aſſigned diſtance. If the glaſs be turned round the perpendicular line QO to the ſides A, HK; the image *a* will appear immoveable, whilſt all the reſt ſeem to turn round with the glaſs.

LEMMA.

300. If a ray of light be tranſmitted through ſeveral contiguous mediums terminated by parallel planes, out of a given medium into the ſame again, as out of air, through water and glaſs, into air again; the emergent ray will be always parallel to the incident ray. The truth of this * is manifeſted by experiments, and the reaſon of it may be illuſtrated as followeth:

Fig. 66. Let AB be an incident ray, W, G two mediums of different denſities, as water and glaſs, and terminated by the parallel planes BP, KO, LF; and let the ray be refracted at B into B*b*, at *b* into *bf*, and at *f* into *f*D: I ſay, *f*D is parallel to AB. Draw the perpendicular B*n* to the refracting ſurfaces; and having from the center B, with the radius B*b*, deſcribed the arch *n b d*, produce AB till it interſects this arch in *d*, and through the point *d* draw *d m* perpendicular to B*n*. Now *m d* is the ſine of incidence, and *w b* the ſine of refraction of the ray AB into the medium W. Let *g a* be the ſine of refraction from air immediately into the medium G, the ſine of incidence being *m d*; through the point *a*, draw the line B*a e*, and B*a e* would have been the refracted ray, if the refractive power of the medium W had been equal to that of G; that is, if the incident ray IK be parallel to AB, B*e* will be parallel to the refracted ray KL.

Now it is reaſonable to ſuppoſe, and it ſeems very evident, that the refraction between the mediums W, G, muſt be equivalent to the exceſs of the refraction of G above that of W, when rays come with a like obliquity out of the ſame medium into each; that is, the ſines of incidence and refraction out of W into G, are as *w b* to *g a*. And upon this ſuppoſition, the refracted ray *bf*, will be parallel to B*e*. Draw *b i* perpendicular to the refracting ſurfaces, and from the center *b* with the radius *bf*, deſcribe the arch *i f h*; produce B*b* till it interſects this arch in *h*, and draw *hk* parallel to *of*. Now becauſe *w b* : *g a* :: [a] *k h* : *of* and the ∠ *o b h* = [b] ∠ *w* B *h*, the ∠ *o b f* is = *w* B *e*; therefore *bf* is parallel to B*e*; but B*e* is parallel to KL, and conſequently *f*D is parallel to LM; but LM is [c] parallel to IK, and IK is [b] parallel to AB, and ſo *f*D * is parallel to AB. *Q. E. I.*

[a] Hyp.
[b] Conſt.
[c] 67. I.

In

* See *Newton's* Lect. Optic. par. I. ſect. 2. N°. 33.

In like manner, a ray EF immerging first into the denser medium G, will emerge at x out of W, in the line xH parallel to EF. Produce EF to t; and let the sines of incidence and refraction from air to glass, and from air to water, be respectively as the perpendiculars or sines t, r, s. Now Fr is the refracted ray; and it would have been Fsy, if the refraction of G had been equal to that of W. At r the ray is refracted from Frv into rx; and for the same reason that bf is parallel to Be, rx is also parallel to Fy, and xH is parallel to EF. *Q. E. D.* If EF be parallel to Df and to ON; Fr will be parallel to fb, and rx will be parallel to bB and to OP.

301. *Corol.* Hence, if a ray passes through ever so many refracting mediums terminated by parallel planes, it will have the same inclination to the surfaces of the last medium, as if it had suffered but one refraction in going immediately out of the first medium into the last. Thus bf is parallel to Be and KL, and rx to Fy and OP. And the case will be the same, though the number of different mediums be increased to infinity, and the intervals of their parallel surfaces be as much diminished; so that the ray may be continually refracted into a curve line.

PROP. XXV.

302. *The proportion of the sine of incidence to the sine of refraction of one and the same sort of rays out of one medium into another,* is, *as the rectangle under the sines of incidence out of the first medium into a third medium, and out of that third medium into the second; to the rectangle under the sines of refraction out of the first medium into the third, and out of the third medium into the second.*

Let as before, ABbfD be the course of a ray from a third medium through the two refracting mediums W, G into the same third medium again; and let SBn, Zbi, VfT, be perpendicular to the refracting surfaces at the points B, b, f. Also, let the proportion of the sine of incidence to the sine of refraction out of the third medium, as air for instance, into the medium W, be as F to f, i. e. S.∠ ABS (= wBd) : S.∠ wBb :: F : f; and out of air into the medium G, as G to g, i. e. S.∠ TfD : S.∠ Vfb :: G : g; and let the sine of incidence to the sine of refraction out of the medium W into the medium G, be as x to y, i. e. S.∠ BbZ : S.∠ obf :: x : y. I say $x : y :: f \times G : F \times g$. Fig. 66.

Because S.∠ ABS : S.∠ wBb :: [a] F : f :: F × G : f × G; and S.∠ Vfb : S.∠ TfD = [b] ∠ ABS :: g : G :: g × F : G × F; it will be S.∠ wBb : S.∠ bfV (= obf) :: f × G : g × F :: x : y :: wb : ga. *Q. E. D.*

[a] Hyp.
[b] 300. II.

Otherwise

[a] Hyp. Otherwise thus. Because $wb : md :: {}^{a}f : F$; and
$md : ga :: {}^{a}G : g$; therefore
$wb : ga :: f \times G : g \times F :: x : y$. *Q. E. D.*

And thus the refractions between any two mediums, and a third medium being known, the refractions between them both may be readily found. As if the sine of incidence of any ray out of water into air, be to its sine of refraction as 3 to 4; and the sine of incidence of the same ray out of air into glass, be to its sine of refraction as 17 to 11; the sine of incidence of that ray out of water into glass, will be as 3×17 to 4×11, or as 51 to 44: Or if the sines of incidence and refraction between air and glass be as 3 and 2, the sines between water and glass will be as 9 and 8. For $3 \times 3 : 4 \times 2 :: 9 : 8$.

303. *Schol.* If a medium terminated by parallel planes, be contiguous to another medium whose refracting sides are inclined planes; and a ray of light be transmitted through both the mediums, and another homogeneal ray whose incidence is parallel to the former, be transmitted through that medium only whose sides are inclined; the two emergent rays will be parallel to one another, and inclined to the incident ray.

Let W be the medium, whose refracting sides are parallel planes; and let the inclined sides of the medium G be K*r*, LR. Now if the incident homogeneal rays be AB, IK, parallel to one another; because the rays *bf*, KL incident upon the plane side LR [b] are parallel; the emergent rays will be also [c] parallel, but inclined to LM, *f*D, because LR is inclined to LF. If the emergent rays be conceived to flow back again, so as now to become the incident rays, [d] they will proceed along the same tract they did before, and so the emergent rays BA, KI will be parallel. In both cases, therefore *&c. Q. E. D.*

[b] 300 II.
[c] 16. I.
[d] 13. I.

It is almost needless to observe, that the case will be different if the three refracting sides are all inclined to one another. Let the prism RST be conceived to consist of two mediums RS*v*, TS*v*, of different densities; it is manifest that the emergent rays will be inclined to those in the figure, whether those within are perpendicular to S*v*, as *cd*; or are inclined to it either way, as *ef*, *rs*.

Fig. 64.

§. 2. *Of vision through lenses; and of finding their focal lengths by experiments.*

The effects of concave and convex lenses, are, in the general, analogous to those of convex and concave speculums respectively; but as speculums and lenses have some distinct properties, they deserve to be explained apart. PROP.

PROP. XXVI.

304. *An object seen through a concave lens, always appears erect, as it doth also thro' a convex lens, if the geometrical image is not betwixt the lens and the eye: If the focal lengths of the lenses are but small; the image through a concave lens appears diminished, and nearer to the lens than the object; and through the convex lens, it appears magnified; and if the rays come to the eye diverging, the image will appear farther than the object.*

That the image, in the above cases, does necessarily appear erect, is self-evident: And as the eye can never come to the center A of the lens, but must be at some distance behind it, suppose at G; the object seen by the naked eye, and its image through the lens always form different angles, these angles being as PGR, to *p*G*r* or *i*G*o*. And consequently, in the first case, the object appears diminished, and in the second magnified; and when the focal length AF is but small, and the object, as in *fig.* 68. is between F and the convex lens; the image through the concave lens appears nearer to it, and through the convex lens farther from it, than the object. The reasons of all which phenomena are very manifest. *Q. E. D.* Fig. 67, 68, 69.

305. *Schol.* 1. When we look through lenses having long focus's, the image appears pretty nearly at the same distance off, that the object does to the naked eye. For in this case, the differences of the divergencies of the refracted rays and of the unrefracted ones, are quite imperceptible; and so, there being nothing here to direct us otherwise, we naturally look for the object thro' the lens, in the same place the naked eye saw it just before. Again, this deception as to the place of the image, is farther favoured by the difference of the visual angles; the visual angle being diminished by the concave lens, and inlarged by the convex one, assist the imagination, in both cases, to bring the image nearer to the object. In these cases, the apparent magnitudes of the object to the naked eye, and through the lenses, are as the visual angles PGR and *p*G*r*. But in looking through lenses having short focuses, if the geometrical image by the convex lens is erect, and at a moderate distance off, the true places of the images are perceived, at least very nearly; and then the apparent magnitudes of the object and its image, are nearly as their [b] true magnitudes; that is, as PR and *pr*. And I think it holds universally, as well as can be judged by experiments, and as has before been taken notice of, that *The apparent magnitudes of objects and their images, are as the rectangles under their apparent distances and visual angles respectively.*

An estimate of the proportion in which objects appear magnified or diminished by lenses.

[b] 198, 218, & 221. II.

Lenses having long focus's do not sensibly alter the apparent magnitudes of objects.

306. *Corol.* 1. Hence, lenses having long focuses, have scarce any sensible effects in magnifying or diminishing objects; and the nearer they are brought to the eye, the less will be their effect; because at the center of the lens, the object and its image subtend the same angle. Nevertheless such lenses may be useful spectacles, as their causing a different divergency of the rays of light may contribute towards a more distinct vision.

Objects appear darker through concave, and brighter thro' convex lenses.

307. *Corol.* 2. An image seen through a concave lens appears darker, and through a convex lens brighter, if it be beyond the lens, than the object; neglecting in this last case the consideration of the loss of light by the lens. If *ab* be the breadth and place of the pupil; it can receive no more light in the first case from the point P, than what is contained in the slender pencil P*st*; but in the latter case, the pencil P*st* is much broader than the former. In the one case the rays diverging more, and in the other less; the pencils entering the eye in the first case are rarer or contain fewer rays, and in the latter they are denser or contain more rays, than if these pencils had come directly from the object.

Fig. 67, 68.

A concave lens placed close to the eye causes indistinctness.

308. *Schol.* 2. An object seen through a concave lens placed close to a common eye, always appears indistinct; and this indistinctness will be the greater, the shorter is the focal length of the lens. But if the eye recedes back to a certain distance from the lens, the object will become distinct.

When the focal length of the lens is so long, that the image appears as far as the object; the indistinctness is owing to that mistake which causes a wrong conformation of the eye. But by retreating farther from the lens, the proportion of the true and apparent distances of the image become nearer to an equality; and so the indistinctness vanishes by degrees.

Fig. 67.

Again, when the focal length of the lens is short, the object appears indistinct, because the rays diverge too much. Thus, if *qa* be the nearest distance of distinct vision, and *ab* the smallest breadth that the pupil of the eye can be contracted to; the reason is manifest, why vision will be distinct when the eye is at *a*, and that it must be confused, if the eye approaches nearer to the lens than the said distance A*a*.

An object seen through a concave lens may be indistinct through the want of light.

309. *Schol.* 3. If the eye continues to recede farther from the lens than the point *a*, the object will continue distinct, if it be sufficiently bright. For the image being near a concave lens having a short focus, does also always appear near it, and not far from its true place; the lens itself, in this case, being as it were an object, to which the eye conforms according to the different distances. But as the eye recedes

farther from the lens, the image becomes continually darker; and it may become so dark, as to be indistinct upon that account.

310. *Schol.* 4. If a convex lens, having a short focus, is not placed at such a distance from an object, as to cause its erect image to be as far from the lens, as the nearest distance of distinct vision; the object will be confused to an eye placed close to the lens. As if *qa* be the nearest distance of distinct vision; the image *pqr* will be necessarily indistinct to an eye placed nearer the lens than the point *a*. But if the eye retreats to *a*, the said image will become distinct, and so it will continue as the eye recedes farther back, but diminished in its brightness. But if A*q* is not less than the nearest distance of distinct vision, the object will appear the distincter the nearer the eye is to the lens, because then it will receive the more light.

An object may be indistinct through a convex lens by being placed too near it.

Fig. 68.

311. *Schol.* 5. The geometrical image *pqr*, of a strait or plane object PQR placed perpendicular to the principal axis of the lens, is [c] concave towards a concave lens, and [c] convex towards a convex lens. This curvity of the image, appears very sensible to both eyes looking through a broad lens having a short focus; but to one eye it is not so apparent; and sometimes the image will appear bent the contrary way, as *wxy*.

A flat object seen through a lens appears spherical.

Fig. 67, 68.

[c] 239. I.

In all cases, the middle of a broad image appears distinctest; and our not being so well able to distinguish different degrees of distance with one eye alone, the different distinctness of the middle and extream parts, seems to be the medium that causes our mistake as to the true figure of the image. In the first case, the image being diminished and obscure, seems to be what leads us to fancy that the rays from the extream parts, which are the most obscure, diverge from a greater distance than those towards the middle. And in the latter case, the greatness of the image seems to be what causes our falling into a contrary mistake; that is, the indistinctness of the extream parts seems to be now * imputed to their being too near, as they were before to their being too far, for distinct vision. But in both cases, by looking more attentively, these mistakes are frequently corrected.

312. If the eye be between the lens and the image *pqr* of an object PQR, that object will appear erect, magnified, and confused; and the farther the eye is from the lens, or the nearer it is to the image, the greater will be the confusion.

Vision thro a convex lens when the object is beyond the principal focus.

* It is true we are not sensible or conscious of these reasonings, but this is no more than what often happens to us in other cases. Ideas excited by our sensations are frequently productive of others that were originally acquired from reasoning; and being thus once acquired, they follow the others so quick as not to allow us time to reflect upon them, or to distinguish any interval betwixt their growth and the sensation that causes them.

Fig. 69. Thus, if the eye be at G, it ſees the points P, R, by rays converging towards *p*, *r*; the middle rays in theſe pencils being the lines G*p*, G*r*, which produced backwards are the lines G*i*, G*o*; and ſo, the middle rays of the pencils proceeding from every point of the object PQR, will be refracted to the eye at G in lines, which if produced backwards, will lie orderly between G*i* and G*o*; and therefore the object appears erect, where-ever the point G is taken between the lens and *pqr*. Again, the viſual angle *i*G*o*=*p*G*r* being greater than the angle PGR; the image appears magnified, if it appears as far as the object, which whilſt the eye is pretty near the lens, is generally the caſe. The object to a common eye appears indiſtinct, becauſe the rays of the ſeveral pencils are united in the eye before they reach the retina; and ſo upon the retina the rays from the adjacent points of the object are mixed together; as in the caſe of viſion of remote objects to ſhort-ſighted perſons. As the eye retreats towards *q*, the confuſion increaſes; and before it has got to *q*, the object will be quite loſt, and nothing will be viſible but a glare of light. During this retreat, the viſual angle is conſtantly increaſing, and that the faſter, the nearer the eye has got to *q*; this angle being always equal to *p*G*r*, wherever the place of the eye or the point G is taken. As the confuſion increaſes, the object will ſeem to quit its place, and approach towards the lens, at which it will at laſt ſeem to ſtick. In this caſe, as we are deprived of the ordinary means by which we judge of diſtances, the greater confuſion and the greater viſual angle it ſeems are the cauſes of ſuggeſting the nearer diſtance; as in naked viſion, the confuſion increaſes the more we approach a near object; though in the two caſes, the *foci* of the pencils, after their refractions into the eye, are on different ſides of the retina. But the lens itſelf being an object, between us and which, we perceive nothing; the object looked at through the lens, appears no nearer than the ſurface of that lens.

As the eye retreats beyond *q*, a large, confuſed, and inverted image will begin to appear: During this retreat, the image will continue all the way inverted, and the confuſion will gradually leſſen, till the eye has got as far as the neareſt diſtance of diſtinct viſion, which ſuppoſe to be *qa*. When the eye is at *a*, or farther back than *a*, the image will appear diſtinct, if it appears alſo in its true place, otherwiſe not. The apparent place of the image is at firſt generally near the lens, and the eyes conforming themſelves accordingly, viſion is neceſſarily indiſtinct; but if the object is bright enough, by looking attentively towards the true place of the image, we ſhall ſoon perceive it in that place, and then, and not till then, it will appear diſtinct; the image will alſo appear convex towards the eye, according to its true figure, if

if the object is a plane facing the lens. The reason of all the above phenomena are very manifest from an inspection of the figure; and it is also manifest, that the field of vision through a given aperture *g h* is constantly decreasing as the eye retreats backwards from the lens; and when it is a pretty way off, it will be difficult to find the inverted image, for the eye must be pretty near the axis before it can meet the rays. At whatever distance the eye is from the image it appears erect or inverted; and its apparent magnitude, as in other cases, will be as its visual angle multiplied by its apparent distance.

313. The image *q* of a small object Q, will appear double, as at *m* and *n*, when the axes *d v*, *e v* of the eyes are directed to a point G beyond *q*; which is a case that frequently happens, and the reason of the phenomenon is very obvious from an inspection of the figure, and from what has been [d] before explained. The image may also appear double when the spectator is before it, if the lens is broader than the distance between the two pupils of the eyes. This case is the reverse of the preceding, the image *s*, *t* being now on contrary sides of the optic axes *d v*, *e v*, as when we look short of the object: But this phenomenon of a double image seldom happens. When an image appears double. [d] 59. II.

Some farther observations relating to vision through lenses, will occur under the following propositions *.

PROP. XXVII.

314. *A long object placed nearly parallel to the axis of a convex lens, and whose farthest end is within the principal focus, appears through the lens protracted to a great length: Also a series of small objects placed one behind the other in a line near the axis, the remotest being within the focus of the lens, appear likewise extended to great distances.*

The geometrical image of the long object AB or CD, after refraction by the lens O whose principal focus is F, is [a] *a b* or *c d*: And the geometrical images of the equal objects AC, GG, HH, II, KK, LL, BD, placed parallel to, and at equal distances from, one another, are [a] *a c*, *g g*; *h h*, *i i*, *k k*, *l l*, *b d*. In many cases, the divergencies of the rays in the pencils entering the eye, are not alone sufficient to give us just ideas of the places of the focus's of these pencils, especially when these focus's are remote from the lens: But in this case, being farther assisted by the continuity of the images of the several parts of the long object, or of the series of the other objects; the apparent places of the said images become thereby protracted to greater Fig. 70. [a] Const.

* See also prop. XIX, XX. in the preceding section, where the parallel cases of vision in speculums have been explained.

greater distances, so as to be nearer the geometric places, than if one part of the long object, or one of the objects placed cross the axis, had been seen singly. Thus, the successive changes undergone by the eye in order to see the several parts of the image *a b* distinctly, or the different distinctness of these parts if the eye remains at rest, contribute to protract the whole apparant length; that is, the image of the remotest point B will in this case appear farther, than if the point B only had been seen. But the apparent image will not exceed the object in length above a certain proportion, which will be the greater, according as the lens is deeper, and the object is nearer its focus. Also, although the apparent magnitudes of the images of the parts of the object continually increase, as these parts are nearer to the focus; yet, after a certain limit, they increase much slower than the geometrical images of those parts. If the lens be broad enough to admit both eyes to see through it at once, the phenomena will be still more agreeable to the theories of the geometrical images.

Greater indistinctness of an image causes a greater apparent distance.

315. *Schol.* 1. If the object AB be placed farther from the lens, so that the point B may be nearer the focus F, or even beyond that focus, the apparent image will be still longer; and that not merely because the geometrical images of the several parts will be farther from the eye, but also because they will be indistincter and fainter. A greater indistinctness of an image causes it to appear farther, as in like cases of [b] naked vision: And when an image is distinct, it will seldom appear farther than the object is, when equally distinct to the naked eye.

[b] See 185, & 188. II.

Phenomena of a box with painted scenes explained.

316. *Schol.* 2. The annexed figure represents the section of a box, in which are placed several scenes as AC, GG, HH, II, KK, LL, BD, *&c.* on which are drawn the representation of a camp, a colonade, or rows of trees, *&c.* These scenes are open in the middle, excepting the last which is whole; that next the eye at AC having the widest opening, and the openings of the rest are gradually diminished, that so the picture on each side may be seen. The front RB of the box is open, but covered with a thin canvas, or oiled paper, to hide the pictures within, and at the same time to admit sufficient light upon them. The box should stand upright on account of enlightening the scenes; for which reason it would be convenient to have the bottom wide enough for a support, without other help. A convex lens may be fixed at the top *r*; but the representation will be more natural, and also more surprising, if the lens be fixed in the side, as at O; a plane speculum or looking-glass RS, being fixed at half right-angles to the axis of the lens. For then the images of the pictures will appear in an horizontal plane; and prenotions of distance from the visible length

Fig. 71.

length or depth of the box being thus removed, the imagination has a greater ſcope in increaſing the apparent diſtances of the images.

In this figure, the focal length of the lens O is rF; and the ſcenes are ſo placed that their geometrical images are at equal diſtances from each other; the diſtance of the image of the remoteſt ſcene BD being = O b or O d *fig.* 70. Theſe diſpoſitions of the ſcenes have been here obſerved, for the ſake of giving an idea of the places where their geometrical images do fall: But it will be beſt to place the laſt ſcene BD nearer to the focus F, or even a little below that focus; and ſo the reſt reſpectively lower down, as experience will direct. For by this means the images of the remoter ſcenes becoming more indiſtinct and faint, they'll appear farther off; and according to the reſpective diſtances the ſeveral ſcenes ſhould appear at, their places in the box muſt be varied, ſo that no general rule for placing them can be here given. Things being properly diſpoſed, if the pictures are good and the ſubject well choſe, the phenomena will be ſurprizing enough. Inſtead of having the ſcenes, the ſides and end of the box might be painted; the box for this purpoſe being made a little taper, otherwiſe the fartheſt end would appear too wide. But the artiſt muſt have regard to the effect of the lens, and conſider the true diſtances of the images of the ſeveral parts, otherwiſe he will be liable to miſtakes, which will hurt his deſign. The ſame regard ſhould be alſo ſhewed both in painting and placing the ſcenes.

The effects of the little ſhew-boxes ſold at the toy-ſhops, having artificial fruits, flowers, *&c.* ſtuck to their ſides, are owing alſo to the cauſes above explained.

PROP. XXVIII.

317. *To explain the phenomena of pictures ſeen through a convex lens.*

If a print or picture, placed upright againſt the wainſcot, be viewed through a convex lens, it will appear a little magnified, and a little improved. A lens uſed for this purpoſe muſt not be too deep; otherwiſe the field of viſion will be too ſmall, and we ſhall be too near the picture. For a ſmall print, a lens of about a foot focal length may be uſed; and for a larger, a lens of about two or three foot focus will anſwer better. But ſuch lenſes not magnifying much, and the adjacent objects to the picture being ſeen at the ſame time or known to be very near; the leſs divergencies of the refracted rays are not ſufficient, in the preſent caſe, to overcome this prejudice.

Let

Let the ſame picture be included in a box, ſo as to be detached from all other objects, and the effect will be very different. A proper box for this purpoſe may be a *camera-obſcura*, which will be deſcribed towards the latter end of this ſection. In this place, it is ſufficient to obſerve that the box may be made on the ſame principle with that *fig.* 71. in which BD repreſents the picture at bottom, RS a reflecting plane; O the lens through which the picture is ſeen erect, and directly before the eye. By this artifice, and the excluding all other objects, the prejudices from prenotions of diſtances are removed; and being here aſſiſted by the painter's art, the picture, if it be a proper one, will be ſurpriſingly improved and magnified.

When the picture is placed at BD *fig.* 71. the diſtance of its geometrical image from the lens is $=$ O*b* or O*d* *fig.* 70. But it will be better to place the picture ſtill nearer to the principal focus F. Now the refracted rays being nearly parallel, they affect the ſight thus far as if they came from a remote object; the image alſo contains fewer viſible parts, than if it was really nearer, and ſo this cauſes ſomewhat of the ſame kind of indiſtinctneſs as we have when we ſee remote objects. We may alſo have the ſame kind of obſcurity, and this will be partly occaſioned by the loſs of light by the refraction of the ſpeculum and refraction of the lens; and the proper degree of obſcurity upon the whole may be obtained, by illuminating the picture in the requiſite proportion. All theſe cauſes conſpire together to make the images appear a great way off, and conſequently to appear proportionably large. The painter alſo, by repreſenting the remoter objects ſmaller, fainter, and leſs diſtinct; enables us to diſtinguiſh, as it were, ſome differences of apparent diſtances. And now the different lights and ſhades, and the reſpective inclinations of different planes, have their due effect, as in naked viſion, in exhibiting the reſpective parts and figures in their due poſitions and attitudes. And ſo upon the whole, inſtead of a picture upon a flat ſurface, we have a lively proſpect of the ſcene repreſented; and we ſee, as it were, real objects upon their own baſis, and detached one from another.

The repreſentation will be ſtill in many caſes better, if the lens be broad enough to admit both eyes to ſee through it; and by retreating a little back from the lens, the viſual angles of the images will be increaſed, and ſo therefore will be their apparent magnitudes.

The apparent image in ſome caſes will be alſo deeper or more hollow, if the print itſelf be made a little concave or hollow †.

318.

† I have not been able to learn when it was firſt obſerved, that pictures are improved by a convex lens; nor have I met with any attempt to explain that phenomenon. The box with the open

318. *Schol.* 1. A convex lens fixed in a frame to be held in the hand, and having before it a plane speculum inclined at the proper angle, or moveable upon a hinge so as to be readily set to this angle, will make a portable apparatus for the pockets, and may be useful for viewing prints we casually meet with abroad; the inclined speculum being also advantageous here, for the reasons above given. But yet I think, the same speculum and lens when thus held in the hand, do not answer near so well, as when they are fixed or placed at the end of a box; and that for the reasons given in the preceding. A portable apparatus for viewing prints.

319. *Schol.* 2. Prints and small pictures may be so greatly improved, by a convex lens, that it is well worth the while of those who have collections by them, to have a proper apparatus for the purpose. But the prints must be judiciously chose: They should not represent objects that have great differences of apparent distances; nor objects that are too near, or too small. The picture of the inside of a church, for instance, because of the faintness and smallness of the figures in the middle, will appear through the lens a little concave; but not near so natural as it does in the concave speculum, the point of sight here seeming much too far off. For the geometrical image of the whole plane being really convex towards the eye, is a cause that counter-acts the painter, and does in part frustrate his design; and the reason why it does not wholly overthrow it, is the smallness of the differences of the divergencies of the rays from different parts of the image of the whole; they being all of them nearly parallel when the image is far off, as it always ought to be. But although these differences, small as they are, have their effects; yet are they less sensible than the smallnesses of the middle figures; and they cease altogether to be sensible, when the real objects have nearly the same apparent distances from the point of sight. Again, when the objects are near, as within the breadth of a room; the distinctness with which they appear to the naked eye, cannot be expressed in the picture; and the picture itself, in this case, can be but little improved by the lens. Also when some of the objects represented are very small in proportion to the rest; their pictures appear through the lens so indistinct and obscure, as to have a bad effect. What prints appear best through a convex lens.

In

open scenes, *fig.* 71. was first brought here from *Holland* in or about the year 1737; and either in that year or the following, Mr. ELLICOTT by the *Royal Exchange* had one made without the bottom F, for the exhibiting of small prints in a book, *&c.* But the *shew boxes* now so common in our shops, and the lens and looking-glass in frames joined by hinges, were not introduced here till about the year 1742 or 1743; although I think that *shew boxes* somewhat analogous to those now in use, were carried about here by the poor *Savoyards*, many years earlier.

In general, the objects to be represented should not be too remote, or too near the point of sight, but at such a moderate distance as to appear from thence tolerably distinct; and there should not be too great a disproportion betwixt their magnitudes or visual angles; the prints also should not be drawn from too small a scale. There are a great variety of landskip prints or pictures, and also some historical ones, that will be greatly improved by the above method of viewing them; and several of those that receive the least advantage from the convex lens, will be mostly improved by the concave speculum; and *vice versâ*. So that by having a concave speculum and a convex lens with the proper apparatus, we may, by one or other of them, greatly improve almost all sorts of drawings or prints, if they are not too small or too big. When they are very small, it is not so easy to magnify them sufficiently for representing nature; and if they are too big, they cannot all be seen at one view; so that a regard should be also had to the sizes of our prints.

PROP. XXIX.

320. *The eye and an object being placed in the axis of any lens, and neglecting the breadth of the pupil: The visual angle of the object to the naked eye, will be to its visual angle through the lens or to the visual angle of its image; as the rectangle under the distance of the eye from the principal focus of the lens, and that of its conjugate focus from the object; to the rectangle under the focal length of the lens, and the distance between the eye and the object, very nearly.*

Fig. 72, to 75. Let A be the lens, F its principal focus, and QAF its axis; QP an object facing the lens, the point Q of which is in the axis: And let O be the eye whose conjugate focus is o; that is, let the point o be such, that any ray as Pa tending directly towards or from o, will be refracted to O. From the extream point P of the object to the point o, draw the line P$a o$; and from the center of the eye O, to the refracting point a, draw the line Oa and produce it to x; from O draw also the strait line ObP. Now AOb or QOP is the visual angle of the object to the naked eye at O; and AOa or QOx the visual angle of its image seen from the same place; for the ray Pa is [a] refracted into aO; and QA goes directly to O unrefracted. I say,

[a] Const.

$$\angle \text{QOP} : \angle \text{QO}x :: \text{OF} \times o\text{Q} : \text{AF} \times \text{OQ}, \text{ nearly.}$$

When the above angles are but small, as in vision through lenses the case generally is, Ab, Aa, may be taken without sinsible error for their measures;

measures respectively. And because of the similarity of the respective right-angled triangles, it will be,

$Ab : QP :: OA : OQ$; and

$QP : Aa :: oQ : oA$; therefore,

$Ab : Aa :: OA \times oQ : OQ \times oA$. But $OA : oA ::$ [b] $OF : AF$; [b] 227. I.

and substituting the like proportionals, it will be,

$Ab : Aa :: OF \times oQ : AF \times OQ$. *Q. E. D.*

321. *Corol.* 1. When the object is very remote, it will be,

$Ab : Aa :: OF : AF$; for then oQ and OQ being infinitely great, the ratio betwixt them is a ratio of equality; and therefore,

$(OF \times oQ : AF \times OQ ::)\ OF : AF :: Ab : Aa$.

[321.] *Corol.* 2. If the eye O be in the focus F of a convex lens; it will be, $Ab : Aa :: OA\ (FA) : OQ\ (FQ)$. For o being now at an infinite distance; oQ, oA, are in a ratio of equality. Fig. 73, to 75.

PROP. XXX.

322. *The image of an object seen through any lens will appear under the same visual angle, and in the same position in both cases, if the eye and the object change places, the lens remaining fixed. That is, if the object be translated from* Q *to* O, *and the eye from* O *to* Q; *the lens in both cases remaining fixed at* A; *the visual angle of the image to the eye at* Q, *will be the same as it was when the eye was in* O; *and according as the image did appear erect or inverted before, it will appear also the same now.* Fig. 72, to 75.

Retaining the same things and references as in the four preceding and corresponding figures, let q be the conjugate focus of the point Q; that is, let the point q be taken such, that $Qf : QA :: QA : Qq$. Also, let the object QP translated to O, be Op; and strait lines being drawn from q to p and from Q to p, let the points where these lines cut the lens be respectively, v, y; and let, as before, the lines Ay, Av, be considered as the measures of the visual angles of the object and its image respectively. Fig. 76, to 79.

Now, as in the preceding prop. it will be,

$Ay : Op :: QA : QO$; and

$Op : Av :: qO : qA$; therefore

$Ay : Av :: QA \times qO : QO \times qA$. But $QA : qA ::$ [a] $Qf : Af$; [a] 227. I.

and therefore, $Ay : Av :: qO \times Qf : QO \times Af$.

† Fig. 72 to 75. Now, † $Ab : Aa ::$ [b] $OF \times oQ : QO \times AF$. But because $AF =$ [c] Af, $QO \times Af = QO \times AF$; and therefore what remains to be proved is, that $qO \times Qf = OF \times oQ$.

[b] 320. II. [c] Const. Because, $Qf : Af ::$ [d] $AF : Fq$;

[d] 227. I. and, $Af : fo ::$ [d] $OF : AF$. Therefore,

$Qf : of :: OF : Fq$; and,

$Qf : (Qf \pm of)\ Qo :: OF : (OF \pm Fq)\ Oq$. Therefore $Qf \times qO = OF \times oQ$; and consequently, $Ab : Aa :: Ay : Av$; that is, the proportion of the visual angle of the object to the visual angle of the image, is the same whether the object be at QP and the eye at O, or the object be at Op and the eye at Q. But the visual angle of the object is the same in both cases, and therefore the visual angle of its image is also in both cases the same. *Q. E. D.*

Again, we are to shew that the apparent position of the image is not changed, by shifting the places of the eye and the object. This is evident by comparing the correspondent figures; or it may be proved thus: If the lens be concave, the image is always erect. And when the eye O is between the convex lens A and its principal focus F; the object will appear erect, whatever be the distance AQ of the object from the lens. For if AQ be greater than Af, the inverted image will be necessarily behind the eye, and so the object appear erect. If the object be placed at Op, it will then have an erect image, and therefore will appear erect, whatever be the distance AQ of the eye from the lens. After the same manner, if the object PQ be between the focus f and the lens it will have an erect image: And if the object be translated to O, and the eye to Q, the object will still appear erect, because o, the conjugate focus to O, is farther from the lens than Q.

Fig. 72, 76. Fig. 73. Fig. 77. Fig. 74. Fig. 78.

Fig. 75. When the distances AQ, AO, of the object and the eye from the lens are such, that the point q, which is the conjugate focus to Q, is between O and the lens; the image appears inverted: And so it will if the eye be translated to Q, and the object QP to Op. For it being, $qF : FA ::$ [d] $Af : fQ$; and $OF : FA ::$ [d] $Af : fo$. The geometrical image is inverted, because AO is greater than AF; and the point o, which is the image of the point O of the object, does necessarily fall between f and Q, because OF is greater than qF. Therefore, *&c. Q. E. D.*

Fig. 79. [d] 227 I.

323. *Schol.* By the transposition of the eye and the object, there will be necessarily a difference as to the distinctness in the two cases, excepting when the lens is in the midway between Q and O; and the apparent magnitude of the object may be also different, notwithstanding the equality of the visual angles in the two cases. These differences

rences may not be much, when the lens is concave; but they may be very considerable, when the lens is convex. When the eye is at O, the object QP appears indistinct, because it is seen by converging rays; and its apparent magnitude will be greater or less, as it appears farther from or nearer to the lens. But when the eye is at Q, the object will be distinct, especially if the distance AQ is not too great: In this case also, the apparent magnitude of the image will be generally greater than in the former, as the apparent distance is generally greater; and the difference may be very considerable. A difference of distinctness and apparent magnitude, will be also observable between *fig.* 74, 78; and also betwixt *fig.* 75, 79; according to which of the distances O*q*, or Q*o*, is the propereſt for distinct vision: And when the image appears in its true place, it will appear bigger from O than from Q, if O*q* be greater than Q*o*; and reversely if O*q* be less than Q*o*. Fig. 73. Fig. 77.

PROP. XXXI.

324. *Any lens and the eye placed in its axis, being both fixed, while an object from being close to the lens is moved farther from it along that axis: If the lens is concave, or the eye is nearer a convex lens than its principal focus; the visual angle of the image will constantly decrease, as the object recedes from the lens: But if the eye is farther from a convex lens than its focus, the said visual angle will constantly increase, till the object has got to the conjugate focus of the eye; and after the object has passed that focus, the visual angle of its image will decrease perpetually.*

CASE 1. Let A be the lens, whose focus is F; O the eye, and *o* its conjugate focus. Now it is manifest that the point *a* where the strait line P*a o* cuts the lens, constantly approaches towards A, as the object QP recedes farther from the lens; and therefore the visual angle AO*a* of the image, continually decreases as QP recedes from A. *Q.E.D.* Fig. 72, 73.

CASE 2. If, while a line QP is moved perpendicularly along the axis AO, from A towards *o*, the point P be supposed to approach nearer towards Q, so as to keep constantly in the line *a o*; or, if the said line QP be supposed to decrease perpetually till it comes to *o*, so that the points Q, P, are always in the lines A *o*, *a o*: The visual angle AO*a* of the image of QP, will remain invariable; and therefore the visual angle of the image of an invariable object QP will constantly increase, as QP recedes from A towards *o*; and the image will also appear erect. When the object is at *o*, its image will be close to the eye at O. If the axis A *o*, and the line *a o*, be indefinitely produced; the visual angle Fig. 74, 75.

angle of the image of an object whose extreams Q, P are constantly in the said lines, will be the invariable angle AO*a*: And therefore the visual angle of a given object, constantly decreases after it has past *o*, the farther the object is moved from the lens; and the image will also appear inverted. Therefore, *&c.* *Q. E. D.*

Fig. 72, to 75. Visual angle of the image always reciprocally as *o*Q.

325. *Corol.* 1. The tangent of the ∠ Q*o*P or A*oa*, is reciprocally as *o*Q, and directly as A*a*: For *o*Q is the co-tangent of that angle to the invariable radius QP; and A*a* is its tangent to the invariable radius A*o*. And therefore the tangent A*a* of the visual angle AO*a* of the image, is always reciprocally as the line *o*Q; that is, as *o*Q is increased or diminished, the tangent of the ∠ AO*a* is diminished or increased in the same proportion. And so when *o*Q is infinitely great, or infinitely small; the said tangent will be accordingly infinitely small, or infinitely great.

326. *Corol.* 2. Because the visual angle of the object is always reciprocally as OQ, and that of its image in the reciprocal proportion of *o*Q; as the object recedes from a concave lens, the visual angle of the image decreases faster, than that of the object would do to the naked eye, if the lens was taken away. And when the eye is between a convex lens and its principal focus, the visual angle of the object to the naked eye, decreases faster than that of its image through the lens. But as AQ increases, the ratio's of *o*Q, OQ approach nearer to an equality; so do also the ratio's of the decrements of the above angles, approach nearer to an equality. In *fig.* 74. the visual angle of the image is increasing, and that at a great rate, while the visual angle of the object is decreasing. And in *fig.* 75. after the object has past *o*, its visual angle decreases slower than that of its image; and that in the proportion as OQ increases slower than *o*Q.

Fig. 72.

Fig. 73.

Fig. 68. To an eye placed in the focus of a convex lens; the visual angle of the image will be the same, at all distances of the object from the lens.

327. *Schol.* If the eye be placed in the focus *f* of a convex lens; the image of a given object PQR will subtend the same visual angle at the eye, wherever in the axis the said object be placed. For conceiving the ray *fv* to be returned back to the lens, it will be refracted in the line *v*PB parallel to the axis; and so wherever in the indefinite line *v*PB the point P be taken, its image will be somewhere in the line *fvp*. After the same manner, a ray proceeding from R will be refracted to *f*. Also the tangents of the visual angles of the object, and its image, will be reciprocally as *f*Q, *f*A; for A*v*, QP are in this case always equal, and therefore *f*Q, *f*A are the co-tangents of the above angles. But the distinctness, apparent place and magnitude of the image, will yet be variable.

PROP.

PROP. XXXII.

328. *The diſtance between a lens and an object being fixed, while the eye retreats back from the lens along the axis: If the lens be concave, or the object be nearer a convex lens than its focus, the viſual angle of the image will conſtantly decreaſe: But when the object is farther from a convex lens than its principal focus, the viſual angle will conſtantly increaſe till the eye has got as far back as the geometrical image; and after the eye has paſt that image, the viſual angle will perpetually decreaſe.*

This prop. is the converſe of the preceding. Let A be the lens, F its principal focus; QP the object, and qp its image. Now when the image is before the lens, as in *fig.* 80, 81. it is manifeſt that the viſual angle qOp conſtantly decreaſes, as the eye O recedes farther from A: juſt as the viſual angles of objects ſeen by naked viſion diminiſh, as the eye retreats from them. When the image is behind the lens, as in *fig.* 82, 83; ſince any ray proceeding from P, does after refraction proceed to the focus p; it is manifeſt that the viſual angle AOB under which the object is ſeen through the lens, is in this caſe alſo always $= \angle qOp$. And conſequently, till the eye has got to the inverted image, the viſual angle conſtantly increaſes; and after the eye has paſt the image, the ſaid angle conſtantly decreaſes. Therefore, *&c. Q. E. D.* Fig. 80, to 83.

329. *Corol.* 1. The tangent of the viſual angle qOp or AOB, is always reciprocally as Oq; or the co-tangent of the $\angle qOp$ is always directly as Oq. And therefore when Oq is infinitely increaſed or diminiſhed, the angle qOp will be the leaſt or greateſt poſſible.

330. *Corol.* 2. As the eye recedes from a concave lens; the viſual angle qOp of the image, decreaſes faſter than the viſual angle QOP of the object, ſeen by naked viſion: And the contrary happens, when the object is between a convex lens and its principal focus. Becauſe in the firſt caſe, the diſtance Oq of the image is leſs, and in the ſecond caſe greater, than the diſtance OQ of the object. But as the diſtance AO increaſes, the ratio's of the decrements of the above angles, will approach nearer to an equality. When the image is behind the eye, its viſual angle increaſes the faſter, the nearer the point O of the eye approaches towards it; and after the eye has paſt the image, the viſual angle at firſt will decreaſe very faſt, and afterwards ſlower by degrees.

Fig. 68. To all distances of the eye from a convex lens, when the object is in the focus; the visual ∠ of the image will be invariable.

331. *Schol.* If an object be placed in the principal focus F of a convex lens; its image will appear under the same visual angle, at all distances of the eye from the lens. For in this case, the rays P*s*, P*v*, P*t*, &c. will be refracted into the rays *sa*, *vf*, *t*G*b*, &c. all parallel to one another, and to the ray *vf*; and so wherever in the axis the eye be placed, as at G, *f*, or *a*, &c. the visual angles AG*t*, A*fv*, A*as*, &c. will be all equal. In this case also, the image will always appear erect.

But here also, the distinctness, apparent place and magnitude of the object will be variable.

When the object is in one focus, and the eye in the other focus of a convex lens; the visual ∠ of the image, is double that of the object.

332. *Corol.* When Q is in F, the ∠A*fv* = ∠QAP: For A*v* = QP, and AF = *f*A. And therefore if the eye be in *f*, the visual angle of the image, will be double the visual angle of the object, seen by the naked eye from the same place.

PROP. XXXIII.

333. *The eye and an object remaining fixed in the same places, while a lens is moved between them, so that its axis is always in the line joining the eye and the same point of the object: If the lens be concave, the visual angle of the image will be least of all, when the lens is in the middle point betwixt the eye and the object; and the visual angle will continually increase, as the lens is moved from the middle towards either extream: If the lens be convex, and its focal length greater than* $\frac{1}{4}$ *of the distance betwixt the eye and the object; the visual angle will be greatest of all, when the lens is in the middle; and it will continually diminish, as the lens is brought nigher either to the eye or the object: But if the focal length of the convex lens, is less than* $\frac{1}{4}$ *of the distance betwixt the eye and the object; the visual angle of the image will be then least of all, when the lens is in the middle betwixt them.*

Fig. 84, to 91.

Let O be the place of the pupil of the eye, whose breadth is here not regarded; Q the object; A the lens in the middle between the eye and the object; that is AO = AQ; the points F, *f*, the principal focus's of the lens when it is placed at A; and G, *g*, these focus's when the lens is moved to *a*. When the lens is in A, *o* is the conjugate focus to the eye, and *v* that focus when the lens is at *a*; that is, the incident rays whose focus are at *o*, *v*, respectively, have their focus at O after refractions.

2 Now

Now $OF \times fo = {}^{a}OG \times gv = {}^{a}AF^2 = {}^{a}aG^2$; and so, [a] 227. I.

$OF : OG :: vg : of$. And when the lens is in $\frac{A}{a}$; it will be,

visual ∠ of the image : visual ∠ of the object :: [b] $\frac{AF \times OQ}{aG \times OQ} : \frac{OF \times Qo}{OG \times Qv}$. [b] 320. II.

The two first of these rectangles are equal, because $aG = {}^{c}AF$; [c] Const. and they are also invariable: Therefore the visual angle of the image when the lens is in A, or at any other point a, will be reciprocally as the rectangle $OF \times Qo$, or $OG \times Qv$. Again $OF = {}^{c}Qf$, and $fg = {}^{c}FG$.

334. CASE 1. *The distance between the eye and an object being fixed, while a concave lens is in motion from either of them towards the other; I say the visual angle of the image will decrease continually, till the lens has got into the middle, where the said angle will be least of all; and after that it will increase continually, and that at the same rate that it decreased before.*

For $OF : OG$ * $\sqsubset Qg : Qf$; and therefore $OF : OG \sqsubset Qv\ (Qg - vg) : Qo\ (Qf - of)$. Wherefore $OF \times Qo \sqsubset OG \times Qv$; and consequently the visual angle of the image is less when the lens is in A than when it is in any other point a; and it is manifest that as the point a approaches nearer to O, the ratio of $OF \times Qo$ to $OG \times Qv$ continually increases. Therefore, *&c.* *Q. E. D.* Fig. 84.

335. CASE 2. *When a convex lens, whose focal length is greater than one fourth of the distance between the eye and the object, is placed in the middle between them: I say the visual angle of the image is then greatest of all; and that this angle continually decreases, as the lens is brought nigher either to the eye or the object.* This case, as to the constructions of the figures, admits of the six following varieties.

1, 2. When both the focus's F, G, are before O; or when they are both behind O, and the focus g is beyond Q. Fig. 85, 86.

We have $OF : OG \sqsubset Qg : Qf$; and therefore $OF : OG \sqsupset Qv\ (vg - Qg) : Qo\ (of - Qf)$.

3, 4. When the focus's F, G, are both behind O, and the focus g is nearer the eye than Q is; or, the focus G only is behind O. Fig. 87, 88.

It will be, $OF : OG \sqsupset Qv\ (vg + Qg) : Qo\ (of - Qf.)$ Therefore in each of these four cases, $OF \times Qo \sqsupset OG \times Qv$; and consequently the visual angle of the image is greatest when the lens is in A; and

* $\sqsubset$ signifies that the quantity before it is greater than that which follows, and $\sqsupset$ signifies the contrary: The expression referred to signifies that the ratio of OF to OG is greater than the ratio of Qg to Qf; the reason whereof is, because $FG = fg$, and $OF \sqsupset Qg$, OF being $= Qf$. The like is to be understood in the expressions that follow. See also note * page 46.

and it is manifest that it decreases continually, as the point a approaches towards O.

Fig. 89. 5. When O is in F, Q will be also in f. And it will be, when the
[a] 332. II. lens is in $\frac{A}{a}$; *visual ∠ image : visual ∠ object* :: $\frac{QF : QA}{aG \times OQ : OG \times Qv}$ :: [d] 2 : 1.

But $aG \times OQ = 2aG^2$, because $OQ =$ [c] $2aG$; and $OG \times Qv = OG \times gv + OG \times gQ$. But $OG \times gv = aG^2$; and therefore $aG \times OQ \sqsupset 2OG \times Qv$. Therefore the visual angle of the image when the lens in A, is greater than when it is in any other point a. *Q. E. D.*

Fig. 90. 6. When O is in G; it will be when the lens is
in $\frac{A}{a}$; *visual ∠ image : visual ∠ object* :: $\frac{AF \times OQ : OF \times Qo}{OQ :}$ [c] Oa.
[a] 327. II.

And $OQ : Oa :: OQ \times Qo : Qo \times Oa$. But $Qo : Oa \sqsupset AF : OF$; For $AF : OF ::$ [c] $Af : fQ ::$ [a] $Ao : (AO)\ AQ :: fo\ (Ao - Af) : Af = Oa$. Wherefore $AF \times OQ : OF \times Qo \sqsubset OQ \times Qo : Qo \times Oa$; and consequently the visual angle of the image is greatest, when the lens is in A. *Q. E. D.*

336. Case 3. *When the distance between the eye and the object, is greater than four times the focal length of a convex lens; the visual angle of the image will be least of all, when the lens is in the middle between the eye and the object.*

Fig. 91. For $OF : OG \sqsubset Qg : Qf$; and therefore
$OF : OG \sqsubset Qv\ (Qg - vg) : Qo\ (Qf - of.)$ Wherefore
$OF \times Qo \sqsubset OG \times Qv$; and consequently the visual angle of the image is least when the lens is in A. *Q. E. D.*

In all the above cases, if the point a be taken betwixt A and Q; the case will be the same as if the eye and the object changed places, whilst the point a retained its former position; and which ever side of A the point a be taken, if the distances from A be equal, the visual
322. II. angle of the image will be the [f] same in both cases. Therefore, &c. *Q. E. D.* †

Fig. 84, to 90. 337. *Schol.* 1. In the two first cases the object appears always erect, at whatever distance from the eye the lens be placed. This is self-evident, *if the lens be concave; and when the lens is convex, because* $OF \sqsupset FA$, if Q is beyond f, o will be also beyond Q; and therefore the image of Q will be behind O, whence the object necessarily appears erect.

Fig. 91. 338. *Schol.* 2. In case the third, the image will continue inverted till the lens has got into a certain point B, so that $Qb : QB :: QB : QO$, the point b being the focus of the lens when it is in B. For then the image

† When $OQ = 4\,AF$, and $AQ = AO$, the image of Q will be in O, and therefore invisible.

image of Q will be in O; and so it will also be, if the point B be taken as far distant on the other side of A. During the motion of the lens from A to B, the visual angle of the image was continually increasing; after this the image will appear erect, and its visual angle will decrease as the lens approaches nearer to O.

To find the point B; take OC = 2AF, and betwixt AO, AC, find the mean proportional AB, which gives the point B required. Because

AB : AO :: [a]AC : AB :: BC (AB—AC) : BO (AO—AB). [a] Hypoth.

Therefore, QB (AB + AO) : AO :: OC (BC + BO) : BO; and

$\frac{QB}{2} : \frac{QO}{2}$ (AO) :: $\frac{OC}{2}$ ([b]bB) : BO :: QB : QO :: Bb : BO. [b] Const.

Wherefore, Qb (QB — Bb) : QB :: QB (QO — BO) : QO. *Q. E. D.*

339. *Schol.* 3. In some cases, the visual angle of the inverted image may be equal to the visual angle of the object seen by naked vision. For whenever QA : Aq :: OQ : Oq; the angles QOP, qOp are equal. Fig. 83. For, QP : qp :: AQ : Aq; and when QP : QO :: qp : qO, the triangles QOP, qOp are equiangular. This can never happen but when the eye and the object, are each of them farther from the lens than twice its focal length; and the distance of either of them from the lens being given, to find the place of the other when the said angles are equal.

Let AG and AH, be each equal to twice the focal length AF, of the lens. Say OG : GA :: AH : HQ, and by this proportion we have either the point O or the point Q.

Because OG : GA :: AH : HQ; it will also be,

OA (OG + GA) : GA :: AQ (AH + HQ) : HQ.

And OQ (OA + AQ) : AQ :: AQ (GA + HQ) : HQ.

But AQ : Qq :: [c]QF : QA. [c] 227. I.

Therefore by multiplying, OQ : Qq :: QF : HQ. Again,

Oq (OQ — Qq) : OQ :: AF (QF — QH = FH) QF :: [c]fq : Af.

Therefore,

Oq : OQ (:: AF : QF :: fq : Af :: AF (= Af) + fq : QF + Af) Aq : AQ.

Q. E. D.

If the ratio of OQ to Oq, be greater or less than the ratio of AQ to Aq, or of QP to qp; the visual angle of the object, will be accordingly less or greater than that of the image.

We have now determined the ratio's of the visual angles of the object and its image, in all the cases that can happen: But the apparent magnitudes will not be always as these angles; and it will not be amiss therefore, to add some farther observations concerning the phenomena themselves. There will be some difference also in the proportions of

the visual angles, on account of the aberrations of the lenses; but it would have been needless to have burdened the above propositions, with these considerations.

When a lens has the least effect.

340. When any lens, either concave or convex, touches the object, it then diminishes or magnifies the least possible; the object and its image being then nearly coincident, and the nearest to an equality that it is possible for them to be. When the lens is brought as near the eye as may be, without offending it, which least distance betwixt the lens and the cornea, may be about one eighth of an inch; the visual angles of the object and its image are so nearly equal, that their difference is scarce sensible, unless the lens has a short focus, as not exceeding a few inches. But yet in this case, if the focal length of the lens does not exceed two or three feet; an object seen through a concave lens will be sensibly diminished, and an object not farther from a convex lens than its focus, will be sensibly magnified by it; and these differences of apparent magnitudes, are generally more owing to the differences of apparent distances of the object and the images, than to the difference of the visual angles, though these also contribute their share; that is, because the image seen through a concave lens generally appears nearer, and through a convex lens farther, than the object.

Fig. 83.

341. An inverted image, when it subtends the same visual angle at the eye that the object does, appears less, and generally much less, than the object. For the image in this case, always appears nearer than the object, its apparent place being generally at or near the lens; and if it be seen in its true place, as at qp, its apparent magnitude will be still less.

Apparent magnitude through a concave lens, when the lens, the object, or the eye are in motion.

Fig. 67.

342. To a given distance between the eye and an object, a concave lens generally diminishes the most, when it is brought nearest to the eye that distinct vision will admit of. Thus, the given distance between the eye at h, and the object PQR, being aQ; if the lens A be brought so near to a, that aq is the least distance of distinct vision, and AQ be greater than aA; the apparent magnitude of the image will be generally the least it can, for that distance of the eye and the object. For if the lens be moved nearer towards the object, although the visual angle will be [a] diminished till the lens gets to the middle between a and Q, yet the image will be enlarged; as if AQ = Af, the the image of PR = HK will be = bk. But the apparent distance of the image from the eye, if the lens is not very flat, increases at least nearly, at the same rate that the true distance aq is increased; and

[a] 334. II.

5

and therefore, whilst the real magnitude of the image is increased, its apparent magnitude must also increase.

Again, if while the lens and the object remain fixed, the eye retreats farther back; and if during this retreat, the image appears in the same place, as is generally the case at least nearly, it will appear all the while of the same magnitude; and that for the same reason, that objects at moderate distances from the naked eye, appear still of the same magnitude while these distances are varied. But if the image should seem to approach nearer to the lens, this will cause it to appear somewhat diminished.

If while the eye and the lens remain fixed, the object be moved farther off; the apparent magnitude of the image will be constantly diminished. For whilst its visual angle is diminished, its apparent distance is scarce at all increased. In this case, the geometrical image is diminished, whilst its apparent and true place are nearly coincident.

Apparent magnitude through a convex lens, is chiefly affected by the visual angle.

343. In vision through a convex lens, the visual angle of the image has frequently a greater share in the idea we form of its apparent magnitude, than when we look through concave lenses; and that because, when the image is far beyond the object, the divergencies of the rays alone are not sufficient means for distinguishing its true place; and the apparent distance of the image, when it is before a convex lens, is seldom so far as its true distance.

Apparent magnitude when the object and a convex lens are both fixed, the eye being in motion.

344. The object and a convex lens being fixed at a given distance, less than the focal length of the lens: As the eye recedes farther back, from being close to the lens, the apparent magnitude of the image will generally increase for a little way; for the visual angle will be diminished in a [a] less proportion, than the apparent distance is increased. If the geometrical image is a good way farther than the object, its apparent magnitude may continue to increase, till the eye has got as far as it can see the whole object thro' the lens; and the object being seen to fill a greater part of the lens, contributes also to this phenomenon. But when the object is pretty near the lens, that is, when its image is not far beyond it; after the eye has got to a small distance from the lens, the apparent magnitude may vary insensibly; because the increase of the apparent distance, and decrease of the visual angle, become nearer to a ratio of equality; or the increase of the one and decrease of the other, may for a while continue exactly proportional, as the image may seem to move a little nearer to the object. Or the apparent magnitude of the image may decrease, by its seeming to approach so towards the object, that the increase of the apparent distance shall be in a less proportion, than the decrease of the visual angle. And hence, it

[a] 330. II.

it is easy to account for all the different phenomena, that may happen by placing the object at different distances from the lens, towards its focus; or by using different lenses.

If the object be placed in or very near the focus, the apparent magnitude will increase all the way as the eye recedes from the lens. For the visual angle of the image [c] remains the same, whilst by retreating from the lens, its apparent distance is increased. And if the object is placed beyond the focus; the apparent magnitude will be necessarily increased, as the visual angle will be increased, till the eye has got as far back as the image. After the eye has past the image, its apparent magnitude will remain the same, if it be seen constantly in its true place; but if it seems to hover about the lens, as the case frequently is, its apparent magnitude will decrease, as the visual angle will diminish faster, than the apparent distance is increased.

[c] 331. II.

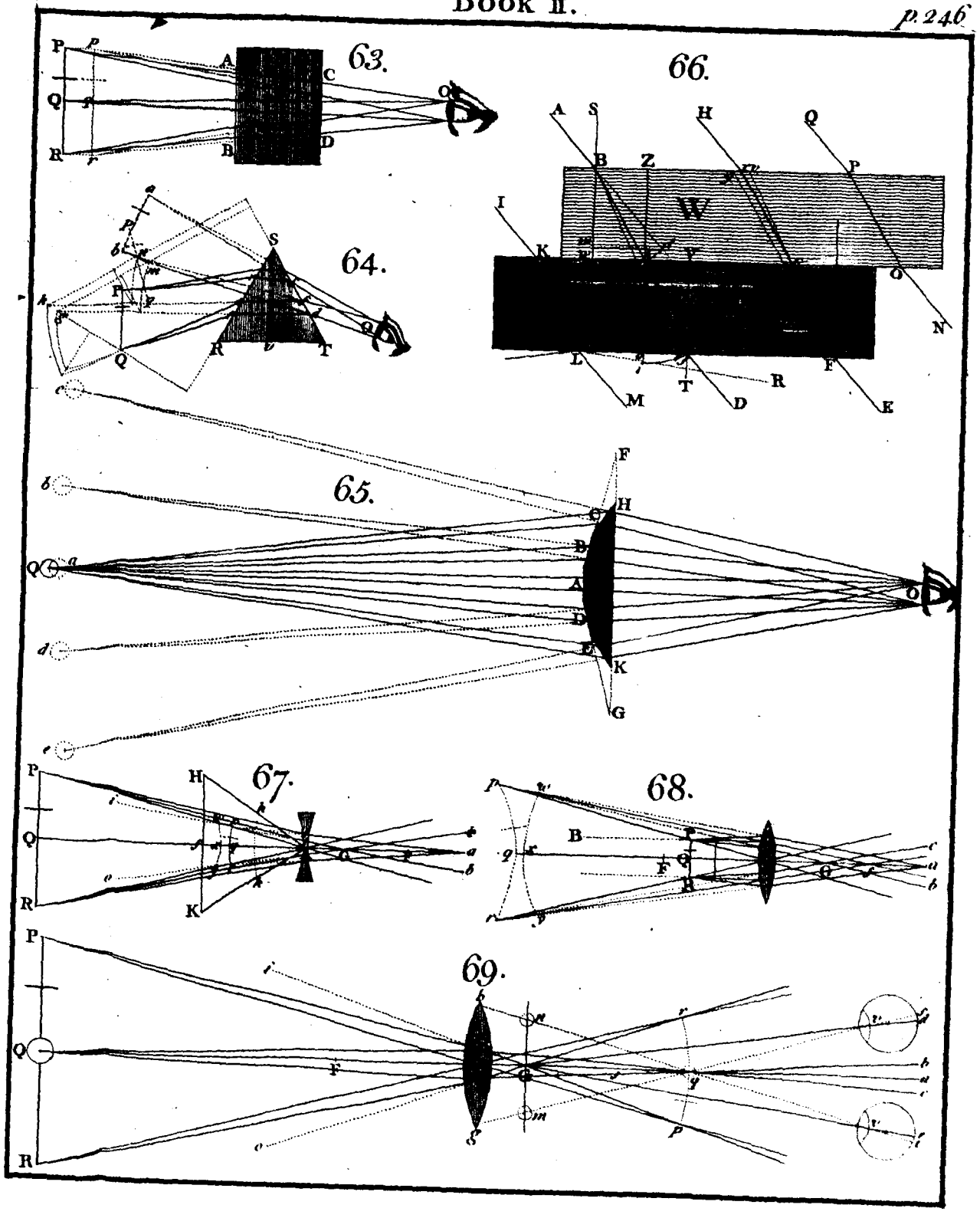
The eye and the lens being fixed, while the object is in motion.

345. The distance between the eye and a convex lens remaining invariable, whilst the distance betwixt an object and the lens is continually increasing: If the eye and the object are pretty close to the lens, the apparent magnitude will continue, for a while, pretty much the same; for in this case, the increase of the apparent distance and decrease of the visual angle, will be nearly proportional: But if the object be moved so far, that its apparent distance through the lens, increases slower than its true distance; the apparent magnitude will decrease. If the station of the eye be nearer the focus of the lens, the phenomena will be different; and the apparent magnitude of the object, as it retreats from the lens, will constantly increase, at least till it has got to a certain distance off. Because here, the visual angle decreases slower than the apparent distance increases; and when the eye is in the focus, that angle [d] continues invariably for all distances of the object from the lens. If the eye be without the focus, as the object is moved from the lens, the apparent magnitude will increase still faster: For in this case, the visual angle itself will be increased; as the image of a point P moving parallel to the axis A q, will be always somewhere in the line fvp; and therefore when the eye is at a point a, further from the lens than f; the farther is the point p, the greater will be the visual angle qap. And whether the eye be within or without the focus, if it be not far within it, the apparent magnitude will increase after the object has past the focus; for the visual angle continually increases, as the image behind advances forwards towards the eye; and till the image has got before the eye, the apparent distance of the object is never less than that of the lens.

[d] 327. II.

Fig. 68.

In this and the preceding, there will be some difference in the phenomena, on account of the aberrations of the lens: Also there will be


63.
64.
65.
66.
67.
68.
69.

be frequently a difference in the apparent place of the image, according as the eye is vigorous or fatigued, &c. and that will cause a difference in the apparent magnitude.

346. It has been [e] shewed, that when the distance between the eye and an object, is less than four times the focal length of a convex lens; the visual angle will be greatest of all, when the lens is in the midway between them: The apparent magnitude in this case, will be also greatest of all, when the lens is not very near either to the eye or to the object, but not always when it is in the middle betwixt them. For as the lens is held at different distances from the eye, the apparent distance of the object will be different; and the apparent magnitude will be greatest of all, when the rectangle under the visual angle and the apparent distance, is greatest. The apparent distance is commonly greatest of all, when the lens is so placed, that the object is near its principal focus: And when the distance between the eye and the object, is much above twice the focal length of the lens, as three times or more that length; the visual angle varies but slowly, as the lens is moved a little either way from the middle. And hence, when the distance between the eye and the object, is sensibly greater or less than about twice the focal length of the lens; the apparent magnitude is commonly greatest, when the lens is accordingly nearer the object or nearer the eye, than the middle. But in looking at an object, we naturally place the lens so, as upon the whole the object appears best; and this it does not always when it is most magnified. For if the eye is suited for vision at common distances, as in all the above observations were supposed; the object must be placed a little within the principal focus, that it may appear to the best advantage.

Phenomena when the eye and object are fixed, while the lens is in motion.

[e] 335. II.

347. From the preceding observations about vision through convex lenses, it is manifest that an object may appear bigger through a lens of two or three feet focus, than through a lens of half that focal length; according as the proportions of the distances betwixt the eye, the object, and the lenses are differently circumstanced. But yet there is another cause that may produce these phenomena. An image seen with both eyes through a broad convex lens, will sometimes appear two or three feet or considerably more, beyond the object, if the lens has not a very short focus: And generally, if the focal length of the lens does not exceed a certain limit as about arm's length; the longer it is, the farther off beyond the object the image may be perceived. Thus, through a lens of a foot focus, the apparent distance of the image beyond the object, may be proportionably greater than if we look through a lens of six inches focus; the distances betwixt the eye,

An object may be more magnified by a shallower lens than with a deeper, and why.

the object and the lenses, being proportional in both cases. The reason whereof seems to be, that the flattest lens being farthest from the object, the eyes adapt themselves the easier for vision at a greater distance; and the rays being refracted more uniformly by the flattest lens, may probably be another reason of our being the better able to distinguish the places, from whence they diverge to the eyes. If the proportion of the apparent distance through the flattest lens to that through the deepest, be greater than that of the visual angle through the deepest to that through the flattest lens; the object will appear biggest through the flattest lens. These phenomena happen only in particular cases, and not frequently.

Magnifying and diminishing powers of lenses, reciprocally as their focal lengths, nearly.

348. Hence it appears, that the proportion of the apparent magnitudes of an object seen through two different lenses, cannot be exactly assigned by having given the focal lengths of these lenses, and the distances of the object from them. However, it may be reckoned in the general, that the magnifying powers of convex lenses, and the diminishing powers of concave lenses, are reciprocally as the focal lengths, nearly. Let an object be placed successively in the focus's, or at similar distances from the focus's of two convex lenses; the eye being also placed successively in the other focus of each lens: The visual angles of the images in these two cases, will be reciprocally as the focal lengths of the lenses; and therefore if the apparent distances of the images from the lenses, were equal in the two cases; the apparent magnitudes would be reciprocally as the focal lengths of the lenses: But as the image seen through the deeper lens, will hardly ever appear so far as it will through the other; so the ratio of the apparent magnitudes through the two lenses, will be nearer to an equality than that of the visual angles.

The apparent magnitude of a remote object, seen through two concave lenses of short focus's, will be very nearly in the reciprocal proportion of their focal lengths: The diameters of the images will be exactly in that proportion; and if the eye be at proper distances, they will appear very nearly in their true places, that is, in the focus of each respective lens. But when the object is near, the deeper lens will not diminish so much more than the other, as in the above proportion.

PROP.

PROP. XXXIV.

349. *To determine the visible area, or breadth of the field of vision, through a given * aperture of a given lens; the place of the eye, and of the indefinite plane or object, being also given.*

Let A be the lens, whose center is A, and focal length is AF; AB the semi-aperture of the lens, O the place of the eye, and PQ the indefinite object. Having drawn the indefinite image pq, draw the line OB, and produce it till it cuts the image in p; through the point p draw the axis Ap, and produce it, as the case requires, till it cuts the indefinite object in P. Now it is manifest that pq, contained between the axis OQ and the line OBp, is the breadth of the visible image through the semi-aperture AB, and that PQ is the part of the object whose image is pq. Therefore PQ is the visible area through the semi-aperture AB. *Q. E. I.* Fig. 80, to 83.

The above construction, regards only the extream rays that enter the center of the pupil; but if the breadth of the pupil be considered, the visible area will be somewhat larger. Let, as before, O be the center of the pupil, and Ob that semi-diameter of it which lies on the contrary side of the axis that the point P is of. By drawing the line bB till it cuts the image, and a line from the center of the lens through this point of intersection till it cuts the object, we have the visible area Qx, as before.

350. *Schol.* 1. Hence the visible area is greatest when the eye is close to the lens: And the lens and object being fixed; the part in view through a given aperture, will decrease perpetually while the eye recedes from the lens, unless the image is behind the eye; in that case it will decrease only till the eye has got to the image, and after that it will increase perpetually. But notwithstanding, as the eye recedes from O farther back; the decrease of the visible area in *fig.* 80, 81, and the increase of it in *fig.* 83, has but a small limit. For, supposing the pupil contracted into a point, the image pq in both cases will be equal to AB when O is at the greatest distance; that is, when OA, OB are parallel. Whence by making qp = AB, and drawing the axis ApP; we have the smallest field in the one case, and the largest in the other, that can possibly happen. In *fig.* 82. when the eye is in q, qp cannot exceed Op the semi-diameter of the pupil: But in this case indeed, nothing can be seen on account of the indistinctness. Field of vision the greater, the nearer the eye is to the lens.

351.

* By the aperture of a lens or speculum is meant so much of it as is open and not covered.

Fig. 81. Field of vision = aperture of a convex lens, when the eye is in its focus.

351. *Schol.* 2. When the eye is in the focus *f* of a convex lens, the visible area of an object placed any where in the axis on the other side, will be equal to the aperture AB. For in this case, if the ray OB or *f*B be supposed to return backwards to B, the refracted ray BP will be parallel to the axis AF; and therefore where-ever PQ is placed in the indefinite axis A*q*, the ray PB will be refracted to O, and the visual angle *p*O*q* will be invariable.

Fig 80, to 83.

352. *Schol.* 3. If the places of the lens, the eye and the object be given; and also the breadth of the object, to find the least aperture of the lens through which it may be seen. Find the image *qp* of QP, and draw the lines *q*O, *p*O; and that part of the lens which is included between them, as AB, will be the aperture required. In like manner, if QP and AB are both given; and it be required to find the point O, where the rays QA, PB intersect after refraction by the lens. Produce *q*A, *p*B till they meet, and that will give the point required.

An object seen through a lens moved up, or down, or sideways, seems also to move, and which way.

353. *Schol.* 4. If a lens held before the eye be moved quickly up, or down, or sidewise; an object seen through it will also seem to move or dance, at the same time. And if the lens is concave, or the eye is behind the inverted image of a convex lens, as in *fig.* 83; the object will seem to move the same way that the lens is moved: In the other cases, as in *fig.* 81, 82; the apparent motion of the object seen through a convex lens, is the contrary way.

Fig. 80, to 83.

From the point P of the object, draw the line P*a*O to the center of the pupil O. Now let the lens be moved upwards till its center A is in *a*; and the point P which before was seen in the line OB, will be now seen in the line O*a*; P*a*O being now the axis of the pencil of rays proceeding from P to O: And consequently P appears in a different place from what it did before; and for the same reason it will seem to move, and also every other point of the object, all the while the lens is moved. The phenomena will be same, if while the lens is fixed, the spectator moves his head.

How to find whether a flattish glass is a lens.

354. *Cor.* And hence we have a ready way of finding whether a flattish glass, has its sides truly flat and parallel; or whether the glass is a concave or a convex lens. For if the glass has its sides flat and parallel, the apparent place of the object will remain invariable while the glass is moved before the eye; because the rays are refracted alike by every part of it. And if while the glass is moved, the object seems to move, the glass has a focus; and by observing whether the motions of the object and the glass are the same way or contrarywise, we know whether the glass is a concave or convex lens.

355.

355. *Schol.* 5. If the diſtance AO of the eye from a concave lens be ſuch, that AB = B*a*; the field of viſion through the given aperture AB, will be twice as wide, as it would be to the naked eye looking through an aperture of the ſame ſize, and at the ſame diſtance from it. And the point P will continue viſible to the eye at O, through the ſmall aperture AB, while the center of the lens is moved throughout the whole ſpace A*a*. But the point P, ſeen through the convex lenſes, *fig.* 81, 82, continues viſible through the large aperture AB, only while the center of the lens moves through the ſmall ſpace A*a*: And in the figures laſt referred to; the field of viſion through the given aperture AB, is only about half the breadth of what it would be through the ſame aperture, if the lens was taken away. And hence it is, that an object continues viſible through a concave lens, while either the lens or the eye is moved ſideways conſiderably, or whilſt the axis of the lens is conſiderably inclined: And this motion may be continued much longer before the object is loſt, than it could if the lens be removed whilſt the ſame aperture remained. And the contrary happens in looking through a convex lens. Fig. 80.

PROP. XXXV.

To find the focal lengths of lenſes by experiments.

This propoſition may be ſolved by different methods, as followeth.

356. The focal length of a pretty broad glaſs lens, may be obtained near enough for ſome purpoſes, thus: Meaſure exactly the thickneſs of the lens in the middle and at the edge, and alſo its breadth; then it will be, By the thickneſs and breadth of the lens.

As the difference of the above thickneſſes, is to half the breadth; [a] *ſo is half the breadth, to the focal length required.* [a] 202. I.

By a proper gage made for the purpoſe, the thickneſs of a lens may be obtained to the 500th part of an inch; and its breadth might be alſo meaſured to a tolerable exactneſs. But leſt the figure of the lens ſhould be faulty near the edge, the thickneſs ſhould be taken at ſome diſtance therefrom, and alſo in two places diametrically oppoſite; half the ſum of theſe two thickneſſes is to be deemed the thickneſs at the points where they were taken, and the diſtance betwixt theſe points is to be reckoned the breadth of the lens. By this method, an eſtimate may be made how the lens is centered, and whereabouts is its vertex; and it may be uſeful on ſome occaſions, as

it may always be put in practice: But if great exactness be required, this method is not to be depended upon; because a small error in the *data*, may produce a great one in the focal length.

By having the radii of the sides.

357. If we have the radii of the sides of the lens given; its focal length may be found by the rules prop. XXIV. or prop. XXV. of Book I. according as the lens has one side plane or both spherical; remembring for *m*, *n*, if the lens is of glass, to use the numbers 31, 20, or * 17, 11, as being more exact than the numbers 3, 2. But this method in practice is very precarious, as it is difficult to find the radii of the sides of the lens to a sufficient exactness: And even the focal lengths of two lenses formed by the same tools, will be different; so that the workmen themselves before trial, do not know the exact focal lengths of their lenses, though they know to what spheres their tools were formed. In practice therefore, we need only use the following rule, according as the centers of the sides are on different sides, or on the same side of the lens;

As the sum or difference of the radii of the sides, is to one of them;
[b] 192. I. [b] *so is double the other, to the focal distance from the lens.*

If the curvities of each side is the same, the radius of that curvity may be deemed the focal length. And if one side of the lens is plane,
[c] 179. I. the focal length may be reckoned equal [c] to twice the radius of the spherical side.

The focal length of a convex lens may be also found by the following methods.

Focus of a convex lens found by the image in a dark room of a given object.

358. Let the room be darkened so that no light comes into it, excepting through a hole in one of the window-shutters: The lens being placed against and close to this hole, move a white paper behind it, till the picture of a particular object, directly facing the lens, appears most distinct, and there fix the paper; then measuring the distances betwixt the paper and the object each from the lens, there will be a sufficient data for finding the focal length required.

Fig. 83.

Let QP be the object without doors, A the lens in the window-shutter, *qp* the place of the paper where the image of QP is distinctest, and QA *q* the axis of the lens. Having measured the distances
[d] 227. I. QA, *q*A, it will be, $Qq : QA :: {}^{d}QA : QF$; and $QA - QF = AF$ the focal length required †.

359.

* If these numbers be admitted, a double convex or concave lens, whose thickness is inconsiderable and whose sides have equal curvities, will have its focal length to the radius of its curvity as 11 to 12; as will appear by applying the rule 185. I. t being neglected.

† The distance betwixt a fixed object without doors and the window-shutter, being once accurately measured; the focal lengths of other lenses may be afterwards found with little trouble, unless A *q* be greater than the breadth of the room.

359. *Schol.* 1. In ſtrictneſs the focal length of any lens[e], is the di- [e] 215. I.
ſtance from its principal focus to the reſpective focal center. And in the example above, the diſtances betwixt the focal centers of the lens, [f] 226. I.
ſhould have been [f] ſubtracted from Qq; and for QA, we ſhould have taken the diſtance from Q to the neareſt focal center. But in theſe ſort of experiments, thoſe niceties would be ſuperfluous.

360. *Schol.* 2. In the above experiment, there will be a certain part of the axis, extending on each ſide the geometrical focus q, within which where-ever the paper is placed, the picture as to ſenſe will be equally diſtinct. Having found nearly the place of q; move the paper from thence both ways along the axis, till an indiſtinctneſs begins to be perceptible in a particular ſmall part of the picture; and having noted theſe two places, the middle point between them may be taken for the focus q. In making this experiment with lenſes of about 12 foot, 8 foot, and 4 $\frac{1}{2}$ foot focus, the lengths of the axis within which the picture of a remote object appeared equally diſtinct, were reſpectively about ‖ 6 or 8 inches, 3 or 4 inches, and 1 or 1 $\frac{1}{2}$ inch. I could not determine the focal length of the flatteſt lens, or that having the longeſt focal length, to leſs than about 2 or 3 inches; the next I could define its focus within about an inch; and the focal length of the deepeſt lens, or that having the ſhorteſt focal length, I could aſcertain to leſs than half an inch. The apertures of each of theſe lenſes was about 1 $\frac{1}{2}$ inch; and this being ſo ſmall in proportion to the focal lengths of the two flatteſt lenſes, is a reaſon why, by this experiment, their focal lengths could not be aſcertained ſo near in proportion, as that of the deepeſt lens.

361. *Schol.* 3. If the focal length of the lens does not exceed 2 or 3 feet, it may be found without darkening the room, by holding the lens at ſuch a diſtance off, that the image of the window-ſaſh may be diſtinct upon the oppoſite wainſcot; then computing as above, the window-ſaſh being now the object.

And for common purpoſes; when the focal length of a lens is but ſhort, this length may be reckoned the diſtance from the lens, to the place where the image of a remote object appears diſtincteſt. For, if the object be diſtant from the lens a 100 or 1000 times the focal length; the image will be beyond the principal focus, only the [g] 100th [g] 227. I.
or 1000th part of that focal length.

362. The focal length of a lens may be found, and that without computation, by making the flame of a candle the object, thus: Move the

Focal length found by the image of a candle, without computation.

‖ Theſe meaſures will be variable, in uſing the ſame lens and object, according to different brightneſs of the object, and different darkneſs of the room. Alſo there will be a difference, if the ſame lens be uſed with different apertures, the other circumſtances being alike.

the lens or the candle, and the paper for receiving its image, so, that, when the image is distinctest, the lens be exactly in the middle betwixt the other two; then half the distance betwixt the object or its image and the lens, is the focal length required. Thus, if $Qq = 2AQ$, AQ must be $= 2AF$. Or, if of the lens, the candle and paper for receiving its image, any one be moved while the other two are fixed; having measured the respective distances, the focal length of the lens may be found, as has been [h] before shewed. Or, if a small hole, of about a $\frac{1}{4}$ or $\frac{1}{8}$ of an inch broad, be made in the window-shutter of a darkened room, and a lens and a paper be held behind this hole at proper distances; the place where the image of the hole is distinctest, may be determined very critically; and so the focal length of the lens may be found, as above.

Fig. 82.

[h] 358. II.

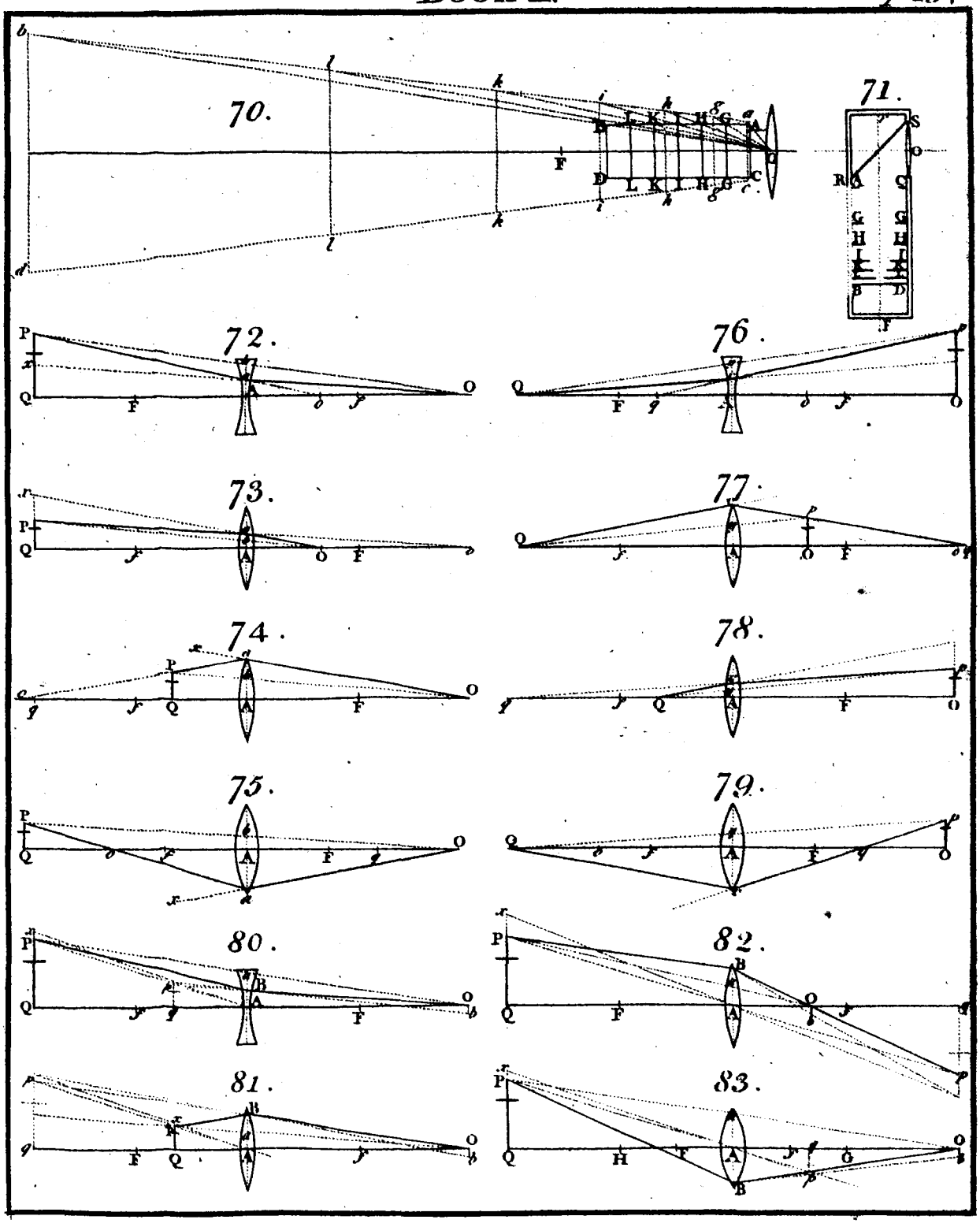
By the sun's image.

363. Place the lens so that its axis may point, as near as may be, towards the sun; then holding a paper behind it, the burning point, or where the sun's image is smallest, and where its limb appears distinctest, is the focus of the lens.

This method of finding the focus's of lenses that are broad in proportion to their focal lengths, such as common spectacles and reading-glasses, is sufficiently accurate. But for lenses having long focus's, unless they are so long as to exhibit the solar spots, the above method will not answer at any rate: For the sun's image being only, as it were, a glare of light, without any distinguishable parts; we can perceive no difference as to distinctness, even at the limb, while the paper is moved either way along the axis to considerable distances. And the different magnitudes of the white circle upon the paper, as it is moved along the axis, will not be sufficient to determine the place of the focus: For if the lens be no broader than the sun's image at the focus, the stream of light passing through it will be a cylinder; and if the lens is not so broad as above, the magnitude of the white circle will increase all the way, as the paper is moved farther from the lens. These inconveniencies may be removed by the following method.

Fig. 92.

364. Let A be the given lens, whose axis is Ah, and principal focus is v or the center of the circle sn; and let snv be the sun's image, or let the $\angle sAn$ be equal to that subtended by the sun's diameter. Let the lens be covered with thin past-board or opaque paper, in which are two round holes a, b, near the edge of the lens: These holes should be at the same distance from the edge, and in a line passing through the middle or the vertex of the lens.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The lens being thus covered, place it so that its axis may point directly towards the sun; then through the holes *a*, *b*, there will issue the streams of light *s a n*, *s b n*; the rays belonging to the upper and lower limb of the sun, having their *foci* respectively at the points † *n*, *s*. Now if a paper be held behind the lens, these streams of light will form two white spots or circles upon it, which gradually approach nearer together, as the paper is moved toward *v*; at a certain point *d* their limbs will touch; after this they will incroach one upon another, and the part common to each will be brighter than the rest; at *v* these circles will coincide; if the paper be moved farther, they will begin to separate, and at *h* their limbs will just touch. But as it is difficult to determine precisely, when the two circles are perfectly united; observe where their limbs are at a small and equal distance asunder, as at *o* and *p*; then bisecting *o p* we shall have the focus *v* required. If that point be called *x*, where the lines *d a*, *h a* produced either way, as the case may require, would intersect; it will be $dv : vh :: {}^{1}xd : xh$; [1] 3.VI. *Eu.* and so $xd + xh : xd :: dh : dv$. The holes *a*, *b*, should be close to the edge of that aperture which the lens is to have when applied to its use; and they should be also of a proper size; not too small, for then the white circles will be dark and ill defined; and to make them large would be disadvantageous, as their centers must be nearer together. The experiment will succeed better, if the room be darkened. Instead of having the paper with the holes in it; the lens might be left open, and covered only with two round opaque circles: But the holes do better.

By this method the focal length of a lens may be found, I think, more accurately than by either of the preceding ‖. By making the experiment with the lenses before mentioned of about 12, 8, and $4\frac{1}{2}$ feet focal lengths; I could determine these lengths respectively within about $\frac{3}{10}$, $\frac{2}{10}$, $\frac{1}{15}$ or $\frac{1}{20}$ of an inch. The limits betwixt the places where I could perceive the images to separate, were respectively about 3 inches, 2 inches and $\frac{3}{10}$ of an inch; and had the flatter lenses been wider, these limits would have been more contracted.

The focal length may be also found in the same manner, by exposing the lens to the moon, and observing where its image is least; or where the bright spots coalesce, if the lens is covered as above. But we cannot be so exact here as with the sun, especially if the focal length

† The incident rays *k a*, *k b*; and *l a*, *l b*; proceed respectively from the upper and lower limbs of the sun; *l a*, *l b*, being parallel to A *s*, and *k a*, *k b* to A *n*.

‖ Other methods for finding the focal lengths of the object glasses, of telescopes, and of microscopic lenses, will be shewed in the next book.

length exceeds two or three feet; and if the lens be covered the holes must be large.

An apparatus for holding the lens.

365. In making the above experiments; the lens and the paper for receiving the images, should have proper supports; so that they may remain fixed at any given station, while the respective distances are measured.

Fig. 93.

The annexed figure represents an apparatus, I made use of, for holding lenses of various sizes. A is the pillar of a wooden candlestick, on which slides the wooden bar *c d*, which may be stayed at any height by means either of a spring or a wooden wedge at *c*, bearing against the pillar. The lens is to be placed between the upright and wooden pieces *a*, *b*; the piece *b* being moveable on *c d*, till it touches the edge of the lens. The fore and inside parts of *a*, *b*, and also the upper surface of *c d*, have narrow and thin ledges for the lens to bear against; and for confining it on the other side, are slips of wood sliding on the inside surfaces of *a*, *b*. These slips have a motion of about half an inch, on account of receiving a thick lens, or a lens in its cell; and they are confined at proper places, by a screw in each going through slits made across in *a*, *b*, and having a nut bearing against the out-side of the said pieces *a*, *b*. This little apparatus I had made so as to hold any lens not wider than six inches: And that the lens might be so placed as directly to face any object, or that its axis might have any elevation; the bar *c d* turns stifly in a round hole made in the socket sliding on the pillar A; and as a guide to point it aright, there is a short index fixed to the top of the immoveable piece *a*, having on it two sights to guide the eye, or to let the sun shine through. An † *ox-eye* fixed to a window looking easterly or westerly, so as it may face the sun when he is low, will be very convenient for finding the focal lengths of such lenses as may be readily screwed to it. Or a short tube fastened to the window, and having without a || plane speculum which may be so inclined as to reflect the sun's rays directly upon the lens, will do still better.

The paper for receiving the light should be parallel to the lens; and how to support it in this manner, there may be different methods, which it would be needless to describe ‡.

To

† See 397. of this.

|| See the description of the solar microscope in the next book.

‡ Having mislaid the above described apparatus, I have since got made a more convenient one for finding the focal lengths of lenses not exceeding two feet. Within about eight inches of one end of a strait wooden bar 32 inches long, was fixed across a short bar, having sliding checks, &c. for confining lenses of different sizes, like *c d*, *a*, *b*, before described. The long end of the bar had a groove in it, in

To find the focal length of a concave lens.

366. Let the lens be covered with paper having in it two ſmall circular holes, as at O, v; and on the paper for receiving the light, deſcribe two ſmall circles as a, b, whoſe centers let be at twice the diſtance aſunder of the centers of O, v; and it will be beſt if their diameters are alſo double thoſe of O, v. Then moving the paper to or fro, till the middle of the ſun's light coming through the holes O, v, falls exactly on the middle of the circles a, b; that diſtance of the paper from the lens will be the focal length required. For it is manifeſt that $ab : Ov :: aF : OF$: Or whatever is the diſtance of a, b, it will be, $ab - Ov : aO :: Ov : OF$, the focal length of the lens. Inſtead of the paper with the holes, ſmall patches may be ſtuck on the lens. Fig. 94.

367. The focus may be alſo found by candle-light, thus: Let Q be the place of the flame; c, d, the bright ſpots upon the paper, and q the point where the lines vd, Oc produced backwards would interſect. To find the point q, it will be $cd - Ov : Ov :: Oc : Oq$. Then $Qq : QO ::$ [a] $QO : Qa$, and Oa is the focal length required. Focus found by candle-light. [a] 227. I.

368. The focal length of a concave lens may be readily found, by joining it to a convex one having a ſhorter focus. By joining the lens with a convex one of a deeper focus.

Let q be the image of any object formed by the convex lens A, fixed at A; and let Q be the image of the ſame object, when the concave lens O is joined to A. Having meaſured the diſtances Qq, qO; to find the focal length OF of the concave lens, it will be, as in the preceding, $Qq : QO :: QO : QF$; then $QF - QO = OF$. *Q. E. I.* Fig. 95.

§. 3. *Of viſion when the object or the eye or both, are in a medium denſer than air.*

369. The appearances of objects placed in a denſe medium, whoſe refracting ſurface is plane, to a ſpectator in the common medium of air,

in which was a ſlider divided into inches and parts; over both theſe was an upright piece which moved the whole length, and which might be readily fixed in any place. The ſide of this piece facing the lens was covered with white paper, for receiving the ſun's image; and for meaſuring the exact diſtance between the paper and the lens, at one end of the long ſlider was fixed occaſionally a piece at right-angles, which by moving the ſlider would touch the thickeſt part of the lens, and then the diviſions cut by the paper would ſhew the diſtance between them. This apparatus was fixed occaſionally to the top of a ſtand for carrying a ſmall reflecting teleſcope, by which it might be elevated to any angle.

air, may be easily understood from prop. V. and its coroll. of Book I. And I think the phenomena agree in all cases with the theory, as it has been there explained.

PROP. XXXVI.

370. *To explain the phenomena of vision of objects placed in a spherical dense medium, to a spectator in the medium of air.*

Phenom. Objects placed in such a medium to a spectator without, appear magnified, and that in different degrees according to their distances from that part of the surface which is next the eye; those that are in contact to that point of the surface not being magnified at all, whilst at all other places they will be magnified, and that more and more the farther they are from the said point of the surface; so that when an object is in contact with the remotest point of the surface from the eye, it is magnified most of all.

Fig. 96. Let ADO be the section of a refracting sphere whose center is O; as for instance, the section of a decanter of water; the spectator being on the side A, and somewhere in the axis A*f*. Let F be the focus of rays, coming from without parallel to the axis *f*OF, after their refraction near A; or F the focus of those rays supposing the medium continued beyond F; and *f* the focus of rays within the sphere parallel to the axis DOA, after their emersion near A. And let Q 1, Q 2, O 3, Q 4, Q 5, D*v*, be the diameter of an object, or the diameters of as many equal and similar objects, situated at these respective places. Now to find the points *q* of the respective images *q* 1, *q* 2, O 3, *q* 4, *q* 5, *q* 6; it will be, QF : OF :: [b] AQ : A*q*. Having thus found the respective points *q*; through the extreams 1, 2, &c. of the objects Q 1, Q 2, &c. draw the strait lines O1, O 2, &c. and produce them till they meet *q* 1, *q* 2, &c. drawn parallel to the respective objects; and *q* 1, *q* 2, &c. will be the geometrical images of the said objects, or of their diameters. Or in this case, because the objects are all equal and contained between the parallel lines AD, *dv*; their images, if we neglect the aberrations of the refracted rays, will be contained between the axis *f*AF, and the line *fd* produced, as to 6. For *f* is the [c] focus of a ray *vd* coming to *d* parallel to the axis; and so the rays coming to *d* from the extreams 1, 2, &c. of the several objects will be all refracted into *df*; whence the extreams 1, 2, &c. of all the images are in the line *fd* produced.

[b] 144. I.

[c] Const.

Hence when Q is in A, or the object is in A*d*; *q* is also in A, or the image is equal and coincident with the object. But as the object is

removed from A towards D v, its image increases, and that more and more as it advances towards D; the image being greatest of all when the object is in D v. All which are exactly agreeable to the phenomena. *Q. E. D* ||.

371. *Corol.* 1. The refracting surface A d remaining the same; if the medium be indefinitely continued on the opposite part, the images will still increase in magnitude as the object advances farther from A, until it has got to F. If the object be placed beyond F, vision will be indistinct; because then the refracted rays will enter the eye, placed near to A, converging.

372. *Corol.* 2. The denser is the medium, the more it magnifies objects placed within it: For then the less is the focal distance A f; and consequently the diameter of the object = A d remaining the same, the greater is the angle A $f d$.

373. *Schol.* 1. Whilst the object is between A and the center O, the image is nearer to A than the object; at O they both coincide, and the image is farther when the object is beyond O. And this is exactly agreeable to the phenomena; the semi-diameter OA next the eye, appearing much shorter than the semi-diameter OD, which seems extended as far as the line q 6. By covering the bottom of a decanter of water with small objects, as peas, *&c.* the phenomena here explained will be very remarkable, and quite agreeable to the above theory, if the eye is nearly in the same plane with the objects. But if the eye is much above that plane, so that the remoter objects are seen through the belly of the decanter, whilst the nearer ones are seen through a smaller segment; the remoter objects will appear accordingly so much the more remote: For OD, D e are in a constant ratio; that is, the greater is OD, the greater also in proportion will be D e.

374. *Schol.* 2. The images of a given small object placed at the centers, or at like distances in proportion to the radii from the refracting surfaces, of any spheres made of the same medium, are equal.

All spheres of the same medium, magnify objects having very small breadths, equally.

Let O be the common center of the spherical refracting surfaces AD, $a d$, and F, f their principal foci; and let O y, the breadth of the given object placed any where between the parallel lines AB, DC, be so small, as not to differ sensibly from the tangents of the arches AD, $a d$: I say the images formed by the refracting surfaces AD, $a d$ will be equal, if the distance of the object in each sphere from the respective surfaces AD, $a d$, be as the radii OA, O a.

Fig. 97.

 Through

|| The lines O $a b$, QB, q B b are for illustrating the theory according to Prop. XXI. Book I. This being a different case, from either of the figures referred to in that proposition, but nevertheless falling under the same rules.

Through the respective points D, d, draw the lines FDH, fdh; and where-ever the objects are placed in the respective spheres, their images will be contained between the lines FG, FH, and fg, fh. Let the object placed within the sphere ABO be called X, and its image x; and the object within the lesser sphere abO be called Y, and its image y. Now because,

[a] Hypoth. 144. I. FX : OF (:: [d] fY : Of) :: [b] AX : Ax (:: [b] aY : ay), it will be, Ax : ay :: AX : aY :: [d] FA : fa (:: Ax+FA : ay+fa) :: Fx : fy. But,

[c] 4. VI. *Eu.* FA : Fx :: [c] AD : x; and fa : fy :: [c] ad : y; and because,

FA : Fx :: fa : fy; it will be AD : x :: ad : y. But AD = [d] ad, and therefore $x = y$. *Q. E. D.*

Thus, if the small object be at the center, as Oy, its image in each sphere will be = Ox; and if it be at the peripheries bc, BC, its images will be gh = GH.

375. *Schol.* 3. When the object is so large, that the versed sine av has a sensible magnitude; the image formed by the refraction of the lesser sphere, will be less than that formed by the greater sphere. For supposing the object Oy to be at the center, for instance; it will be, fv : fO :: vd : Ox; and therefore the greater is the ratio of fv to fO, the greater will be the ratio of Oy to Ox. And hence, equal objects having sensible magnitudes, appear somewhat more magnified the larger is the sphere. Add to this, that the greater obscurity of the thicker medium, may somewhat enlarge the apparent distance of the image; and this will proportionably increase the apparent magnitude.

376. *Corol.* And hence, from these two causes conspiring together; the small bores of capillary tubes, or such as are used for mercurial thermometers, appear bigger in proportion the thicker are the tubes.

377. *Schol.* 4. A broad ruler with parallel sides placed upright in the middle of a conical vessel, as a glass of water, will at first sight appear broadest where the glass is narrowest. This deception is chiefly owing, to the breadth of the ruler being referred to that of the glass; the distance that is observed between them, being partly attributed to the figure of each. But holding the glass even with the eye, and looking attentively, the apparent taperness of the ruler will soon vanish.

Fig. 97. 378. *Schol.* 5. The greatest proportion that the breadth of the image of an object placed within a sphere of water, can bear to the breadth of the object, is that of 2 to 1. For the image is greatest, when the object is at the remotest surface BC. Let K be the principal focus of rays coming from without upon the surface AD; and

it will be, AB : AG :: [f]KB : KO :: [g]2 : 3 :: [g]AB : AF. Wherefore FA : FG :: 1 : 2 :: AD : GH. But at first sight one is apt to imagine that the diameter of a round object, as a pea, is magnified much more than in the above proportion. We see that the whole bulk is greatly swelled; the apparent magnitude in this case, being not much less, than as the cube of the diameter, as on account of the distance between our two eyes, we see at once more than half the pea; and by so much as is seen, the idea of the invisible part is readily suggested. And hence, the bulk and diameter are supposed at random, to be magnified in near the same proportion. We commit the same mistakes in naked vision, the reason indeed in both cases being the very same. But if we attentively compare the breadth of the magnified pea, with the extent of two contiguous peas seen by naked vision, we shall soon rectify our former mistake.

[f] 144. I.
[g] Const.

If the refracting sphere is solid glass, then GH : AD :: 3 : 1.

If the object be at the center of a sphere of water, its diameter will be to that of its image as 3 to 4; and if the sphere is solid glass, the proportion between them will be as 2 to 3.

379. To a spectator in a dense medium, as for instance in water, all objects within that medium will be confused; and the confusion may be so great, that one object cannot be discerned from another, as in the case of a very flat eye that cannot discern any thing without spectacles. The case will be much the same, if the objects are without that medium, if its refracting surface is flat, as that of smooth water.

Vision in a dense medium confused.

The reason of which is, that the refraction between the dense medium, and the cornea and the aqueous humour, is less than between air and the cornea; and so the rays not converging enough at their entrance into the chrystaline humour, are not united upon the retina, but have their focus's beyond it. If the refractive power of the dense medium, is equal or greater than that of the cornea and aqueous humour; the rays will accordingly proceed to the chrystaline humour either unrefracted or more diverging.

380. The best way for *divers* is to suit themselves with glasses by experiment; for according as they are *long*, *common*, or *short*-sighted; or as the refractive power of their † *cornea* may be greater or less in proportion to that of their chrystaline humour, they will require glasses of greater or less convexities. But, if the focal length of the cornea in our natural element, air, is known; a lens which will restore distinct vision in a given medium, may be found by the following *prop.*

PROP.

† By the refractive power of the cornea, is properly meant that of the aqueous humour behind it.

PROP. XXXVII.

381. *The focal length of a lens in one medium, being given; to find its focal length in any other given medium.*

Let the medium whereof the lens is made, be called G, one of the mediums wherein it is placed A, and the other medium W. Let the greater and lesser of the sines of incidence and refraction between G and A, be respectively m and n; and between G and W, M and N; and the focal length of the lens in A be $=f$, and in W $=$ F. And it will be, $f : F :: \frac{n}{m-n} : \frac{N}{M-N}$ nearly.

Fig. 98, 99. 1. Let the lens be a plano-convex, with its plane side to the incident rays; Oa the radius of its convexity, af its focal length in the medium A, *fig.* 98; and aF its focal length in the medium W; to find which in the respective mediums, it will be accurately:

[a] 175. I. O$a : af ::$ [a] $m-n : n$; and O$a : a$F $::$ [a] M—N : N; whence

$af = \frac{Oa \times n}{m-n}$; and aF $= \frac{Oa \times N}{M-N}$; and therefore,

$af : a$F $:: \frac{n}{m-n} : \frac{N}{M-N}$. *Q. E. D.*

If the lens be reversed, its focal length in each medium will be still the same, [b] reckoning that length the distance between the focus, and the nearest focal center of the lens.

[b] 177, & 211. I.

And in all lenses, whose sides are both spherical, if we neglect their thicknesses as inconsiderable, it will be,

[c] 185. I. $f : F :: {}^{c}\frac{n}{m-n} : \frac{N}{M-N}$. Therefore, *&c.* *Q. E. D.*

382. *Corol.* Hence if the lens is glass; the medium A is air, and the medium W is water; it will be $f : F :: 1 : 4$.

[d] 35. I. For $m-n : n ::$ [d] $1 : 2$; and M—N : N $::$ [e] $1 : 8$; whence,

[e] 302. II. $\frac{n}{m-n} : \frac{N}{M-N}$ $(:: \frac{2}{1} : \frac{8}{1} :: 2 : 8) :: 1 : 4$.

383. *Schol.* 1. Any lens being given; we may find another lens which being joined to the former, both together shall have a given focal length in any given medium.

Fig. 99. Let aF be the focal length required of the two combined lenses, in the given medium. 1. Find [f] the focal length of the given lens in the given medium W; if the lens has no † focus, that is, if it transmits the rays unrefracted; then it is manifest that aF is the focal length of the lens

[f] 381. II.

† This cannot happen but when the medium of the lens has the same refractive power as the medium W; as is supposed to be case of the aqueous humour of the eye and common water.

lens required in this medium, whence its focal length in air is [f] readily found. [f] 381. II.

If the given lens has a real focus, as g; then say, $gF : ga :: {}^{g}ga : gG$; or if its focus is G, that is, if it transmits parallel rays diverging as from G; it will be, $GF : Ga :: {}^{g}Ga : GH$; [g] 227. I. and aG or aH will be accordingly, the focal length in the medium W of the lens required. Whence in all cases, its focal length in air may be found.

384. *Schol.* 2. Hence having the focal length of the *cornea* in air, we may also have its focal length in any other given medium; and so, from the preceding, suit ourselves with a lens that will restore distinct vision in that medium. A proper lens for divers.

Thus, if the focal length of the *cornea* in air is $= aF$, and its refractive power be the same with that of water; a lens that will procure distinct vision in water, will have its focal length therein $= aF$; and if the lens is glass, its focal length in air will be $= {}^{h} \frac{4}{3} AF$. If [h] 382. II. the focus of the cornea a, when under water be g, or G; the focal length in water of a lens that will restore distinct vision, will be accordingly aG, or aH.

385. *Schol.* 3. If the cornea be not concerned in the change made in the eye, for procuring distinct vision at different distances; the lens above found will enable the eye to see distinctly in water, at all such distances as it can in air: But if the cornea is concerned in that change of configuration of the eye, as most probably it is; the latitude of distinct vision through the above lens in the dense medium, will not be so extensive as it is to the naked eye in air.

386. *Schol.* 4. Hence again, if a person under water had sufficient light, and was exactly fitted with a lens for seeing an object at a given distance distinctly; we might from thence find pretty nearly the focal length of the cornea in air: And if the said person could also determine, how much farther and nearer than the said given distance, he could also see distinctly; we might also make an estimate of the share, which the cornea has in the change of configuration of the eye. But the want of light, and of ability of staying at ease any time under water, are difficulties that would render very precarious, the conclusions drawn from the observations of a person under these circumstances.

§. 4.

§. 4. *Of camera-obſcura's.*

The theory in general of the *camera-obſcura* has been already explained in Book I. *art.* 243 to 246. I am here to add ſome farther particulars, not there taken notice of, with a deſcription of different contrivances for forming the *dark chamber.*

387. The pictures exhibited in the *camera-obſcura* are very entertaining; eſpecially if they repreſent objects in motion, as carriages either by land or water, people walking, the wafting of the boughs and leaves of trees, *&c.* And if all circumſtances are favourable; the pictures ſeem to appear more lively and diſtinct, than the objects do to a naked eye looking at them, from the place where the lens for forming theſe pictures or images was fixed. For the picture being diminiſhed, the rays forming it are condenſed; and ſo the colours, ſhades, *&c.* are proportionally brighter and more vivid. The picture being alſo a near object, is ſeen free from that faint hue wherewith objects without doors are commonly tinged; thoſe rays that occaſion theſe appearances, being as it were abſorbed in the picture, or becoming too feeble, after they are there diſperſed, to be perceived.

Objects at a moderate diſtance make the beſt picture.

388. All other circumſtances being alike, the pictures of objects that are near, as within 5, 10, or 20 yards, appear more vivid than thoſe of objects that are more remote. And univerſally, the pictures will be more diſtinct and pleaſant, when the objects are at ſuch moderate diſtances in proportion to the focal length of the lens, as to exhibit ſmall parts, as the features of a perſon's face, the bricks and tyles of houſes, *&c.* If the light without is favourable, and the ſpectator has been ſome time in the dark; it is ſurprizing how diſtinct and bold the pictures of ſmall objects will appear, that are diminiſhed at leaſt 20 or 30 times; and a perſon may be known, when his features are no bigger than in that proportion. The lights, ſhades and tints being here exquiſitely delicate and perfectly juſt, at the ſame time, that the picture is diminiſhed; the *relievo's* of objects are ſurprizingly bold, and when the natural ones are but moderate, thoſe in the picture ſeem to come up to the real ſize. A diſtant proſpect appears perfect enough, though it does not make ſo entertaining a picture.

The *camera* ſhould be as dark as poſſible.

389. The room wherein the picture is formed ſhould be as dark as poſſible; that is, all light ſhould be excluded from it but what comes thro' the lens; for in proportion as the field about is darker, the objects will appear brighter, as the ſtars do in a dark night. And for this purpoſe, the lens, beſides having its aperture properly limited, ſhould

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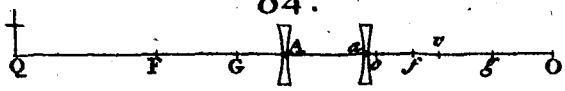

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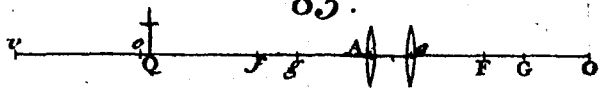

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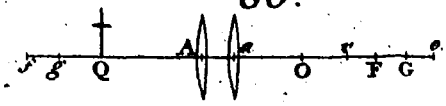

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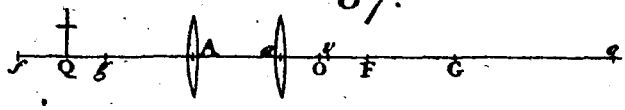

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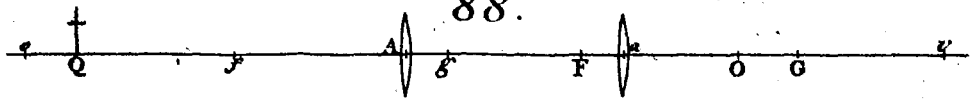

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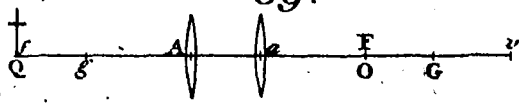

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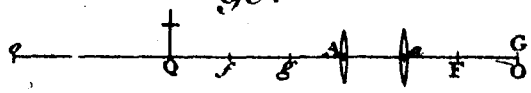

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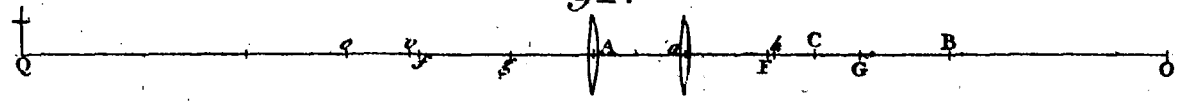

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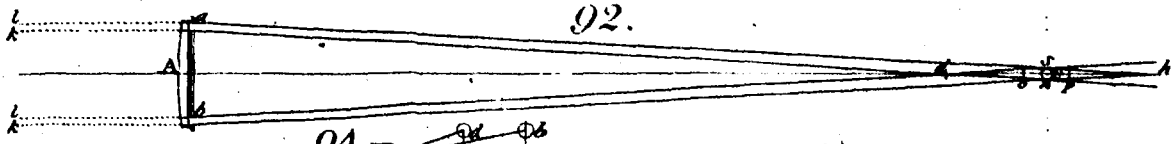

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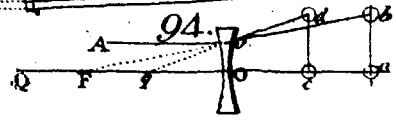

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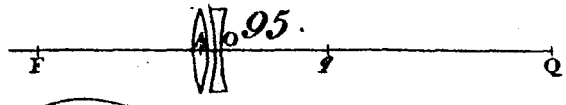

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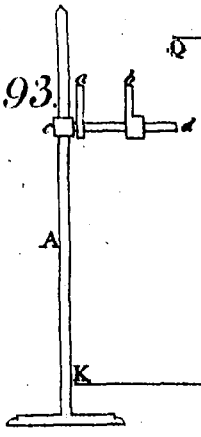

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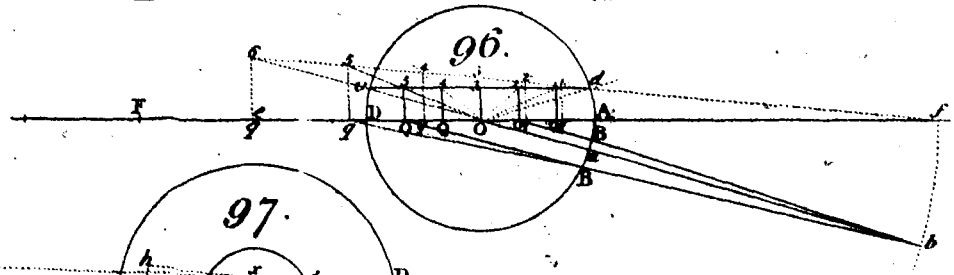

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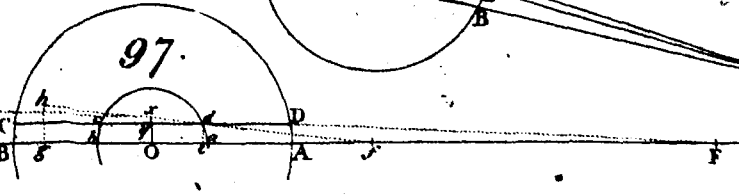

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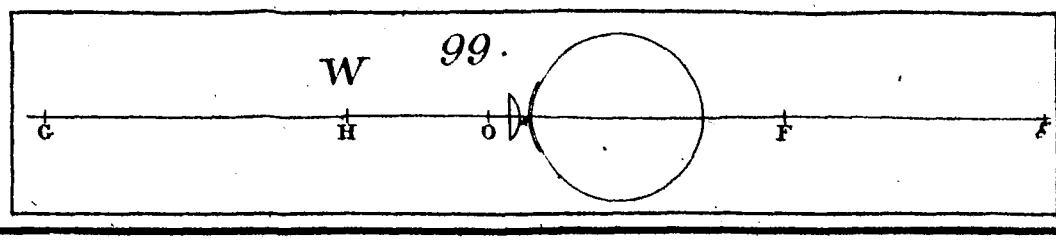
99.



ſhould have a ſhort tube before it, ſuch as will exclude all light but what is contained in the field intended to be taken into the picture. This tube ſhould be black on the inſide; and it would do ſo much the better, if the whole inſide of the dark room was alſo black. The ſpectator himſelf, at leaſt whilſt he looks at the picture, ſhould be alſo in the dark; for by this means, the pupils of his eyes will enlarge, and as they do enlarge, the apparent brightneſs of the picture will increaſe, and being alſo free from extraneous light, the impreſſions upon the retina's will be the more vivid and ſenſible.

And the objects well illuminated.

390. The objects ſhould be alſo well enlightened; otherwiſe, notwithſtanding all the above requiſites have been taken care of, if the weather is dark, or the objects are in the ſhade, the picture will be obſcure, dull, and no ways agreeable. This phenomenon therefore of the camera-obſcura, ſhould never be attempted but in a clear day; and it is beſt of all when the ſun ſhines upon the objects from the oppoſite coaſt; that is, if the proſpect is weſtern, the phenomena will be beſt in the morning or forenoon; if eaſtern, in the afternoon; if northern, about noon. A ſouthern aſpect is the worſt of any for the camera-obſcura, becauſe the ſun is never oppoſite to it; and in this caſe, the experiment will ſucceed beſt either betimes in the morning, or late in the evening; that is, when the ſun ſhines moſt directly upon the objects. In all caſes, a great deal of light is ſtopped by lenſes, as is manifeſt from the vividneſs of the images of objects ſeen in them by reflection: And in the preſent caſe, the light impinging upon the paper, or whatever the obſtacle be at which the picture is formed, is there diſperſed, ſo as to be much rarified before it comes to the eye. And hence it is, that the *chamber* requires to be very dark, and the objects well illuminated.

Proper apertures of lenſes for the camera obſcura.

391. The lens, uſed for this purpoſe, ſhould have a proper aperture. If the aperture be too ſmall, the picture will be dark and obſcure, and upon that account indiſtinct and unpleaſant; and if it be too large, the picture will be indiſtinct on account of the aberrations of the extream rays, and alſo becauſe the room will be too much enlightened by adventitious light entering into it; and this laſt cauſe has no ſmall ſhare in marring the pictures, as it proportionably renders them more obſcure: But when the aperture of the lens is moderate, the vitious rays on account of the aberrations are, in the preſent caſe, at their entrance into the eye very dilute and rare, and therefore ſcarce ſenſible. But the apertures here will admit of ſome latitude; and they may be a little narrower or wider, according as the objects are more or leſs illuminated; and according as they are nearer

or farther. It is beſt therefore to leave the aperture of the lens, juſt wide enough for the darkeſt weather it is to be uſed in, and to have a narrow ring to clap on occaſionally.

By making obſervations as carefully as I could, in a pretty clear day, the objects being at about 150 yards diſtance, and the ſun ſhining obliquely upon them: A lens of 18 inches focal length, I thought did beſt with an aperture of about 1 inch or $1\frac{1}{4}$ inch in diameter, according as the weather varied; and the weather being overcaſt, an aperture of $1\frac{1}{2}$ inch ſeemed to do better than a leſs: And had the objects been nearer with the ſun bright upon them, I doubt not but that an aperture of ſomewhat leſs than an inch would have been beſt. This lens when open was $2\frac{1}{4}$ inches wide, and it ſeemed then to form a picture pretty diſtinct; but on clapping to it a ring having a proper aperture, the difference was very conſiderable. The ſun ſhining upon the objects; a lens whoſe focal length was about 30 inches, required an aperture at the leaſt of $1\frac{1}{4}$ inch; and it would bear at moſt, one of about $1\frac{3}{4}$ inch. The picture was thrown into a box placed in a light room; but had the room itſelf been dark, thoſe apertures would have bore, and even required to have been, conſiderably leſs. In this laſt caſe, an aperture of $1\frac{1}{2}$ inch ſeemed to ſuit very well with a lens of eight foot focus.

From theſe obſervations, an eſtimate might be made of the proper aperture of a given lens, the object being under given circumſtances. But as the focal lengths of the lenſes are increaſed, there will be no great nicety required in ſettling theſe apertures; only I think they ſhould not be taken much wider, if any, than in the ſubduplicate proportion of thoſe lengths: And let us do what we can, the apparent brightneſs of the picture will decreaſe nearly, as the ſquare of the focal length of the lens is increaſed. For this apparent brightneſs will be nearly, as the denſity of the light in the picture, divided by the denſity of the adventitious light in the room. And whatever is the focal length of the lens, the denſity of the adventitious light in the room, will be nearly as the ſquare of the linear aperture of the lens; and to preſerve the ſame denſity of the light in the picture, the linear aperture muſt be as the focal length of the ſame lens. Wherefore, if the ſame aperture be retained, that is, if the adventitious light in the room remains the ſame; the apparent brightneſs of the picture, which in this caſe is as the denſity of the rays that form it, will be reciprocally as the ſquare of the focal length of the lens: The apparent brightneſs of the picture will be alſo in the ſaid proportion, if the denſity of the light upon it be preſerved the ſame; that is, if the linear

linear aperture of the lens be taken as its focal length; for in this case, the adventitious light will be directly as the square of the focal length. And in settling the linear apertures of different lenses, we shall not much mend the matter, by taking any proportion between the simple and subduplicate of their focal lengths; the apparent brightness being here, I think, the chief thing to be regarded, the aberrations not being sensible; and this brightness, as near as I could estimate, is nearly in the proportion above assigned.

392. And hence, without some method of * preventing the adventitious light, from diminishing in so great a degree the apparent brightness of the picture; we shall soon come to a limit, beyond which it would be useless to extend the focal lengths of our lenses. If the picture be received upon paper, a lens of about 8 foot focus, I take upon the whole to be preferable to one that is longer; this will represent objects in a landskip, larger than we commonly see them in pictures; and the picture will be much brighter, distincter and pleasanter, than if the lens had been flatter. A lens of about 8 foot focus, in the present case, has manifestly a very considerable advantage above a lens of 12 foot, and more still proportionably above a lens of a longer focus. As we decrease in our focal length, the images will appear brighter, sharper, and rather more distinct and pleasant, till we come to a certain limit; after this, the picture will be less pleasant and seemingly less distinct, because we shall lose some of the smaller parts of the objects. This limit as to the least focal length, I do not take to be less than 2 or 3 feet; especially if the picture be received upon paper within a room, for in that case, there will not be a convenient space for the spectator to view it in, if the lens be shorter than as above. Upon the whole, I take it, that a lens from about 4 to 6 or 8 foot focal length, is the best size of any for the camera-obscura. But in some cases, a lens of a yet longer focus will do better; and in some others we may descend to so small a length as 12 or 8 inches focus, and the picture will be pleasant enough.

Limit of the focal lengths of lenses for the camera-obscura.

393. In some cases, the breadth of the picture may be about two-thirds of its distance from the lens, and even more if the paper for receiving it be made a little concave; that is, the picture may take in a field or angle of near forty degrees. But in most cases when the field is so wide, the middle half of the picture will be distincter than towards the extreams; and therefore generally it is best not to take in a larger angle than about 20 or 30 degrees.

Extent of the field that may be taken in the picture.

* How this may be partly remedied will be seen a little farther on.

Glaſs poliſhed only on one ſide for receiving the images.

394. If inſtead of the white paper or cloth for receiving the picture, be placed a glaſs having both its ſides ground flat and parallel, and poliſhed only on one ſide; a ſpectator behind the glaſs will ſee the images vaſtly more vivid, diſtinct and bright, than by the reflection from the paper. The rays of light in their paſſage through the rough ſurface of the glaſs, are there broke and diſperſed, after the ſame manner as they were by the paper, and ſo as to paint there as it were the images of the objects from whence they came. But here the rays tranſmitted are not ſo much diſſipated, as they are by the reflection of an opaque ſurface; and ſo the rays being denſer, the ſpectator behind receives a greater quantity in each pencil, than when he ſees the picture upon the paper *. In this caſe, the ſpectator is alſo leſs offended with extraneous light, and it would be eaſy to exclude from him all ſuch light, excepting what falls on the glaſs itſelf, and is tranſmitted through it; and hence, from theſe two concurring cauſes, we may have a good picture by a much ſhallower or a much deeper lens, than when the rays are reflected from the paper.

The phenomena farther explained.

395. It might be expected that if the flat glaſs be taken away, and a tranſparent one be placed in its room, or if nothing be placed there, we might yet ſee ſuch a picture as we did before. We ſhall indeed ſee a narrow image ſomewhere before us; but its true place will not be readily found; and when it is found, it will not be ſo well defined, diſtinct or bright, or ſo like a real object, as what we ſaw on the rough ſide of the glaſs or on the paper. In this latter caſe, the rays diverge to the eye from the rough ſurface as if that was an object, or a picture like what we ſee was painted upon it; and this ſurface being uniform and colourleſs, the rays coming to the eye ſtill retain the original colours, *&c.* of the objects.

Fig. 100.

Thus, the rays PB, PA, PC; QB, QA, QC; RA, RB, RC; proceeding from the extreams and middle of a remote object, are refracted by the convex lens BAC ſo as to unite in the reſpective focuſes *p*, *q*, *r*. Now if the eye be placed behind the image *p q r*, as at O, it is manifeſt that only a very ſmall part of it can be ſeen at once; and upon this account it will ſcarce have the reſemblance of a picture. But if the glaſs GK poliſhed on one ſide be placed in the *foci p*, *q*, *r*; the rays in the ſeveral pencils inſtead of proceeding directly onwards, as repreſented at *p* and *r*, will be inflected various ways, as repreſented at *q*: So that the image *q* will be viſible on either ſide

* In conſequence, and as a fuller proof of this, the ſpectator cannot ſee ſo much of the picture at one view, as he could when it was reflected from the paper; and here he muſt move his head, ſo as to receive the rays in a nearer direction to that which they had when they reached the glaſs.

ſide the axis, within a certain limit EEE. And the rays here having undergone no other modification excepting that of being inflected, the phyſical point *q* retains ſtill the figure and colour of the original point in the object. And as the rays belonging to every point in the image, are inflected in like manner, all diverging different ways from their reſpective focus's in the glaſs; ſo to an eye ſomewhere at E, they exhibit a viſible picture, of a conſiderable breadth, as if it was really painted on the glaſs. In like manner if GK be a white paper, the rays falling on every point of it will be inflected different ways; as if they diverged from the paper, as repreſented by the rays *q*D. But, as before obſerved, there will not ſo many rays come now to the eye from each point of the image, as when they were tranſmitted through the glaſs; and for which reaſon the picture will be leſs bright and vivid.

396. The window-ſhutters of the room intended for a *camera-obſcura*, muſt be made to ſhut very cloſe; and if there be any creviſes, they ſhould be ſtopped by nailing ſlips of black cloth or leather over them. This being done a round hole is to be made in one of the ſhutters, SH, againſt a ſaſh or caſement which muſt be left open during the experiment; and cloſe to the ſaid hole, a convex lens BAC is to be fixed. The paper ſkreen or glaſs half poliſhed GK, muſt have a proper ſupport to hold it in the focus *p q r*, which focus is readily found, by moving the ſkreen to or fro till the picture is moſt diſtinct. Camera-obſcura how made. Fig. 100.

397. For holding the lens, there is a little convenient apparatus to be had ready in the ſhops, called a *Scioptric-ball*, and by ſome an † *Ox-eye*. This conſiſts of a wooden ball BB, confined in a hollow zone of wood CCDD; the flat ſide of which CC, is laid cloſe to the window-ſhutter, againſt a hole made big enough to receive ſo much of the ball as projects forwards; and the whole may be confined by three buttons faſtened to the ſaſh, as *a*, *a*, *fig.* 100, made and placed ſo as to turn over the rim *a a*. The wooden zone conſiſts of two parts Scioptric-ball. Fig. 101.

† This aptly enough may be conſidered as a kind of artificial eye; the wooden zone may repreſent a fruſtrum of the eye's orbit, and the voluble ball within, that of the natural eye. A lens whoſe focal length is about = EA being ſcrewed at the end E of the hole in the ball, may repreſent the cryſtaline humour; and fixing in a ſhort tube at A, a glaſs with parallel ſides and poliſhed only on one ſide, the pictures of objects will be ſeen upon it, as they are upon the retina; and to make the thing more natural, the ſides of this glaſs may have a ſphericity equal to that of the ball. If we ſucceſſively turn the lens towards objects at different diſtances, we ſhall ſee a neceſſity of a different configuration in the eye for viſion at different diſtances; and one way how diſtinct viſion may be reſtored, as by altering the length of the axis; as will appear by drawing in or out the tube at A till the picture is diſtinct upon the glaſs. If a lens having a longer or ſhorter focus be ſcrewed at E, we ſhall have the caſe of a long or ſhort-ſight perſon; and by holding a proper convex or concave lens before that at E, we ſhall ſee the effects of ſuch glaſſes in reſtoring diſtinct viſion.

parts CC, DD, ſcrewed together after the ball is placed between them, and ſo as to admit the ball to turn eaſy round its center. The ball has a large cilindrical hole, at each end of which is cut a female ſcrew, for receiving a lens as A fixed in its cell. By the motion of the ball, the axis of the lens may be turned different ways; and the ſphericity of the zone, prevents any light from being tranſmitted betwixt it and the ball. There are commonly two lenſes, of different focal lengths, fitted to the ball; and by uſing them both at once, there will be in effect a third, having a ſhorter focus than either. When one lens only is uſed, it ſhould be fixed to the end A, that ſo the ſhort tube may exclude ſome of the extraneous light out of the room.

Picture may be ſeen out of another room.

398. If the picture upon the glaſs half poliſhed GK, be at a moderately ſmall diſtance from the partition, dividing the dark room from an adjacent one; a perſon in the adjacent light room, may ſee this picture very well through a hole oppoſite to it in that partition. This hole, which may be ſhut cloſe at other times by two folding-doors black on their inſide, and covered with a picture-frame, or &c. ſhould be big enough to receive part of the perſon's head, ſo that when he looks he may be quite in the dark; the little folding-doors contributing much to this end, and ſomething black may be alſo placed over them. The picture may be thrown to either ſide of the dark room, by a plane ſpeculum properly placed, as may ſuit beſt with reſpect to an adjacent light room. A contrivance of this kind may be entertaining, and the more ſo, as the phenomena will be leſs expected than in the dark room.

How to ſet the picture upright.

399. An inverted picture does not look near ſo pleaſant as an erect one, eſpecially if it repreſents live animals; and a perſon cannot be known near ſo readily in the inverted picture, as after the ſame picture is made upright; though in the former caſe, the images be much brighter and ſharper than in the latter.

If a perſon facing the picture, whether it be upon paper or on the glaſs half poliſhed, looks into a looking-glaſs held before him with its face towards the picture and inclining downwards; the images ſeen in the glaſs will be erect, and in their due poſition as to right and left if the picture be on an opaque ſurface, otherwiſe they will be inverted. In this caſe the looking-glaſs ſhould be pretty large; but as ſuch are cumberſome and ſeldom true, it will be more convenient to place a ſmall plane ſpeculum near the lens, ſo as to inflect the rays before they form the picture. In the annexed figures, PA, QA, RA, are the axes of pencils of light coming from an object without doors, which after paſſing through the convex lens A, are reflected by the plane

Fig. 102 & 105.

plane ſpeculum *s t*, into the lines *s p*, *v q*, *t r*; and the *foci* of theſe pencils being at *p*, *q*, *r*, an image of the object is formed upon the plane G K. In this caſe, the lens and the plane ſpeculum may be both placed in a ſhort ſquare tube, open only at the furtheſt end, and on the ſide facing the ſpeculum. By this means, the room will be much darker, and the picture proportionably livelier, than when the light is tranſmitted directly into it through the lens. The ſpeculum may be ſo fitted in its box, as it may be eaſily inclined to any angle; and being but ſmall it may be made of ‖ metal, which reflects abundantly more light than glaſs; or if it be glaſs, its ſides ſhould be exactly flat and parallel. The tube may ſlip into a ſquare box made in a board fitted to the place made for the ſcioptric-ball; or the tube might be round and fitted to the hole in that ball.

400. In all caſes, the reflecting plane ſhould be ſo inclined as to reflect the principal pencil of rays, or that whoſe axis is coincident with the axis of the lens, nearly at right angles to the plane GK; for by this means, the focus of all the pencils will be nearly in that plane, which is perpendicular to the axis *v q*, and ſo the whole picture will be diſtinct. In *fig.* 102, 103, the reflecting plane makes half a right-angle with the axis QA; and ſo the rays coming along that axis are thrown directly upwards or downwards. In the one caſe the picture is formed upon the cieling, or on a paper parallel to it; and in the other caſe upon a paper on or parallel to the floor; and in both theſe caſes the tube muſt reach a little way into the room. The firſt method is inconvenient, as it is aukward and troubleſome to look directly upwards; and the ſecond method will be alſo inconvenient, unleſs the lens and ſpeculum can be placed high enough, and far enough into the room; to admit the picture to be about the height of a table above the floor, and to give room to the ſpectator to ſtand or ſit betwixt it and the window. If this be done, the picture will lie very convenient for copying; and in this caſe, the figures will be alſo in their due poſition as to right and left. In *fig.* 102, by placing another plane ſpeculum *x* above *s t*, and making an angle with it a little † obtuſe; the picture may be ſhown on a white plane *g k*, inclining a little from the window. The two ſpeculums and the lens might be fitted in the ſame box; and the picture thus formed would be very pleaſant, if both the

‖ That a hard whitiſh, metal well poliſhed, reflects abundantly more light than quick-ſilvered glaſs, ſeems to me very manifeſt; and of this Mr. *Short*, who has had much experience in theſe matters, is very certain. The contrary opinion, which for a long time prevailed, muſt have aroſe from the compariſons having been made with improper metals.

† The angle ſhould be a little obtuſe, that the picture may be a little higher than the ſpeculum.

the ſpeculums are good, and the objects without are well illuminated by the ſun; otherwiſe it will be dark, becauſe of the loſs of light by the double reflection.

Fig. 104. 401. In the annexed figure, the paper-ſkreen GK is ſuſpended obliquely from the cieling. This method gives a very agreeable picture, as the images are not only erect, but alſo in their due poſition as to right and left.

A ſkreen for optical uſes.

A large ſkreen would be uſeful here, and alſo for ſome other optical purpoſes; and it would be alſo convenient to have it portable. The ſkreen may be made of any ſize by ſheets of white paper paſted ſmoothly on cloth, to which ſhould be fixed a ledge and roller, as is commonly done to large maps. The two ſupporters I had made were about 5½ feet high, and in the top of each was a round pin drawn tightly, by a wedge thro' a mortiſe in the end of it, againſt a ſhoulder on the other ſide; at the other and inner end of each pin was a groove, in which ſlid a rail of about ſix foot long. Between theſe two rails was fixed occaſionally the rail of the ſkreen, being there confined by two wedges going through the ends of it; and in the bottoms of the ſaid ſide-rails, the roller turned ſtiffly for bringing the ſkreen ſmooth, being drawn tight in the holes by two wedges as above. Through the groove in the inner end of each of the center-pins, and through one of various holes made in the ſide-rails, went a ſmall peg by which the ſkreen could be ſuſpended at various heights; and by turning the ſaid center-pins, it was eaſy to give the ſkreen any deſired inclination. The whole was eaſily put together, or taken aſunder, ſo as to take but little room when out of uſe.

Fig. 105. 402. In this figure, GK is a glaſs poliſhed only on one ſide, which ſhould be placed rather higher than a perſon's head, that he may ſee the picture by rays coming through the glaſs, pretty directly. The ſpace between KG and the lens ſhould be incloſed on all ſides, ſo that no light can come into the room but through the flat glaſs GK. By this means, we ſhall have the moſt lively picture that can be exhibited in the camera-obſcura, tho' it will be inverted as to right and left; and perſons at a moderate diſtance without, may be known as well as if we looked at them through the window. To put this method in practice in a common room; the lens, plane ſpeculum, and the flat glaſs, ſhould be fixed in a kind of priſmatical box, black on the inſide, which may be readily hooked againſt the upper part of the window, ſo that the tube containing the lens fits to a hole made in the ſhutter to receive it; the diſtance between the lens and GK being

before

before adjusted, according to the distances of the objects that are to be taken in, and the upper window-sash let down. If the objects are lower than the window, the plane speculum *s v t* need be but short. Those who have a spare room or closet facing a good near prospect, by letting such an apparatus stand in it, might at any time agreeably entertain themselves, by only shutting the door upon them; there being something more entertaining in such a picture as this, than in seeing the real life †. Either another window or the bottom of this, might serve to enlighten the room at other times.

403. The entertainment which the *camera-obscura* affords to all classes of people, is what led me to be so particular in its description ‡. And before I dismiss this subject, I shall just observe, that if a concave speculum be placed against a small hole in the window-shutter of a dark room; a picture, like that we have been describing, will be formed on a sheet of paper placed near the principal focus of the speculum. Picture by a concave speculum.

404. One considerable use that the *camera-obscura* has been applied to, is in assisting young ‖ artists to draw in perspective; and for this purpose different sorts of portable boxes have been contrived, the most common of which is that I am now going to describe. But first let it be observed once for all, that all boxes made for this purpose should be close, and black within; and that the spectator also when he looks, should be screened from the light as much as possible. Portable *camera's*.

The annexed figure, represents a vertical section of the box through the middle: ST is a looking-glass received in grooves on each side of the box; and making half a right-angle with the axis Q A *b* of the convex lens A, fixed in the short and sliding tube *d g f e*. GK is a glass polished Fig. 106.

† A compleat thing of this kind, is a little room built by Dr. *Hoadly* at *Chelsea*. The roof is pyramidical like that of a summer-house; and out of the sides of it are projections like those of dormer lights or windows; in each front is placed a lens with a plane metallic speculum fixed at the proper angle for throwing the picture upon a large glass half polished, fixed against the cavity made by the said projection close to the cieling: There is another apparatus of the same kind at top, the lens and little speculum being made to turn all round; so that there are five pictures in the room at once, and which give sufficient light for reading a small print.

‡ *J. Baptista Porta* is, I believe, the first author that takes notice of the inverted pictures of external objects formed upon the wall of a dark room, by rays coming through a small hole in the opposite side. These phenomena he describes at large in his *Magia Naturalis, lib.* xvii. *cap.* 6; and also shews how these pictures will be improved by a convex lens, and made erect by a plane speculum, *&c.* This work consisting of twenty books, Mr. *Molyneux* quotes as being printed in 1589; but *Hartley's Catalogus Universalis* places the first edition at *Francford* in 1591. *P. Gaspar. Schottus* says that four of these books were published at *Antwerp* in 1560, but these books are on subjects no ways relative to the passages here referred to.

‖ It is certain that the camera-obscura is very useful for this purpose, and also that in many cases it exhibits what may be deemed a fine picture; but how far it may be useful to a painter is another consideration, and what is beside my present purpose to discuss.

polished only on one side for receiving the image *p q r*, formed by rays which before their reflection by the speculum ST, tended to form an image *a b c*. LM is the lid turned upwards, and which now serves to screen the picture and the spectator from the light, the better to effect which, there are also folding leaves on each side; so that the whole space LMNB above the box is inclosed, excepting in the front NB and the top MN. The two glasses when out of use, are put into a drawer sliding in the cavity EPCD, with something soft under and between them to prevent their breaking in carriage. Also the pyramidical trunk EFHI is usually made to hook on and off, so as when out of use to be placed in the box. But this seems to be needless.

Another box. 405. As it is difficult to have a large looking-glass that is true, a small one may be made to answer the purpose; as in *fig.* 102, G*ab*K may represent the section of a box, at the bottom of which is fixed at the proper angle the plane speculum *s v t*; the lens A is moveable along with a short tube or drawer to the proper distance, and GK receives the picture as in the preceding.

How far the tube need slide. 406. In all cases, the tube carrying the lens should have a smooth and steady motion; to effect which, the principal bearings should spring against it, and it would be best to have the tube of brass. The tube need not have a motion of above a $\frac{1}{4}$ part of the focal length of the lens, at most; and for common purposes it need not slide near so much. Matters should be so ordered, that when the tube is pushed as far in as it can, the paper or glass for receiving the images, should be then in the principal focus of the lens; and objects that are beyond the other focus of the lens a 1000, 100, 50, 10, *&c.* times the focal length of the lens, will have their pictures in the box distinct, when the tube is drawn out, the $\frac{1}{1000}$, $\frac{1}{100}$, $\frac{1}{50}$, $\frac{1}{10}$, *&c.* part of that focal length. If the pictures of objects that are nearer than about 6 or 8 times the focal length of our lens are wanted, it will be best to use another lens of a shorter focus. By this means the short tube will still serve; for it would be inconvenient here to have the tube long, as it must be proportionably broad.

Fig. 106. 407. The rough side of the glass GK is usually placed uppermost, for the sake of copying with a pencil the picture upon it; and sometimes instead of the glass, is used oiled paper strained in a frame, for the same purpose.

If the board BP be made to slide out, and the glass GK be placed in its stead; the speculum ST being taken away, the lid LM shut, and the glass GK screened from the light coming to it from without;

out; we shall have an inverted picture much sharper and distincter than the former, and also much properer for the imitation of an artist, as the images here are duly situated notwithstanding their being upside down †. A draftsman, I should think, would make no other use of the *camera*, than to assist his imagination in drawing upon another paper he has before him. But when the picture is drawn upon the rough side of the glass or on the oiled paper, as it must be again copied, the error caused by the looking-glass may be then rectified. But for the purpose of drawing, the following construction of a box seems to be much more convenient.

408. The annexed figure is the box *fig.* 106, set upright upon the bottom, neither of the glasses GK or ST being here used. Instead of ST is here used the small speculum *s t*, which reflects the rays coming through the hole or opening *m n*, directly upon the lens A; and which are there refracted, so as to form a picture upon paper placed at the bottom BC of the box. To a spectator with his back towards the objects, this picture will be erect, and the images in their due position as to right and left. For viewing the picture, a slit may be made at I, which may be covered occasionally by two slips of boards sliding to each side: And for drawing, the front BL, which before was the upper part, must be opened; the lid ML supported by the folding leaves on the sides, serving here also to screen the artist from the light, and which with the addition of a piece of loose black cloth thrown over his shoulders, will keep it out effectually. The opening *m n* is easily enclosed, when the tube is used as in *fig.* 106. Fig. 107. A proper box for drawing.

409. If the speculum *s t* is a good one, it is indifferent whether the rays from the objects come to it, or the lens first; but if it be faulty, it will do less harm when placed after the lens, as represented in *fig.* 108; and therefore this last construction, for the purpose of a *camera obscura*, is always to be preferred.

410. The annexed figure is a larger box, made upon the same principles, and answering the same purposes, with the last described. The bottom part of it is the * *shew-box* for holding pictures before the concave speculum; and the top of it is the box, *fig.* 106, set with the face LB downwards, the lid LMN being now taken away. Or it will answer the same purpose, if the small box be set upright, as in *fig.* 107, the bottom BC being taken away. I had the little box designedly made Fig. 109. * 286. II.

† For this purpose of copying, the paper on which the picture is received, may be divided into squares, either by cross lines or by a frame with cross threads or wiers laid on occasionally: This frame would serve for dividing the glass half polished, if it be used instead of the paper; and the paper on which the copy is to be made being divided into squares in like manner; and these numbered; in either case it will be easy to take a correct copy of the principal lines, and the whole may be afterwards finished at leisure.

a cube, that it might fit on either way. By either of these methods, we shall have a large picture on a sheet of paper placed at the bottom of the box, and which will appear very distinct and erect through an opening at LM, or thorough the front MN: The said opening being shut, and the folding doors before [a] described laid open; a piece of loose black cloth thrown over the tops of the doors and the spectator's shoulders, he may easily gather about him, so as to exclude all light from the box; and in this position fall to work with his pencil. If there be crevises left betwixt the box and the edges of the doors when they are open, there should be thin slips of black leather nailed over them. A box made only for this purpose, might spread wider towards the bottom; but as the doors in that case cannot open so conveniently, I think this figure is preferable.

[a] 286. II.

To make the boxes more portable.

411. There may be various methods of putting boxes together, so as to be reducible into a less bulk for the conveniency of carriage. There is one which has obtained in our shops for many years, called a *Book-camera*, from its being made to fold exactly in the shape of a folio book; and to make a kind of *rebus*, it is bound with leather and lettered on the back, in the same manner †. To erect it into a box, one of the lids or cover of the book serves for the back, and the front makes part of the top, to which is joined with hinges a board having its form semicircular, and reaching beyond the bottom; to this top is fixed black leather or cloth to hang over the draftman's shoulders when he draws, and which serves as a front to the box: Two boards fastened by hinges to the ends, turn up and form the sides and also support the top; and by way of a tube for holding the lens, there is fastened to the top a leathern collar, which may be raised or depressed, by an arm coming from the frame of the lens to an upright board or bar sliding in one of the sides; the plane speculum is fixed in a frame so as to be set to any angle, and turning round the lens to different aspects. This box is founded upon the same principles with that *fig.* 107, the paper for receiving the picture being placed at the bottom; and the whole contrivance is not amiss, if care be taken that the parts are so well fitted and proportioned, that the box is steady when in use. But for home purposes, one would prefer a box remaining fast together.

Book-camera.

Another portable camera.

412. In contriving things for carriage, the weight as well as the bulk should be reduced as small as possible, and at the same time there should be as little trouble as may be in fitting the machine for use; and

† Instead of this leathern cover, a naked box would be better, so that it might be sent by a carrier without other package.

and when in use it should be also of a convenient height, if it may be, without a foreign support. I do not know a better method for obtaining all these ends in a portable *camera-obscura*, than the following *. In the annexed figure BD is the section of a square board, to each corner of which is fastened a stiff brass joint, wherein is fixed a round staff, of about the length and size of those used to support surveying instruments, and which like them consist of two parts screwed together. The board BD is of a size just big enough to admit a proper tube for carrying the lens A and reflecting plane *s t*, and which is made to slide smooth in another short and springing tube fixed in the said hole; or instead of these tubes may be used a leathern collar, like that to the book camera: Or rather, the lens should be placed first, as in *fig.* 108. The oblong board GK, which may consist of 2 or 3 pieces joined together either by hinges or pins, may have holes in the corners through which the legs at the joints may screw against shoulders left in the brass part for that purpose; or the legs may be easily fastened to the board by other methods, but the above I think is the best, as it will be a great stiffening to the whole. The legs as low as the board GK, on which the paper for receiving the picture is placed, is covered with a black leathern conical bag; which on the back and two sides may be hooked under the board GK to keep it tight; but on the fore part, the bag must be wider and hang lower, so as to admit a person's head and arm, and cover his shoulders sufficiently to exclude extraneous light from the picture. Fig. 110.

If the machine has four legs, one of them should be made shorter than the rest, and have a sliding foot which may be brought to stand fast upon any uneven floor. Or three legs might be made to answer the purpose here; and in either case, this stand might serve to carry a serveying or levelling instrument, *&c.*

413. To a person who is accustomed to draw by his eye, a camera of about 6 or 8 inches long and 2 or 3 inches wide, which he can easily put in his pocket, may perhaps answer all his purposes †. This may be made to be used like *fig.* 106, and with or without the plane speculum ST. But a painter will use the *camera obscura* with discretion; and there are many cases wherein a convex speculum, or a concave lens, may perhaps answer his purpose much better, Pocket *camera.*

414.

* I had an intimation of this contrivance some years since by Mr. *Dugal Campbel*, one of his Majesty's engineers. But one that he got made being much too slight, did not answer.

† These small *camera's* neatly and conveniently made, are now to be had at our shops. They have been made with a lens of about $1\frac{1}{2}$ or 2 inches focus, and fitted to the heads of canes; but these are much too small to answer any purpose.

Shew-boxes.

414. Any camera-obſcura, wherein the picture is received upon paper at the bottom, may be eaſily converted into a *ſhew-box*, or one for improving prints. And it is pity that the common ſhew-boxes, are not made to anſwer both theſe purpoſes. The ſame lens may ſerve for both; but in the one caſe it muſt have a limited aperture, and in the other, the lens ſhould be broad enough to admit both eyes to ſee, through it, the print or picture at once. All that is required for obtaining theſe two ends, is to have either an aperture, or two lenſes with their proper apertures, to be ſhifted on or off occaſionally.

Fig. 107, 109.

The print to be viewed is to be placed at the bottom of the box with its top inwards, where it is enlightened thro' the front, now left open, either by day or candle-light; but it ſhews much more natural and pleaſant by day-light, as we are not accuſtomed to ſee proſpects by any other: The opening *m n* in the tube *fig.* 107, or the top box *fig.* 109, are to be turned the contrary way to what they are in theſe figures, that the box may ſtand between the ſpectator and the light. For the ſake of ſuiting different eyes, the tube ſhould have a little motion, that the lens may be brought either nigher to or further from the bottom than its focal length. But thoſe who are too ſhort or too long ſighted to ſee diſtant objects without ſpectacles, they muſt uſe the ſame here. The box ſhould be big enough to take in a large print or a book of prints; and it would be of ſome advantage, if two thin blacked boards be faſtened by hinges to the ſides near the top, to be ſet to the margins of the print; ſo as to limit the ſight, as much as may be, to a view of the print only.

In a ſhew-box, it is beſt to place the plane mirror outermoſt, as in *fig.* 107; becauſe any faults in that mirror will be leaſt prejudicial, when it receives the rays after they are refracted by the lens. The field of viſion will be the [b] more extenſive, the nearer the eye is to the lens; but the box *fig.* 107 will exhibit a print ſufficiently large; and therefore upon the whole, that conſtruction as to the placing of the lens and the looking-glaſs, is the beſt of the two for this purpoſe of viewing prints.

[b] 350. II.

Inconvenience of reverſing the pictures, how to be remedied.

415. I have above, referred only to thoſe boxes which had been before deſcribed for other purpoſes, for this alſo of improving prints. But theſe have this grievous inconvenience, that the plane looking-glaſs reverſes the print or picture as to right and left: The remedy is indeed obvious enough; another looking-glaſs will ſet all to rights, by reverſing the reverſion made by the firſt. The looking-glaſſes may have different poſitions; but in all caſes of this nature, it is beſt to incline a plane ſpeculum ſo as to make about half a right angle with the

the principal pencils. This disposition of a speculum will reflect more light than when it stands more oblique, and there will accrue the less damage from the imperfections of its figure. Let one of the plane mirrors be placed like TS *fig.* 109, and let the lens A be placed nigher to it, as in the line EL; then a plane looking-glass placed upright before the lens A, and inclined so as to make about half a right angle with its axis, will exhibit the picture in its due position †. By having the looking-glass before the lens to shift in and out occasionally, it will be very easy to adapt a *camera* like that *fig.* 109, in the manner and for the purpose last described. A box of this sort is now got into our shops, and was first made by Mr. *Nairne*, optical instrument-maker in *Cornhill*, in the year 1753. But this contrivance has its inconveniencies. The field of vision is rather too much contracted: From the almost unavoidable irregularities of the figures of looking-glasses, an indistinctness arises; and from the loss of light at their surfaces, the picture, unless it be strongly illuminated becomes too dark and obscure; so that upon the whole it does not appear so distinct and pleasant as with only one looking-glass.

In the *Gentleman's Magazine* for *April* 1753, we have the description of a box for improving prints by Mr. S. PARRAT of *Killerton*, near *Kirby-Lonsdale* in *Westmoreland*. And I am apt to think, that the shew-box which one Mr. *Johnson* from *Berwick* had last spring at *Charing-cross*, was much of the same sort. Having already dwelt long upon this subject, I might perhaps have contented myself with only referring to Mr. *Parrat*'s own account of his box, if I did not think the following construction to be more advantageous.

416. The box I got made is 16 inches long, 18 inches wide, and 24 inches deep. Within this box, and parallel to the top and bottom, is fixed a partition leaving a cavity above about 10½ inches deep; into this cavity slips another box of the same length with the outer one, so that both together may draw out to the length of about 27 inches, which is the proper distance betwixt my lens and the print to be viewed. The lens is fixed in the end of this inner box, a little nearer the bottom than the top, having behind it an opening about 4 inches wide and 1½ inch deep. The lens is included within a short tube, blacked on the inside, and of such size as to hide all the box, excepting the print itself at the farther end; and for confining the view to prints of different widths, the sides of this tube or box have an angular motion. To prevent the inner box from dropping when drawn out, A new shew-box.

† Or, the lens might be placed first, as in *fig.* 107, and the two speculums, inclined in the manner above described, behind the lens. This disposition of the lens and speculums, will give a distinctier picture than the other, but the field of vision will be more contracted.

out, its lower end reaches down to the table, so as always to support it level. The farther end of the outward box, against which the prints are placed, is fixed to a board sliding smoothly upon the bottom above-mentioned, so as it may be drawn out far enough to take in a large book of prints; and that it may be supported, this end also reaches down to the table, being strengthened by two brackets fixed to it below and to the sliding board. At the ends of the two sides, are fixed two slips or cheeks about $\frac{3}{4}$ of an inch wide, for confining the prints or a book betwixt them and the sliding end; these slips slide in grooves made in a small bar fixed across the top, and another lying over the sliding board, so as to hide all except the print itself; an advantage not to be slighted ‖. For illuminating the prints, about half the top turns up stiffly upon two pegs, so as to stand at any angle, and when upright it drops $1\frac{1}{4}$ inch within the box; the under-side of this lid is covered with white paper for reflecting light upon the print, which together with that falling directly upon it through the aperture now made, will illumine it sufficiently. The whole together is, I think, very convenient for the purpose, and has a better effect than any I have yet seen. The box was made so deep as above described, that when placed upon a common table, a person sitting upon a chair might look into it at his ease, without much stooping. The bottom part, which is every way inclosed and having a door on one side, is big enough to hold several hundreds of prints; and for the sake of distinguishing them into several sortments, there are several sliding shelves which will draw out at the door, which is about $2\frac{1}{2}$ inches narrower than the side, to make room for the brackets before mentioned for strengthening the sliding end. This box is blacked on the inside, and is easily converted into a *camera-obscura*.

A box of a small size.

417. If the bottom part be omitted, the box above described will be reduced to about half the size; and a proper case or chest for holding prints, may now be kept in the inner box. This box, but with some disadvantage, may be yet reduced into a much smaller size: Let the inner box before described, be made no deeper than you would have a drawer for containing prints, and let the outer box be made just big enough to contain this drawer. Let the lid of the case, excepting a slip of about 2 inches wide at the front which should remain fixed to the sides, turn upon joints, and † be supported upright,

‖ These sliding cheeks have fixed to them, and also to the box, slips of black leather, which will allow them to move in the whole 5 or 6 inches.

† The method I used was by two joint rules having their ends turned upon pins fixed to this lid and the sides, and so as to lie within the thickness of these sides. This method may

upright, which may be done various ways. Against a proper aperture made in this lid, the lens is to be fixed; or it may be fixed in a board sliding to different heights, that it may be the better adapted to prints of different sizes.

The drawer also has a lid, which turns upright upon pegs made within about 1½ or 2 inches of the farther or lower end, which end when the lid is upright bears against a ledge or false end fixed across the drawer; and in this position the lid is fastened by a button in the said ledge. Against this lid the prints or books with prints are to be placed, the bottom of a book bearing upon the above-mentioned ledge; and the sides and top being all free from obstacles, a book may be here placed of any size. The prints should be supported at top; for this purpose, let there be two narrow boards sliding upright in grooves on the back of the lid, and to the tops of these sliders, let there be fixed a narrow transom the whole width of the lid: By a spring, screw, or a wedge bearing against one of these sliders, the transom may be supported at various heights for holding prints of various depths. Along this transom, *let there be a thin slip either springing, or turning stiffly, against it for* holding the prints; this slip should turn upon joints, and so as to be easily opened, which may be effected by different methods. Within the upper half of the transom, let there be two slips sliding to each side, the fore-side of their outer ends being in the same plane with the fore-side of the lid or transom; at these ends let there be wooden slips drawn close to them by springs, for fastening the edge of a print. By this means, a very long or very deep print may be held straight at the top; and, were it necessary, it might be also supported at bottom, by two sliders drawing out to the sides, as above; and having grooves in them, or by two slips turning upon joints. For confining books of different thicknesses; there is a narrow board or false end, to be placed occasionally into different correspondent grooves, made a little asunder in the sides of the drawer a little before the lid; and so as the top of the said end, when there placed, may stand about an inch or better above the sides of the drawer. The light is now to come upon the print sideways; and having placed the lens and print at their due distance asunder, move back the table against the pier, or shut so much of the window till you have the proper degree of light; too much light being here very prejudicial, and there is but little danger of having too little.

This little apparatus, when out of use, may be placed as a drawer in a chest or under a table; and it will answer extremely well, if due care

may be useful in many cases, where one thing is to be held at a given angle to another.

care be taken as to the proper degree of light upon the prints: It has one considerable advantage above the box described in the preceding, unless that be made very big, the taking in of very large prints. But the excluding of all objects from the sight, excepting the print itself, and preventing the glare incident to near and bright objects, are advantages not easily counter-ballanced.

Having a lens fixed on a pedestal standing on a table, it would be easy to support prints against a wainscot having a proper light, so as to exhibit the phenomenon we have been considering, without the aid of any box.

The conclusion.

418. I shall now conclude this subject with the following observations:

1. In all cases, a lens for improving prints, should be broad enough to admit both eyes to see through it at once, or about 4 or 4 ¼ inches wide. This enables us the better to judge of the greater distance of the image of the picture, on which the effect greatly depends, as in like cases of naked vision.

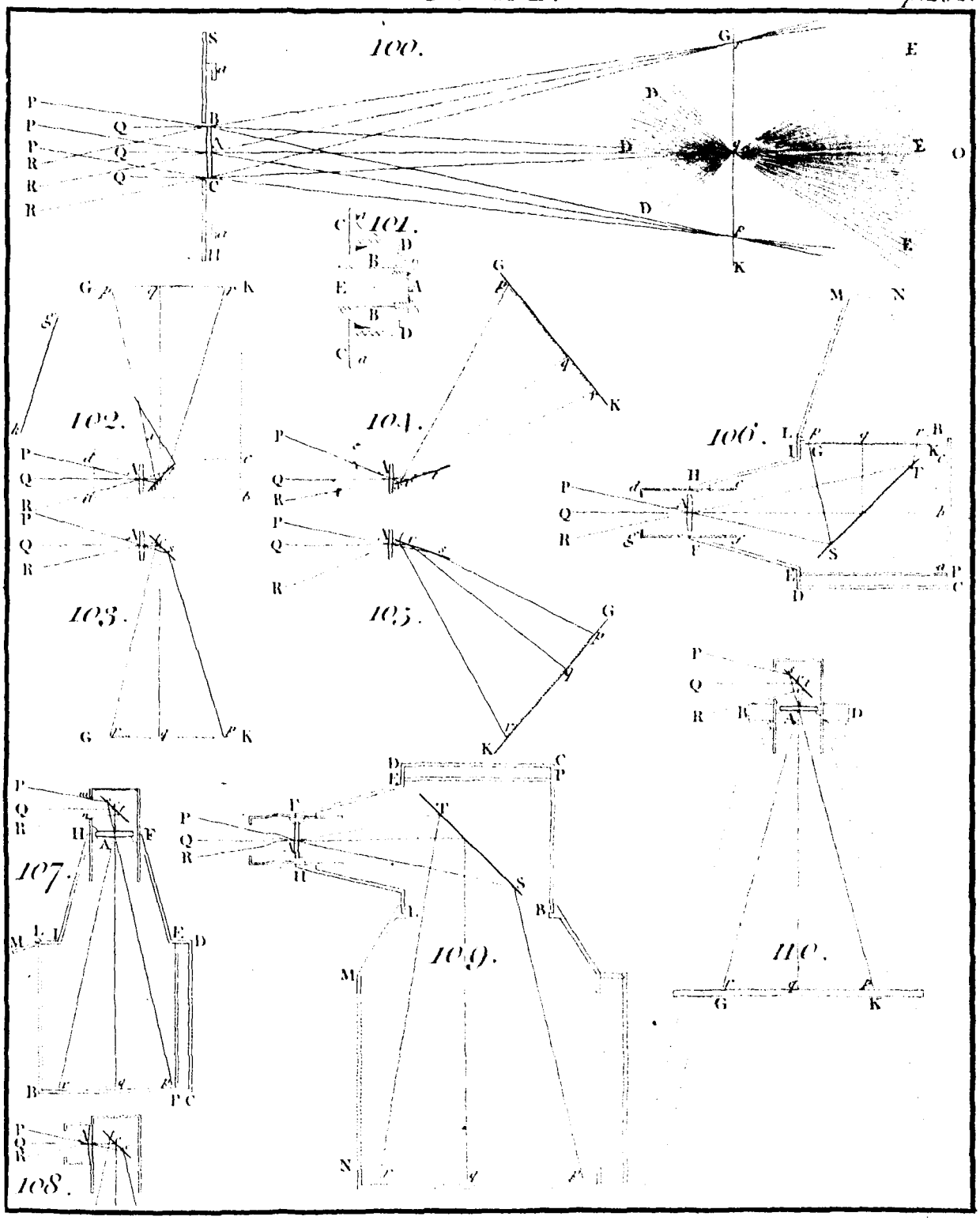
2. For common sized prints, which are about 14 or 16 inches wide, a lens of about 24 or 26 inches focal length, will best answer the purpose. If the lens be much more or much less convex, the prints will accordingly either appear coloured and distorted towards the edges, or the effect of the lens will not be sufficiently sensible. If the prints are much smaller or much larger, the lens should accordingly have a much shorter or longer focal length. So that to a given lens, the prints should be nearly of a size, and also done nearly on the same scale.

3. The prints should be moderately and properly enlightened, and so as not to have any glare, as near objects are apt to have. Upon this the phenomena greatly depends; the degree of light received from remote objects, such as are represented in the print, is usually very moderate. And it may be laid down as a general rule in optics, that a spectator should be skreened, as much as may be, from all light but what he receives from the object he looks at.

In the case before us, the lens increases the visual angle but very little, and yet the print appears greatly magnified, insomuch that the objects represented may appear of their natural size. The principal causes of this phenomenon have been already explained*: And it may be further observed, that the case here seems somewhat analogous to that of viewing objects at different distances by the naked sight: These appearing still of the same magnitude, whilst the first distance or visual angle is varied a great number of times; and to some one of those stations, the circumstances of the print may be exactly correspondent, with perhaps the addition of some favourable advantages.

* 317. II.

FINIS.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First & last signatures washed and deacidified with magnesium bicarbonate. Leaves mended as needed with heat sensitive tissue. Book rebound in full cloth with new acid free end signatures & unbleached linen hinges. Folding cloth box constructed to house & protect book & original boards.

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