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W. AVERELL HARRIMAN, Secretary
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Objective Methods Of Forecasting Winter Minimum Temperatures At Washington, D. C.

By

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OBJECTIVE METHODS OF FORECASTING WINTER MINIMUM TEMPERATURES AT WASHINGTON D. C.

INTRODUCTION

The research described in this report constitutes one of several preliminary projects which have been conducted in the process of designing systematic and objective methods for short range weather forecasting. Developmental work on 24-hour minimum temperature forecasting for Washington was begun in the Weather Bureau by Walter T. Wilson, whose technique required the selection each day of an index station located upwind from Washington at a distance equivalent to 24 hours of air flow on the 3-km. chart. The surface temperature at this index station was then employed in a simple linear regression formula to obtain the forecasted minimum temperature at Washington 24 hours later. Subsequently, however, Glenn W. Brier, in an unpublished paper, showed that results of comparable accuracy could be derived from a least squares equation relating the 0130 E.S.T. temperature at Columbus, Ohio, to the temperature at Washington 30 hours later.

In order to promote the further development of minimum temperature forecasting, the Weather Bureau sponsored research at New York University, in which objective methods for forecasting New York City minimum temperatures were devised. This research was later supported by the U. S. Army Air Forces. In the following pages, reference will be made to the work done at New York University.

It was the purpose of the Short Range Forecast Development Unit to apply the objective temperature forecasting methods developed for New York to Washington, D. C., and to conduct additional studies directed toward the refinement of these methods. In Section I, following, the technique originally developed at New York University, known as the trajectory method, is described. Section II traces the application of this technique to Washington. Sections III-IX discuss the efforts made to improve the accuracy of the forecasts obtained by the trajectory method, including a comparison of the results obtained with forecasts made from observed weather maps, and a test of the assumptions regarding the control of air flow near the ground. The final section sums up the present status of the work and indicates future plans.

This report would be incomplete without an expression of sincere thanks to the many individuals whose cooperation and assistance were vital to the progress of the research. Particular thanks are due to the members of the forecast staff at the Washington National Airport for their wholehearted cooperation throughout every phase of the work.

I. THE NEW YORK UNIVERSITY METHOD OF FORECASTING AIR TRAJECTORIES

The use of objective methods in temperature forecasting has for many years attained a high degree of efficiency in the "fruit-frost" protection service in California and other States. In this type of work, the basic activity is the forecasting at night of the temperature expected the next morning, with most attention given to a consideration of the effects of radiation in lowering the critical frost level during the night.

The methods of temperature forecasting discussed in this report differ from the fruit-frost work primarily in that the time lag involved is considerably longer, making the effect of air movement much more important. Hence, the forecast problem becomes more nearly that of locating on today's weather map the air which is expected over the station tomorrow, noting its temperature, and estimating the modifications likely to occur during its journey.

A method of estimating the mean seasonal point of origin of air coming into New York City was outlined in the first report[1] on the subject, submitted by New York University.[1] For each season of the year, a relationship was shown to exist between the temperature at a fixed station 24 hours upstream and the minimum temperature recorded at New York City the following morning. In a second report [2] prepared by the U. S. Army Air Forces Weather Station at the University, a method was outlined for forecasting the daily 24-hour surface air trajectory into New York, thus varying the index station from day to day.

As described in the second report, the first step in the method is to locate on the 0730 E.S.T. weather map the air expected to arrive at New York City 24 hours later. Next, the modification in temperature of this air during an overland journey is taken as a linear function of three independent variables. These are its current departure from normal,[2] its latitude (representing its distance north or south of New York City), and the difference between its current temperature and the New York City temperature. The constants for the equation relating these variables are determined by least squares. In the case of an air trajectory with any extensive over-water history, recourse is had to simpler empirical expedients.

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[1] See "References" at end of paper.

[2] The present report makes use of the 0730 E.S.T. decade normals as originally prepared by the Weather Bureau for the New York University research program.

The essence of this method is obviously the forecasting of a 24-hour air trajectory. This is done by the following objective technique: the current surface weather map is projected 18 hours ahead in 6-hourly increments, assuming that during the period it will remain under the control of the flow pattern existing at the 3-km. level on the previous night, and that it will not undergo change in shape, i.e., no deepening or filling; surface air trajectories are then constructed on the twofold assumption of geostrophic control of surface winds and a constant friction term.

II. THE TRAJECTORY METHOD AS APPLIED TO WASHINGTON, D. C.

Several modifications in the New York University technique were made prior to its application to Washington, D. C. They consisted, first of all, in extending the time lag to 30 hours, thus making it possible to forecast objectively from the 0130 E.S.T. map the temperature for 0730 E.S.T., 30 hours later. Other modifications are described in the following paragraphs.

Assumptions Made in Forecasting Air Trajectories

In an unpublished paper by Weiss [3], it is shown that during the winter months surface pressure systems in the United States tend to move in the same direction as the air flow at the 3-km. level and with an average speed which is 77 percent of the speed of the air flow at the 3-km. level. This rule was used in forecasting the surface pressure patterns. The rule used herein also states that during the 24-hour period for which the surface pressure pattern movement is forecast, the 3-km. pressure pattern will itself move eastward through 4.5 degrees of longitude.

The surface wind during the first 6 hours of the period is assumed to be under the control of the isobars on the current 0130 E.S.T. surface map. During the second 6 hours it is assumed to be under the control of the 0730 E.S.T. map, and so on. It is assumed that the air flow is directed at an angle of 30° across the isobars toward low pressure and that its speed is 80 percent of the geostrophic wind indicated by the surface isobars. Figures 1 through 5 illustrate the method of forecasting the surface pressure pattern and show how one of these forecast patterns compared with that actually observed. Trajectories computed from both forecast and observed pressure patterns are shown. Detailed instructions for the construction of these trajectories are given in Appendix A.

Development of the Forecasting Equations

In accordance with the assumptions stated above, trajectories into Washington, were constructed for each day of three winters (1942-43, 1943-44, 1944-45), winter being defined as the months of December, January, and February. These will be referred to in this report as "forecast" trajectories. The index station selected each day was the station nearest to the origin of the trajectory for which a 0730 E.S.T. normal temperature was available. As in the New York University method, the trajectories were divided into two groups, those with more than a 6-hour history over water being known as "water" trajectories and studied as a separate group. The "land" trajectories were then stratified according to whether the index station was situated north or south of the latitude of Washington, D. C. For land trajectories the following equation was used:

$$Y = ax_1 + bx_2 + cx_3 + dx_4 + e \qquad \text{(I)}$$

where

Y = observed 0730 E.S.T. temperature at Washington 30 hours hence,

x_1 = current 0130 E.S.T. temperature at Washington,

x_2 = current 0730 E.S.T.[3] normal at Washington,

x_3 = current 0130 E.S.T. temperature at index station,

x_4 = current 0730 E.S.T.[3] normal at index station,

and a, b, c, d, and e are numerical constants. In this equation, as throughout the report, all temperatures are understood to be in degrees Fahrenheit.

This expression is slightly different from that described in the New York University-AAF Weather Station report [2], in which the forecast equations are set up to estimate the quantity, $Y - x_3$, which represents the modification of the index temperature. In addition, the New York University formulas use a single parameter analogous to the quantity, $x_2 - x_4$, which allows only one regression constant or weight to apply to the two variables. The advantage of separating this quantity into two variables is not expressly demonstrated herein, but certain inherent advantages may become apparent from the discussion in Section VI of this paper.

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[3] See Appendix B for explanation of the reasons why 0730 E.S.T. normal temperatures were used in lieu of 0130 E.S.T. normal temperatures.

Fitting 9 months of record to Equation (I) gave the constants shown in the following formulas:

For northern land trajectories:

$$\hat{Y} = .15x_1 + .20x_2 + .31x_3 - .13x_4 + 14.9 \qquad \text{(II)}$$

For southern land trajectories:

$$\hat{Y} = .15x_1 + .51x_2 + .28x_3 + .15x_4 - 1.8 \quad , \qquad \text{(III)}$$

where $\hat{Y}$ is the forecast value of the 0730 E.S.T. Washington temperature, to be distinguished from the actual value. The mean residual from these equations, which were derived only from the land trajectories of the 9 winter months above mentioned, is 5.1°, with a maximum error of 29°. When tested on the forecast land trajectories during the winter 1945-46, the equations gave an average error of 4.0°, with an extreme error of 13°. In the case illustrated in Figures 1-5, the index station selected in making the forecast was Moline, Ill., with a temperature of 12° and a normal of 18°. The Washington temperature and normal were 43° and 30°, respectively. The forecast for Washington, after substitution in Equation (II), was therefore 29°. The temperature observed was 26°.

Formulas for forecasting the temperature when the controls indicate a water trajectory will be discussed in Section V.

III. THE POSSIBLE USE OF MORE ACCURATE PROGNOSTIC PRESSURE PATTERNS

After these results had been obtained, a test was made of the performance of forecasting equations based on the assumption of entirely accurate prognostic pressure patterns. Actual observed weather maps, drawn at 3-hour intervals, were used for the purpose. Trajectories were constructed from these maps for the same nine winter months, and the same assumptions were made regarding friction as in the forecast trajectories. The following equations, to be known henceforth as the "observed map" formulas, were thus obtained.

For northern land trajectories:

$$\hat{Y} = .22x_1 + .46x_2 + .38x_3 - .18x_4 + 4.6 \qquad \text{(IV)}$$

For southern land trajectories:

$$\hat{Y} = .28x_1 + .22x_2 + .29x_3 + .30x_4 + .5 \qquad \text{(V)}$$

The average error of these formulas when applied to the original data (winters, 1942-45) was 4.3°, with a maximum error of 19°. Tested on the winter of 1945-46, they gave an average error of 4.7°, with a maximum of 15°. Since there was but a doubtful increase in the accuracy of "forecasts" made with this type of

equation, even when the pressure patterns were known in advance, further steps were taken to compute trajectories by other methods.

It will be noted that the constants in equations (IV) and (V) differ from those in (II) and (III). These differences are due in part to sampling variation, including the facts that an "observed map" trajectory may select a different index station than the "forecast" trajectory for a given day, and that some cases will be listed as land trajectories when computed from forecast pressure patterns, and as ocean trajectories when computed from observed pressure patterns.

IV. TESTS OF ADDITIONAL TRAJECTORY COMPUTATIONS

Formulas Derived Assuming No Friction

It is questionable whether the sample of air whose temperature is measured in the morning at Washington has actually traveled near the ground under the influence of friction or has traveled instead at some upper level. To examine this point, trajectories were constructed from observed 3-hourly maps for the same 9 months (winters, 1942-45), assuming full geostrophic flow. The following equations were developed.

For northern land trajectories:

$$\hat{Y} = .22x_1 + .13x_2 + .35x_3 - .04x_4 + 12.5 \qquad \text{(VI)}$$

For southern trajectories:

$$\hat{Y} = .45x_1 + .20x_2 + .32x_3 - .02x_4 + 4.8 \qquad \text{(VII)}$$

The average and maximum errors of forecasts based on the original data were 4.6° and 22°, respectively, and on the test data, 4.6° and 19°. These results are very similar to those obtained by the other methods. In addition, it will be noted that the "index normal" (x_4) term is negligible in both (VI) and (VII) and has a negative sign for southern as well as northern trajectories. This conflicts with the rule that modifications due to changes in latitude should be of opposite sign, depending on whether or not the air is moving into Washington from the north (-) or from the south (+). It would appear reasonable to expect a like rule to hold true for the modifications due to crossing the isotherms of normal temperatures. Such an effect can be noted in equations (II), (III), (IV), and (V), which were derived from trajectories in which a friction term was included.

Use of Upper Winds in Tracing Air Trajectories

An additional study was made to discover whether a set of trajectories computed by another method would lead to better temperature forecasts, using the same general type of forecasting equation. To repeat, the method here developed states that the first step in forecasting the temperature at Washington is an attempt to locate on the current map the air which is expected in Washington at the time for which the forecast is made. The modification of this air over the trajectory is then said to be a function of its current actual temperature and current normal temperature and, in addition, a function of the current temperature and current normal temperature at Washington.

In this additional investigation, the air was traced 30 hours upwind from Washington by following the observed pilot balloon data at the first "standard" level, or at the first level above the ground for which upper air wind observations are transmitted by teletype[4]. Index stations were selected from the same three winters of original data and the following equations were derived:

For northern land trajectories:

$$\hat{Y} = .10x_1 + .30x_2 + .53x_3 - .11x_4 + 7.3 \qquad \text{(VIII)}$$

For southern land trajectories:

$$\hat{Y} = -.07x_1 + .76x_2 + .48x_3 + .20x_4 - 10.3 \qquad \text{(IX)}$$

The mean and maximum residuals of forecasts based on the original data were 4.5° and 17°, while test data taken from the winter of 1945-46 gave an average error of 4.3° and a maximum of 18°. These results were disappointing because it had been hoped to obtain a significant reduction in the residuals. The increased weight placed in these formulas on x_3, the term denoting the temperature at the index station, may indicate that this method is coming close to defining the source of the air. Here again, however, some of the differences are assignable to random sampling, etc., as mentioned earlier.

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[4] For an exact definition, see U. S. Weather Bureau Circular N, Instructions for Airway Meteorological Service, Appendix IX, "Code Used in Reporting Pilot Balloon Observations."

V. PARAMETERS TO BE USED WHEN THE FORECAST TRAJECTORY IS OVER WATER

The Water Trajectory Equation

When the trajectory is forecast to originate over the ocean, the lack of an index station immediately requires that a separate set of parameters be devised for use in such instances. In addition, the modifying influences associated with an over-water history are different than those for a land trajectory.

In the New York University technique, water trajectories were treated separately. Satisfactory results were obtained from a simple empirical rule, which consisted in taking as the forecast minimum temperature the average of the mean sea surface temperature of the trajectory and the temperature at New York at forecast time. A similar rule, when applied to Washington, gave results which were not satisfactory.

The following formula was finally developed from 46 cases where the trajectory was forecast to originate over the adjacent ocean and to have at least 13 hours of its 30-hour history over water:

$$\hat{Y} = .46x_1 + .18x_v + .12x_t + .23x_r + 1.3 \qquad (X)$$

where

x_1 = Washington temperature at 0130 E.S.T.,

x_v = temperature at 0130 E.S.T. at the point where the trajectory crossed the coast line from water to land,

x_r = mean sea temperature over the trajectory,

x_t = departure of the Washington 0130 E.S.T. temperature from the Washington 0730 E.S.T. normal temperature.

The mean error of the forecasts, based on the 9 months of original data, was 5.0°, with a maximum of 15°, and on the 1945-46 test data, 5.2° and 20°, respectively.

Actual vs. Forecast Trajectories

An analysis of the trajectories computed by the "observed-map" method described in Section III showed that 27 of the 46 forecast water trajectories were not water trajectories at all if the trajectories computed from observed 3-hourly maps were accepted as correct. An attempt was therefore made to determine whether such erroneous trajectory forecasts could be detected in advance. Examination of the maps in question showed that the presence along the East Coast of a flat high pressure system with its major axis slightly east of Washington, and with no indication of a secondary East Coast storm, would generally result in the non-occurrence of the forecast water trajectory. Figure 6 illustrates this type of situation.

When the "forecast trajectory" formula was applied to these 27 cases on the assumption that they were land trajectories, the average error was found to be 5.7°. The water trajectory formula yielded an average error of 4.3°. The smaller error of the latter is attributable in part to the fact that these 27 cases constitute a greater proportion of the total number of cases used in developing the over-water formula than in deriving the land equations.

An additional example of an erroneous over-water trajectory forecast is shown in Figure 7. The data for substitution in formula (1) are:

$$x_1 = 27;\ x_2 = 30;\ x_3 = 22;\ x_4 = 25.$$

The resulting forecast temperature is 28.2°. If the water trajectory be assumed correct and the corresponding formula (X) employed, the following values are substituted:

$$x_1 = 27;\ x_v = 18;\ x_r = 60;\ x_t = -3.$$

This substitution yields a forecast of 30.4°. The temperature observed 30 hours later at 0730 E.S.T., February 15, 1944, was 36°.

This apparent inconsistency, wherein fairly good results are obtained by the use of variables which are in some instances unrelated to what actually occurred, may be partly accounted for by noting that the straight persistence correlation of the 0130 E.S.T. temperature at Washington with that observed 30 hours later is higher ($r = 0.61$) in the forecast water trajectories than in the forecast land trajectories ($r = 0.40$). These correlation coefficients, however, do not show a significant difference when ordinary tests are applied.

Trajectories Originating Over Land with a Subsequent History over Water

A study of 31 forecast trajectories which originated over land but traveled 6 hours or more over water is of interest. The "forecast trajectory" equation gave the following results:

1. For the 23 cases where the northern land formula was applied, the average error was 5.9°, with a bias of -0.6°, and a maximum error of -11.8°.

2. For the 8 cases where the southern land formula was applied, the average error was 3.8°, with a bias of -2.9°, and a maximum error of -8.1°.

Trajectories determined from observed 3-hourly maps showed 25 such land-water cases, with results as follows:

1. In the 23 cases where the "observed map" formula was used, the average error was 4.6°, with a bias of -3.8°, and a maximum error of -17.6°.

2. In the two remaining instances where the southern land formula was used, the average error was 4.7°, with a bias of +4.3°, and a maximum error of +9.0°.

The sign of the bias should be noted, since it suggests that ignoring the water portion of the trajectory results in forecast temperatures that are too low when the trajectories are from the north and too high when they are from the south.

VI. GEOGRAPHICAL STUDY OF THE TERMS IN THE LAND FORECAST EQUATIONS

The "forecast trajectory" equations previously developed from land trajectory cases separate the trajectories into two groups, (a) those with the index station north of, and (b) those with the index station south of the latitude of Washington, D. C. These have been referred to as northern and southern land trajectories, respectively. It was noted in Section V that a significant fact concerning the regression equations developed for them was a difference in sign of the "index normal" (x_4) term between the northern and southern groups. It was accordingly decided to stratify the cases further by noting the area in which each of the "observed map" trajectories originated. The areas or sectors which were selected arbitrarily for this purpose are shown in Figure 8(a). The data, grouped into classes according to the sector of origin of the trajectory, were then studied separately. The mean Washington 0730 E.S.T. temperatures resulting from air trajectories originating within each area are shown plotted geographically in Figure 8(b), with the lowest such temperatures shown in Figure 8(c). These figures are based on data for the three winters of 1942-43, 1943-44, and 1944-45.

For each area, regression equations of the type ($Y = ax_1 + bx_3 + dx_4 + e$) were computed, the "Washington normal" (x_2) term being omitted since it showed little variation within each class. Still included, however, were the current Washington temperature (x_1), the current index station temperature (x_3), and the index station normal temperature (x_4).

The regression equations derived from this "area method" are listed in Table 1, together with the mean and maximum errors on the residuals of forecasts based on both original and test data. A comparison of these results with the results obtained within each area by using the more general northern and southern "observed map" equations is also shown. It will be noted that the more general equations give slightly better results in the test data. This fact may indicate that the more detailed break-down into geographical areas places too much emphasis on the forecast of the trajectory, which, if incorrect, causes not only the wrong index data to be used but also the wrong forecasting equation.

Nevertheless, these equations are interesting insofar as they show how much weight should be placed on the various elements in the forecast, depending on the origin of the air. Figure 9 shows the way in which the regression constants vary from sector to sector. Note, for example, in 9(a), the emphasis placed on persistence when the origin is close to Washington, as compared with the negligible weight given to the current Washington temperature when the air is moving in from the vicinity of Chicago.

VII. CONTROL FORECASTS

For a check on the quality of forecasts made by the methods outlined in this report, two types of comparisons were made: (1) with forecasts of minimum temperature made by conventional methods (this comparison is described in the following section of this paper); and (2) with extremely simple methods of forecasting described below.

The "Normal Persistence" Equation

One of the most simplified methods that can be used is based on the assumption that the departure from normal temperature at 0730 E.S.T. tomorrow is a function of today's departure from normal temperature at 0130 E.S.T. Figure 10 gives a regression line of this type, computed from the three winters of original data, and shows the distribution of forecasts made during the test winter 1945-46 with the use of this regression line.

Use of Columbus, Ohio, as a Fixed Index Station

Figure 11 shows a similar simplified method in which the 0130 E.S.T. temperature at Columbus, Ohio, is used as the independent variable. Obviously this gives better forecasts than the "Normal Persistence" equation above. As will be shown later, however, it does not give results comparable with either the conventional forecasts or forecasts made by the trajectory method.

VIII. TEST FORECASTS MADE DURING THE WINTER MONTHS OF 1945-46

Table 2 gives a comparison of forecasts made during the 1945-46 winter season by both the "forecast trajectory" technique and conventional methods. It should be remembered that the trajectory technique forecasts the temperature for 0730 E.S.T., while the conventional forecasts are made for the minimum temperature expected for the 12-hour period ending at 0730 E.S.T. The summary at the end of the table shows that the average error for the trajectory method was 4.2°, while that for the conventional minimum temperature forecasts was 3.9°. Tables 3 and 4 give a summary of both methods with respect to their relative ability to forecast 24-hour temperature changes.

A comparison of the errors made by both methods in those cases when the 30-hour change in temperature (from 0130 E.S.T. "today" to 0730 E.S.T. "tomorrow") equalled or exceeded 10 degrees is shown in Figure 12. This comparison although not favorable to the trajectory technique may help in answering the questions of those who doubt the ability of such forecasting schemes to forecast any large changes. (See Table 5.)

A study made to determine whether or not the conventional forecasts could be improved by incorporating the trajectory method showed that if the conventional forecast had been revised by 3° in the direction of the objective forecast, in those cases where the two differed by more than 5°, the average error of all conventional forecasts would have been reduced from 3.9° to 3.6°. In the 29 instances where such a difference occurred, the conventional forecast resulted in an average error of 4.9°. This reduces to 3.8° when the above procedure is applied. Figure 13 shows the frequency of errors made using each method, as well as the frequency of errors for the revised conventional forecasts.

Figure 14 shows the distribution of errors from a comparison of three increasingly complex objective temperature forecasting systems and the conventional methods, when the four were applied to the test winter of 1945-46. As mentioned previously, the "Normal Persistence" and "Columbus" formulas were developed as control forecasts. This comparison is, therefore, an indication of the increased accuracy obtained by the "trajectory" method over that obtained by much simpler schemes.

IX. ATTEMPTS TO INCORPORATE ADDITIONAL FACTORS

The meteorologist who reads this report will immediately wish to point out additional factors which have not been considered in the development of the forecasting equations. Several preliminary attempts have been made to correlate residuals from the "forecast trajectory" equations with other variables, but thus far with little success. The variables included the observed wind speed and cloudiness at 0130 E.S.T. on the night for which the forecast was made and the number of hours that a given trajectory passed over snow cover. These variables have not been correlated with other sets of residuals, however, nor have the other methods of incorporating these data been tried.

X. CONCLUSIONS AND FUTURE PLANS

Although it is apparent that the possibilities of this line of attack are far from exhausted, there are indications in this study of the need for further basic research into the tracing of air currents. The conception of a unique trajectory for the air whose temperature is measured in the morning at Washington or New York may be misleading. Processes of vertical and lateral mixing

may bring into the thermometer shelter at the forecast station air molecules which have originated 30 hours previously at widely separated points. Results already attained, however, indicate the effectiveness of a single index station in providing a reasonable sample of the temperature of the air whose history is to be traced.

With regard to measurements of the accuracy of computed and forecast trajectories, the general northern and southern land equations themselves offer little guidance. Their mean residuals are much alike regardless of the manner in which the trajectories were computed or forecast. No method thus far tried has yielded a mean residual low enough for that method to be regarded as significantly superior to the other methods used in making trajectory computations.

Geographical studies of the forecast parameters provide an outline for the design of an experiment which may reveal how much of the error is due to incorrect trajectory forecasts. In addition, it should be possible to estimate what portion of the error is due to local temperature variations and thus obtain an estimate of the forecasting accuracy which can reasonably be expected in the event that these local sources of variation are not explained.

Future work should be done on a limited number of detailed case studies in an attempt to evaluate further the effects of radiation, cloudiness, wind, and snow in individual cases.

In addition, the maps should be initially stratified on the basis of whether the early morning temperatures are expected to be determined primarily by radiation and other local effects rather than by the effect of large-scale advection or movement of air. Whenever it is indicated that local radiation will be predominant, a forecast could be made of the weather elements at 1930 E.S.T., from which a set of graphs or formulas might be used to forecast the temperatures for the following morning.

It is also planned to read from the 0130 E.S.T. weather maps the zonal and meridional indices by means of which certain errors which have been noticed may be avoided. Considerable error has been found to occur in cases when both computed and forecast trajectories are from the south or west but the predicted warmer temperatures do not materialize because the initially strong northerly flow persists. An attempt should be made to stratify the maps according to such cases, possibly on the basis of selected zonal and meridional indices, and thus to determine the value of using separate equations for various degrees of initial zonal and meridional flow.

The ultimate usefulness of the trajectory method of temperature forecasting depends on its effectiveness in assisting the forecaster to do a particular forecast job quickly and accurately with a minimum amount of attention. Its usefulness in a central or district forecast office would therefore increase if it could be used in providing forecasts not only for one city but for several throughout the district. This aspect of the problem, which might involve the forecasting of several trajectories in a single operation, is therefore being considered.

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Table 1.--Temperature forecasting equations grouped according to area of origin of air trajectories, with comparison of errors made using "observed map" method and area method.

AREA	N*	REGRESSION EQUATIONS USED IN AREA METHOD	ORIGINAL (BASIC) DATA[1]				TEST DATA[2]			
			Area		Obsvd. map		Area		Obsvd. map	
			mean error	max. error	mean error	max. error	mean error	max. error	mean error	max. error
A...	35	$\hat{y}=.61x_1+.14x_3+.01x_4+5.74$	3.0	-9.2	3.5	-11.8	6.7	-10.8	7.3	-12.4
B...	14	$\hat{y}=.47x_1+.39x_3-.39x_4+15.45$	3.8	-8.4	4.3	-11.5				
C...										
D...	19	$\hat{y}=.13x_1+.39x_3-.26x_4+20.70$	3.3	-9.5	4.3	11.6				
E...										
F...	43	$\hat{y}=.32x_1+.23x_3+.33x_4+5.70$	4.2	13.6	4.2	17.6	4.8	13.4	4.1	11.4
G...	43	$\hat{y}=.07x_1+.60x_3-.28x_4+22.11$	4.5	-11.7	4.6	-15.2	4.0	8.5	3.9	8.0
H...	17	$\hat{y}=.09x_1+.47x_3-.22x_4+26.83$	3.4	-8.4	4.1	-11.0	5.1	8.6	5.1	-11.6
I...	15	$\hat{y}=.35x_1-.59x_3+1.05x_4+.47$	4.6	10.4	5.5	19.1				
J...										
K...	17	$\hat{y}=.66x_1-.09x_3+1.27x_4-25.88$	4.1	-8.0	4.4	-12.9				
L...	11	$\hat{y}=.45x_1+.14x_3+.49x_4-1.32$	1.7	3.7	1.9	5.5				

*N = number of cases used in deriving regression equations.

1 Data from winter months, 1942-1945.

2 Data from winter months, 1945-1946.

Table 2.--Daily comparison and summary of test forecasts made during the winter of 1945-46

DATE	CONVENTIONAL FORECASTS OF MINIMUM TEMPERATURE			TRAJECTORY FORECASTS OF 0730 E.S.T. TEMPERATURE		
	Forecast	Observed	Error	Forecast	Observed	Error
1945	°F.	°F.	°F.	°F.	°F.	°F.
December 1	30	30	0	32	30	2
2	32	38	-6	33	39	-6
3	43	42	1	40	43	-3
4	31	31	0	38	31	7
5	24	33	-9	34	37	-3
6	33	40	-7	32	40	-8
7	40	38	2	39	38	1
8	36	35	1	38	37	1
9	38	42	-4	39	45	-6
10	25	27	-2	33	27	6
11	18	21	-3	22	22	0
12	20	22	-2	26	25	1
13	26	24	2	31	29	2
14	28	30	-2	30	31	-1
15	25	23	2	23	23	0
16	5	16	-11	21	16	5
17	15	17	-2	17	17	0
18	25	25	0	27	25	2
19	22	22	0	24	22	2
20	15	19	-4	27	23	4
21	15	19	-4	24	24	0
22	20	17	3	25	17	8
23	16	14	2	23	16	7
24	23	28	-5	24	29	-5
25	34	35	-1	39	35	4
26	30	35	-5	38	36	2
27	29	31	-2	31	32	-1
28	33	34	-1	39	37	2
29	35	30	5	40	36	4
30	35	40	-5	31	42	-11
31	31	33	-2	34	33	1

Table 2.--Continued

DATE		CONVENTIONAL FORECASTS OF MINIMUM TEMPERATURE			TRAJECTORY FORECASTS OF 0730 E.S.T. TEMPERATURE		
		Forecast	Observed	Error	Forecast	Observed	Error
1946		°F.	°F.	°F.	°F.	°F.	°F.
January	1	25	22	3	22	23	-1
	2	21	24	-3	30	28	2
	3	34	30	4	37	32	5
	4	35	38	-3	39	38	1
	5	45	53	-8	47	57	-10
	6	55	57	-2	63	63	0
	7	58	53	5	60	53	7
	8	46	42	4	53	42	11
	9	43	46	-3	46	54	-8
	10	40	36	4	39	36	3
	11	39	37	2	37	40	-3
	12	37	34	3	34	34	0
	13	25	26	-1	29	26	3
	14	32	30	2	34	31	3
	15	25	19	6	21	19	2
	16	18	23	-5	28	25	3
	17	22	31	-9	29	33	-4
	18	32	36	-4	34	41	-7
	19	17	20	-3	21	20	1
	20	26	19	7	30	29	1
	21	25	31	-6	29	31	-2
	22	20	14	6	17	14	3
	23	27	18	9	33	21	12
	24	33	27	6	27	35	-8
	25	25	37	-12	34	37	-3
	26	17	23	-6	22	24	-2
	27	15	12	3	24	12	12
	28	24	34	-10	35	34	1
	29	30	37	-7	32	39	-7
	30	42	42	0	41	50	-9
	31	28	30	-2	33	30	3

Table 2.--Continued

DATE	CONVENTIONAL FORECASTS OF MINIMUM TEMPERATURE			TRAJECTORY FORECASTS OF 0730 E.S.T. TEMPERATURE		
	Forecast	Observed	Error	Forecast	Observed	Error
1946	°F.	°F.	°F.	°F.	°F.	°F.
February 1	29	31	-2	33	34	-1
2	15	23	-8	25	23	2
3	15	21	-6	36	22	14
4	28	31	-3	32	32	0
5	34	31	3	36	33	3
6	35	32	3	30	32	-2
7	32	29	3	25	29	-4
8	31	38	-7	38	39	-1
9	40	38	2	45	38	7
10	30	24	6	35	24	11
11	20	24	-4	26	24	2
12	28	35	-7	32	38	-6
13	44	46	-2	41	63	-22
14	29	24	5	36	24	12
15	15	21	-6	30	22	8
16	40	40	0	33	40	-7
17	35	35	0	35	37	-2
18	29	30	-1	34	30	4
19	32	35	-3	39	39	0
20	27	29	-2	33	29	4
21	28	32	-4	33	33	0
22	42	36	6	40	40	0
23	35	36	-1	39	36	3
24	28	26	2	25	26	-1
25	27	35	-8	31	36	-5
26	44	57	-13	44	57	-13
27	35	37	-2	36	37	-1
28	32	31	1	37	32	5

SUMMARY

	CONVENTIONAL FORECASTS	TRAJECTORY FORECASTS
	°F.	°F.
Average Error	3.9	4.2
Maximum Error	-13	-22
Bias	-1.4	0.2

Table 3.--Comparison of conventional forecasts of the 24-hour minimum temperature change with the observed minimum temperature change at Washington National Airport, December 1, 1945 to February 28, 1946.

	Conventional forecast (°F.)									
	-20°	-19° to -15°	-14° to -10°	-9° to -5°	-4° to +4°	+5° to +9°	+10° to +14°	+15° to +19°	+20°	TOTAL
Observed change -20°	1	1								2
-19° to -15°		2	1							3
-14° to -10°	1		2	7						10
-9° to -5°		2		8						10
-4° to +4°				6	27	3	2			38
+5° to +9°					9	4	1	1		15
+10° to +14°					2	5				7
+15° to +19°						1		1		2
+20°						1	1			2
TOTAL	2	5	3	21	38	14	4	2	0	89

Table 4.--Comparison of trajectory forecasts of the 24-hour change in 0730 E.S.T. temperature with the observed change at Washington National Airport, December 1, 1945, to February 28, 1946

	Trajectory forecast (°F.)									
	-20°	-19° to -15°	-14° to -10°	-9° to -5°	-4° to +4°	+5° to +9°	+10° to +14°	+15° to +19°	+20°	TOTAL
Observed change -20°	3	1								4
-19° to -15°		1	2							3
-14° to -10°		1	3	4	4					12
-9° to -5°				4	4					8
-4° to +4°				4	18	6	1			29
+5° to +9°				1	6	9	3	1		20
+10° to +14°					2	5	1			8
+15° to +19°						1	1			2
+20°					1	1			1	3
TOTAL	3	3	5	13	35	22	6	1	1	89

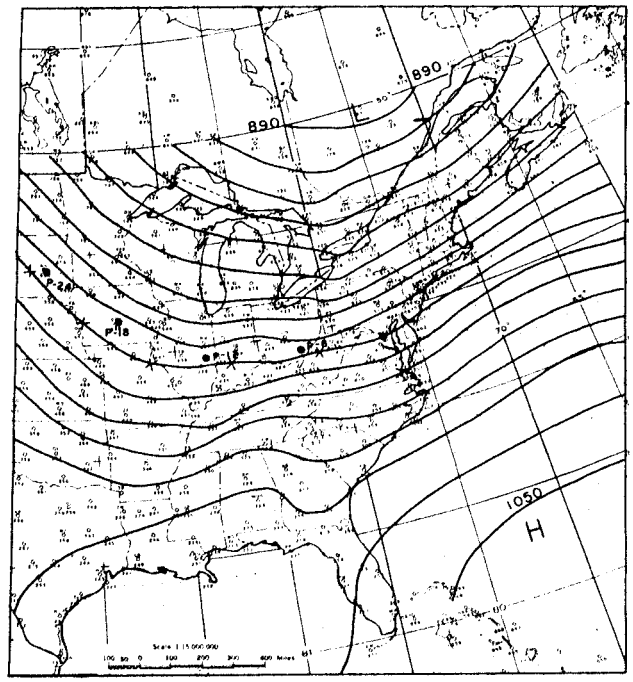


Figure 1.--700-mb. contour pattern at 2300 E.S.T., January 12, 1946. (Method of determining the points marked P_6, P_{12}, etc., is given in Appendix A.)

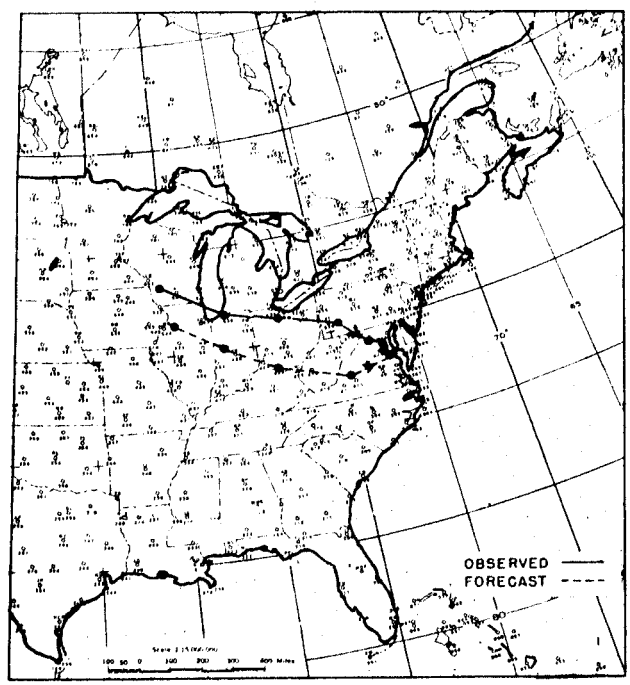
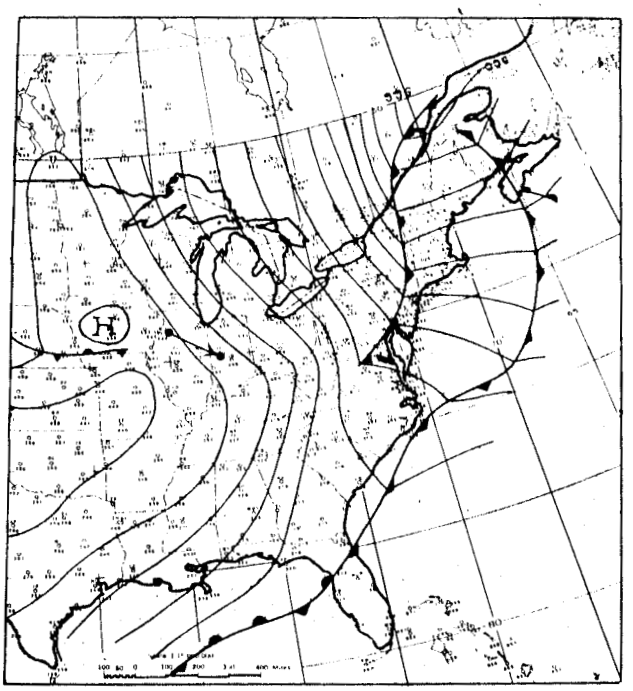
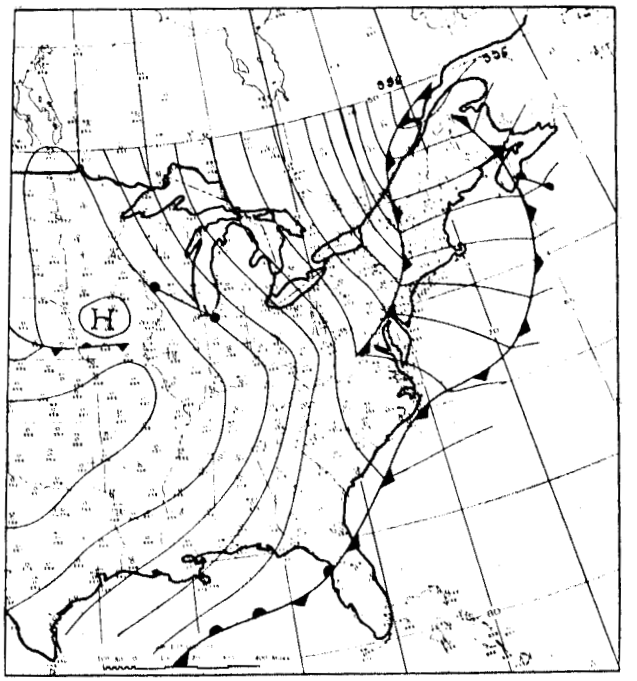


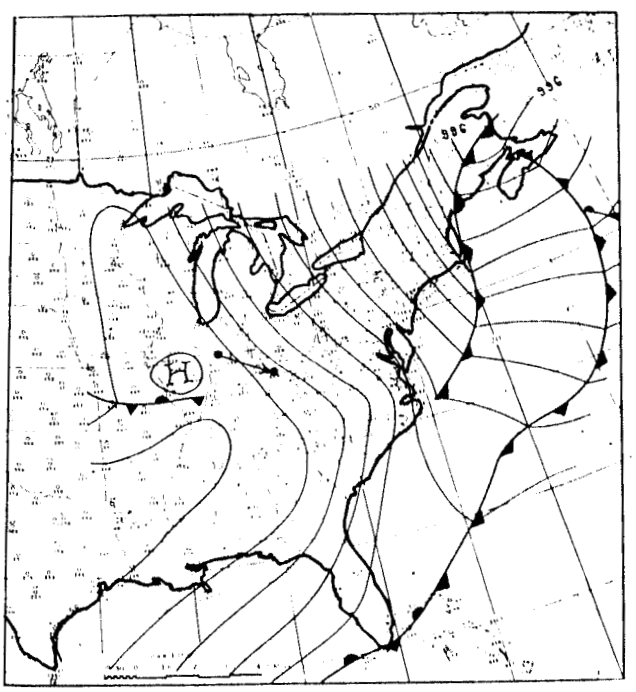
Figure 2.--Plotted comparison of observed and forecast trajectories into Washington for the 30-hour period, 0130 E.S.T., January 13, 1946, to 0730 E.S.T. January 14, 1946. (See Figures 3, 4, and 5 for details.)



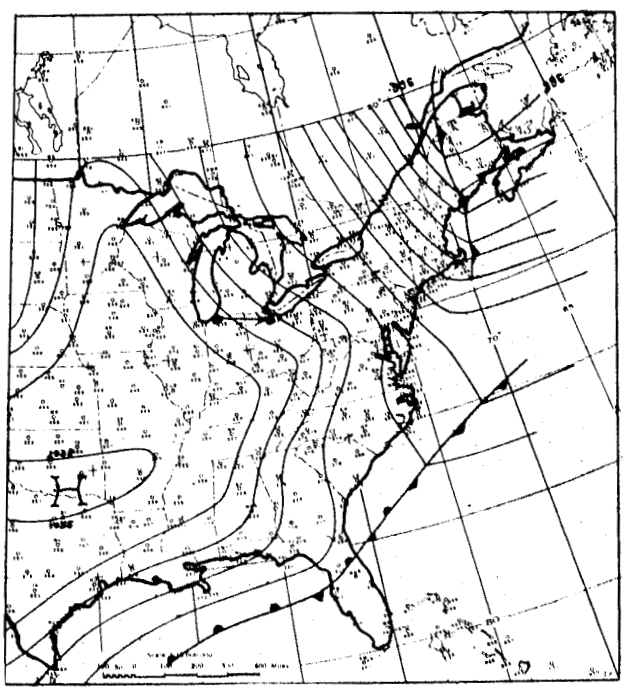
(a) Forecast trajectory, 0130-0730 E.S.T., January 13.



(b) Observed trajectory, 0130-0730 E.S.T., January 13.

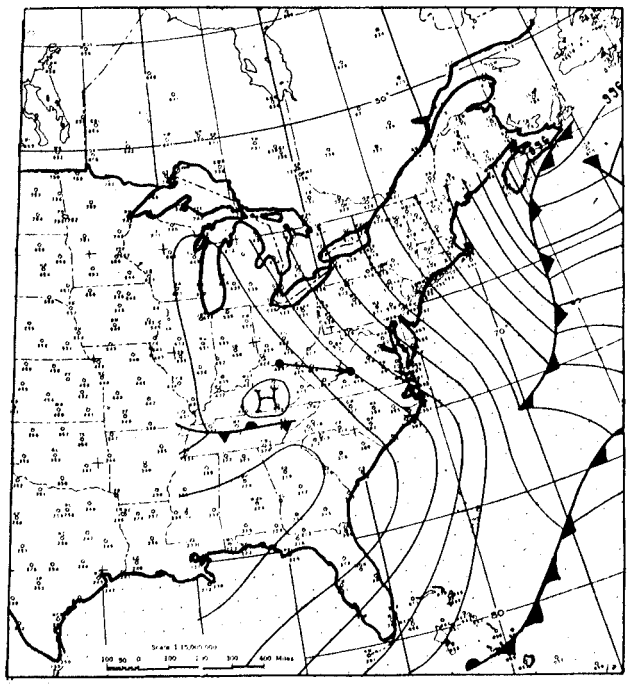


(c) Forecast trajectory, 0730-1330 E.S.T., January 13.

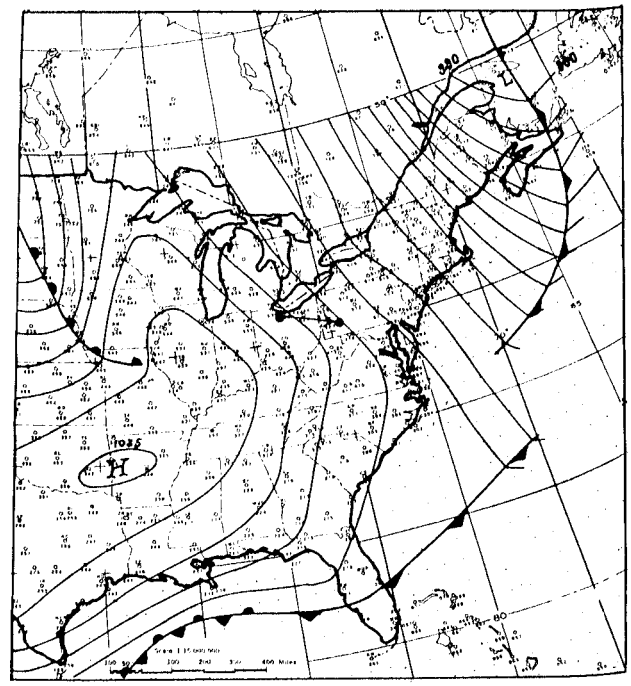


(d) Observed trajectory, 0730-1330 E.S.T., January 13.

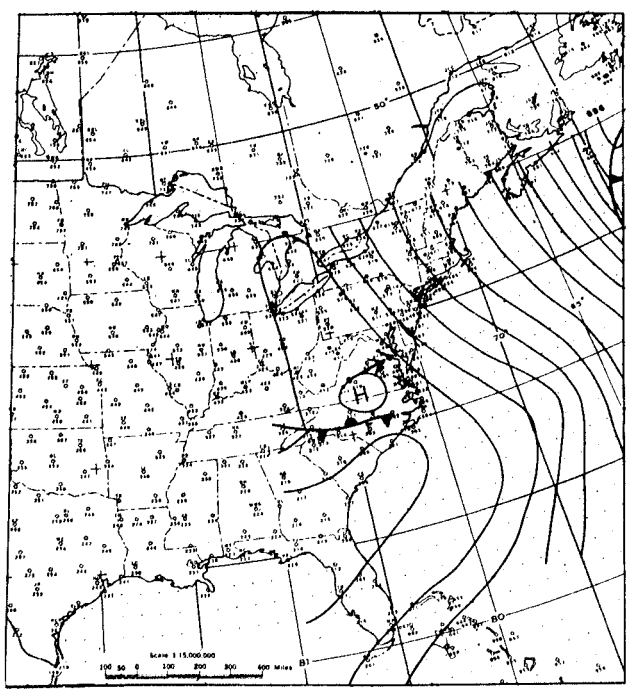
Figure 3.--Forecast (left) and observed (right) trajectories for the first 12 hours of the 30-hour forecast period.



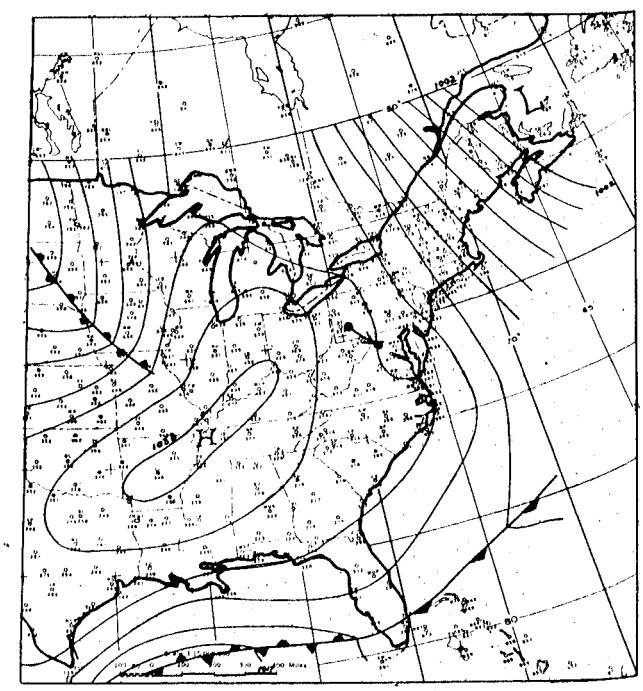
(a) Forecast trajectory, 1330-1930 E.S.T., January 13.



(b) Observed trajectory, 1330-1930 E.S.T., January 13.

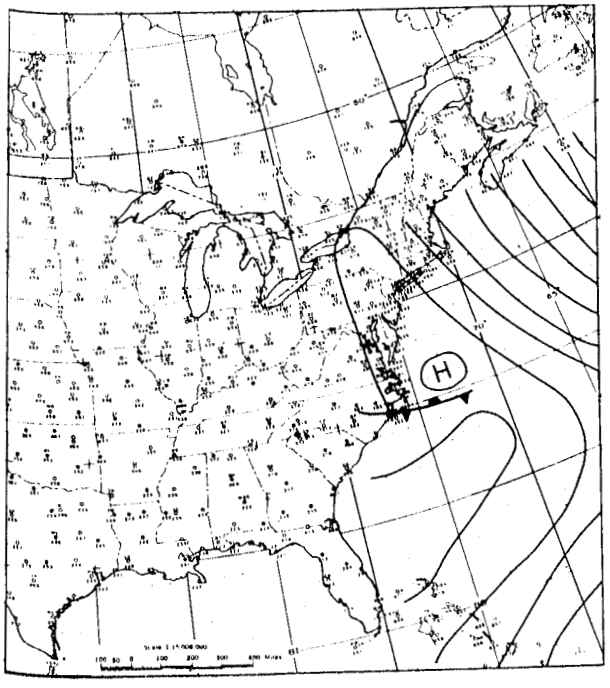


(c) Forecast trajectory, 1930 E.S.T., January 13, to 0130 E.S.T., January 14.

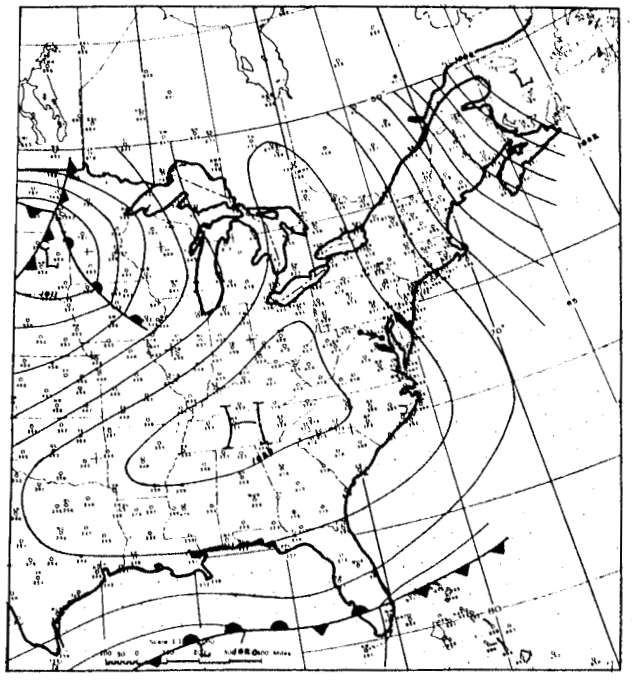


(d) Observed trajectory, 1930 E.S.T., January 13, to 0130 E.S.T., January 14.

Figure 4.--Forecast (left) and observed (right) trajectories for the second 12 hours of the 30-hour trajectory.



(a) Forecast trajectory, 0130-0730 E.S.T., January 14.



(b) Observed trajectory, 0130-0730 E.S.T., January 14.

Figure 5.--Forecast (left) and observed (right) trajectories for the final 6 hours of the 30-hour trajectory.

72123

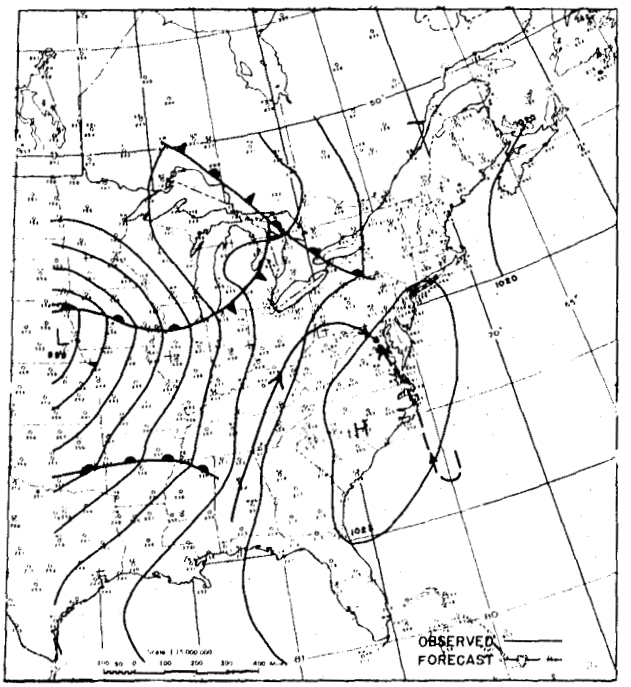
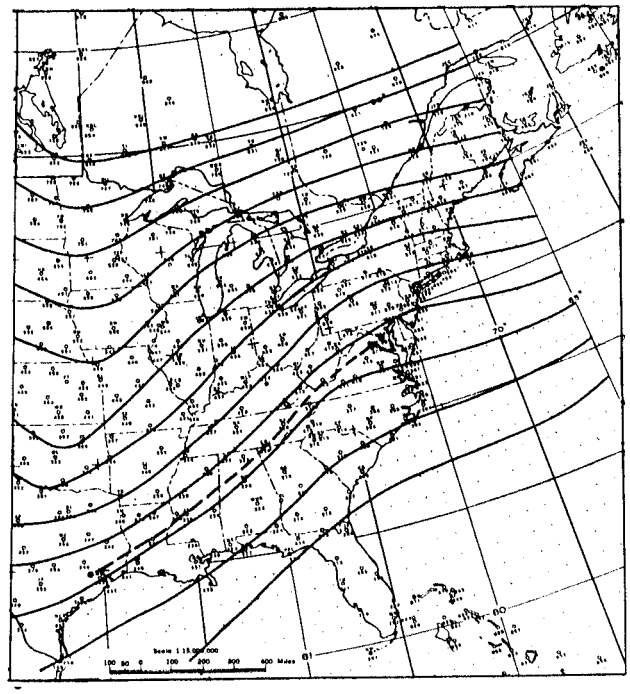
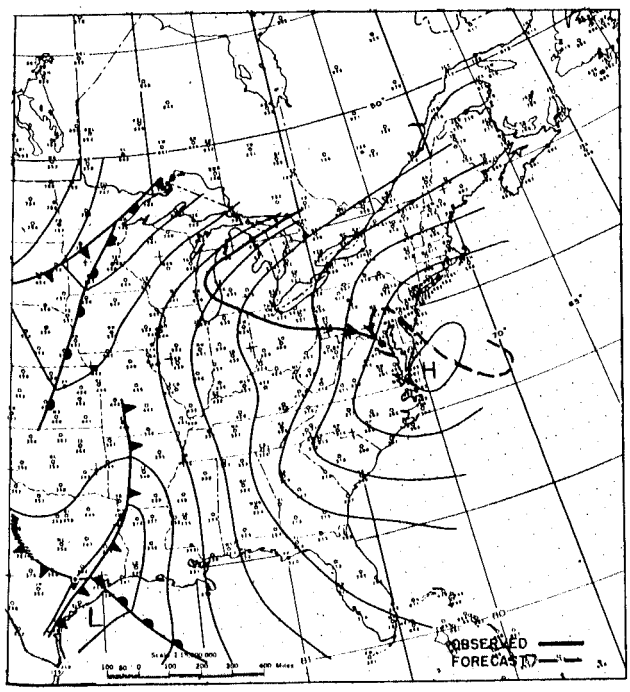


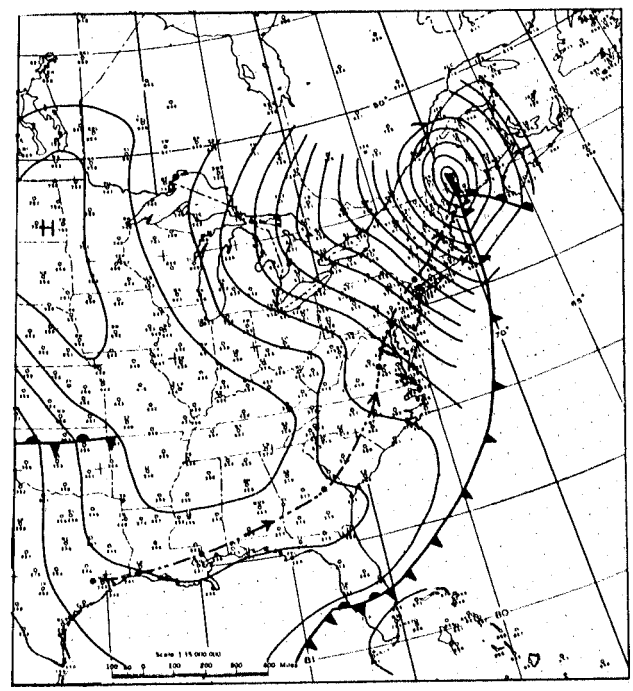
Figure 6.--Illustration of erroneous water trajectory forecast, associated with a high pressure system along the East Coast at 0130 E.S.T., February 15, 1945.



(a) February 13, 1944, 2300 E.S.T., 3-km. map.

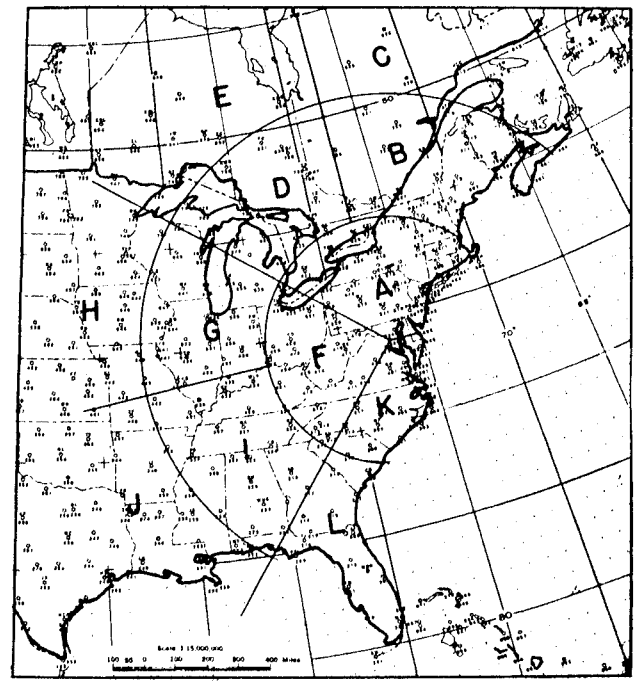


(b) February 14, 1944, 0130 E.S.T., surface map.

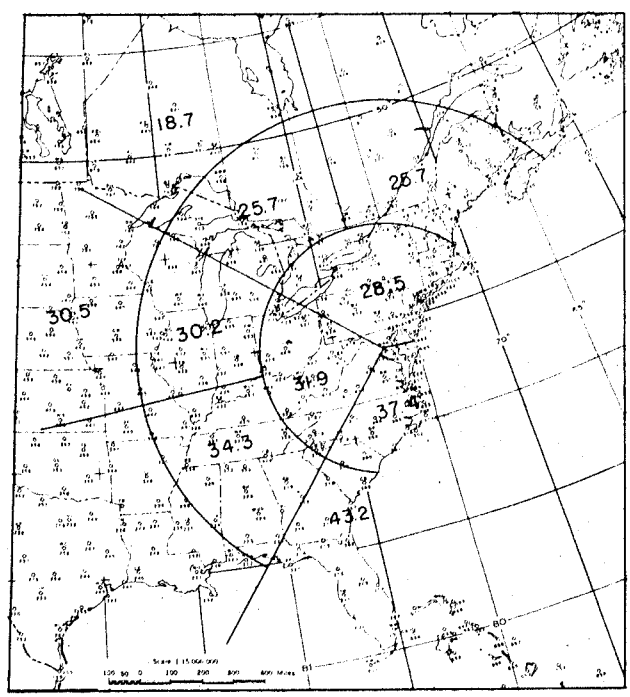


(c) February 15, 1944, 0730 E.S.T., surface map.

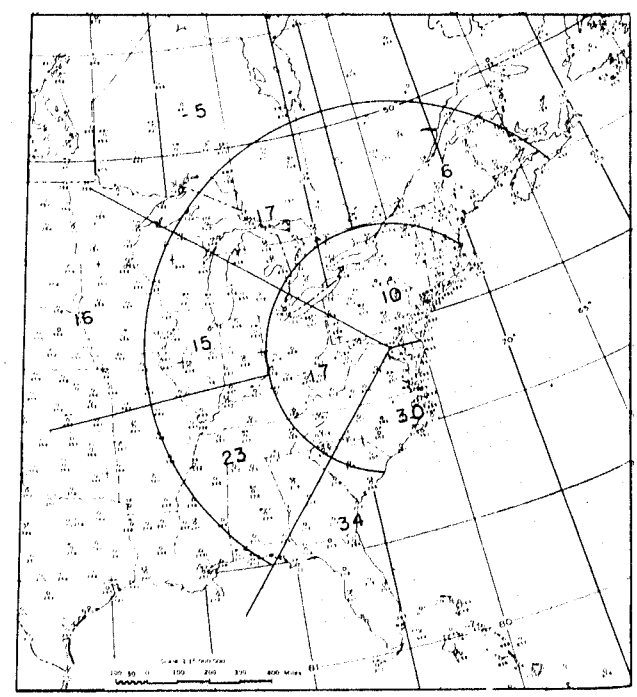
Figure 7--The 3-km. pressure map, showing (a) the forecasted path of the surface low pressure system used to obtain the forecast trajectory, shown on surface map (b); the observed trajectory shown in (b) differs from the forecast trajectory because of the unpredicted 30-hour deepening of the Low, the observed path of which is shown in (c).



(a)

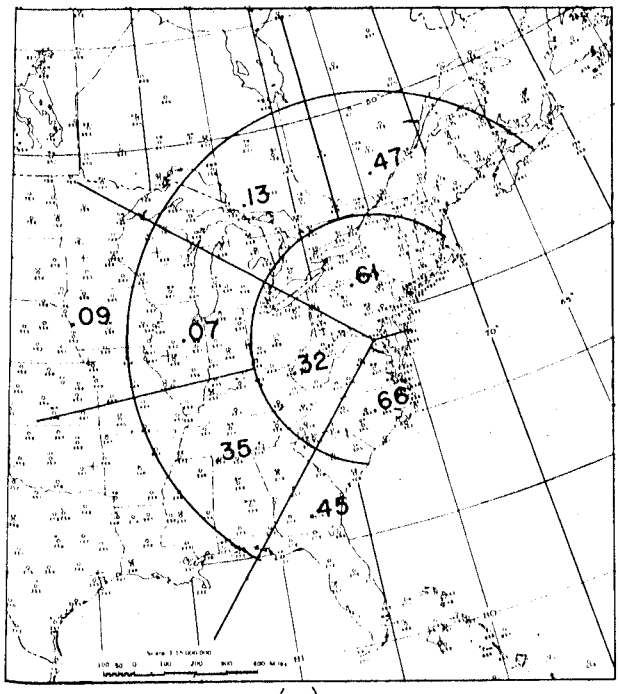


(b)

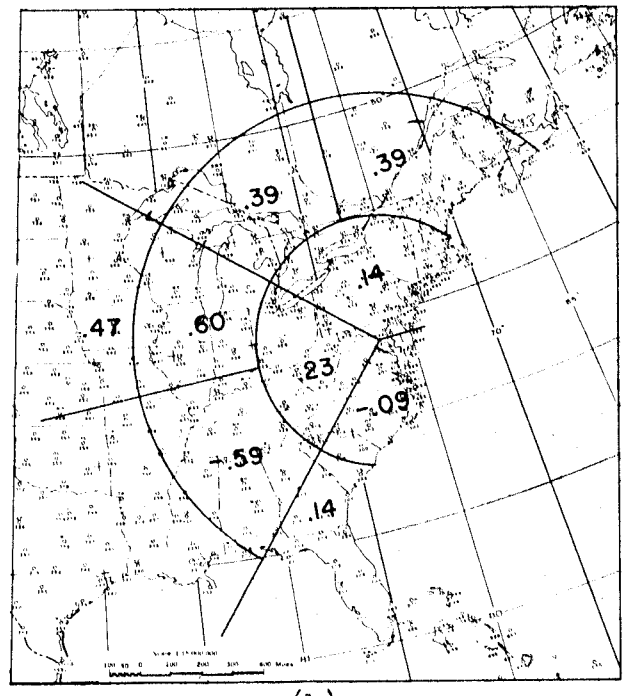


(c)

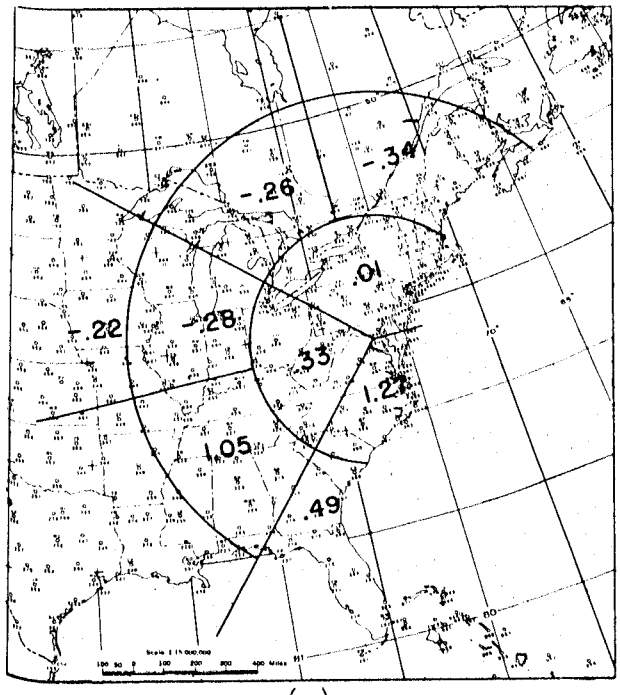
Figure 8.--(a) Geographical origins of "observed map" trajectories, classified by area.
(b) Mean 0730 E.S.T. temperatures (°F.) at Washington, D. C., for "observed map" trajectories originating in designated areas.
(c) Lowest 0730 E.S.T. temperatures (°F.) at Washington, D. C., for "observed map", 30-hour trajectories originating in designated areas.



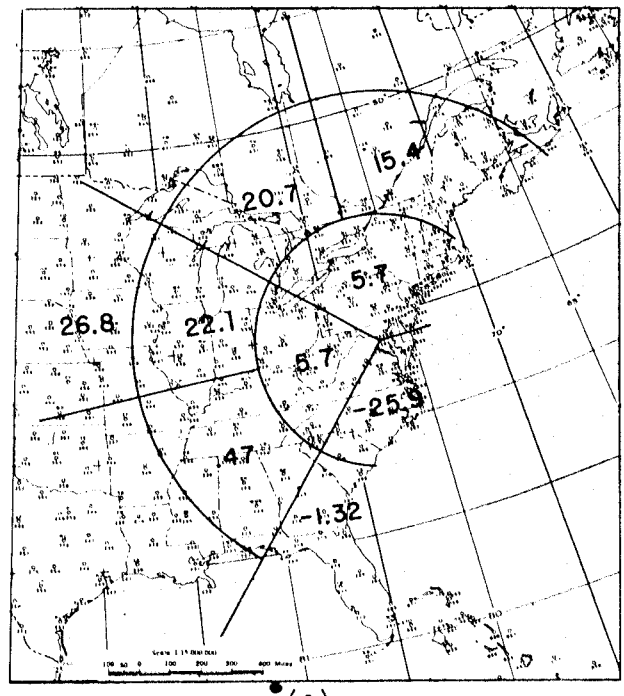
(a)
Partial regression coefficient for x_1.



(b)
Partial regression coefficient for x_3.



(c)
Partial regression coefficient for x_4.



(d)
Regression constant (d).

Figure 9.--Geographical distribution of regression constants in "land" forecasting equation, $y = ax_1 + bx_2 + cx_3 + d$.
(Explanation of terms used in equation is given in Section 6.)

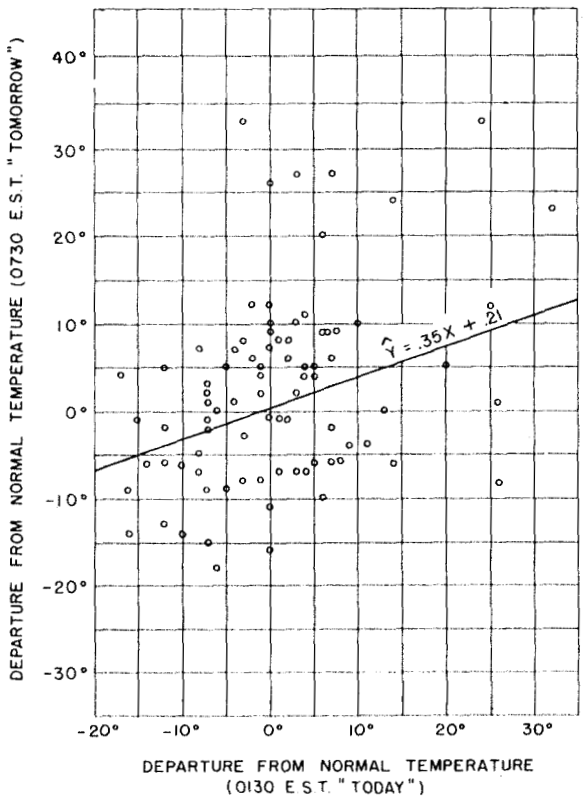


Figure 10.--Distribution of forecasts made during the test winter 1945-46, compared with the regression line for the "Normal Persistence" forecasting equation, derived from data of three previous winters.

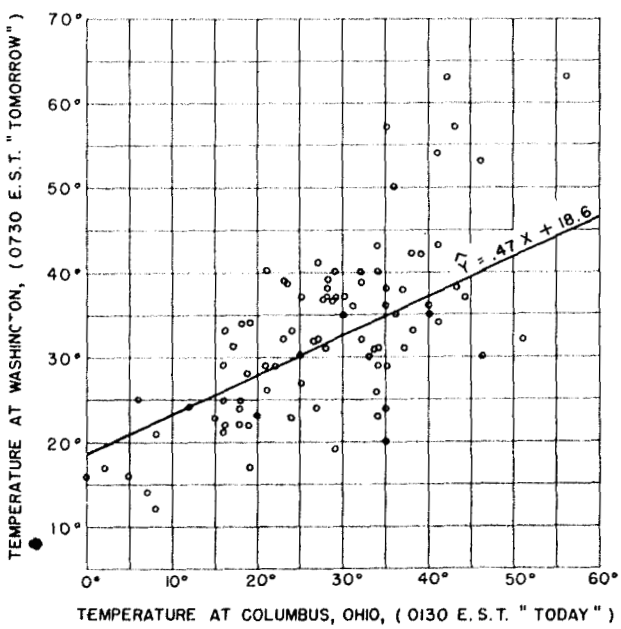


Figure 11.--Distribution of forecasts made during the test winter 1945-46, compared with the regression line for the "Columbus" forecasting equation, derived from data of three previous winters.

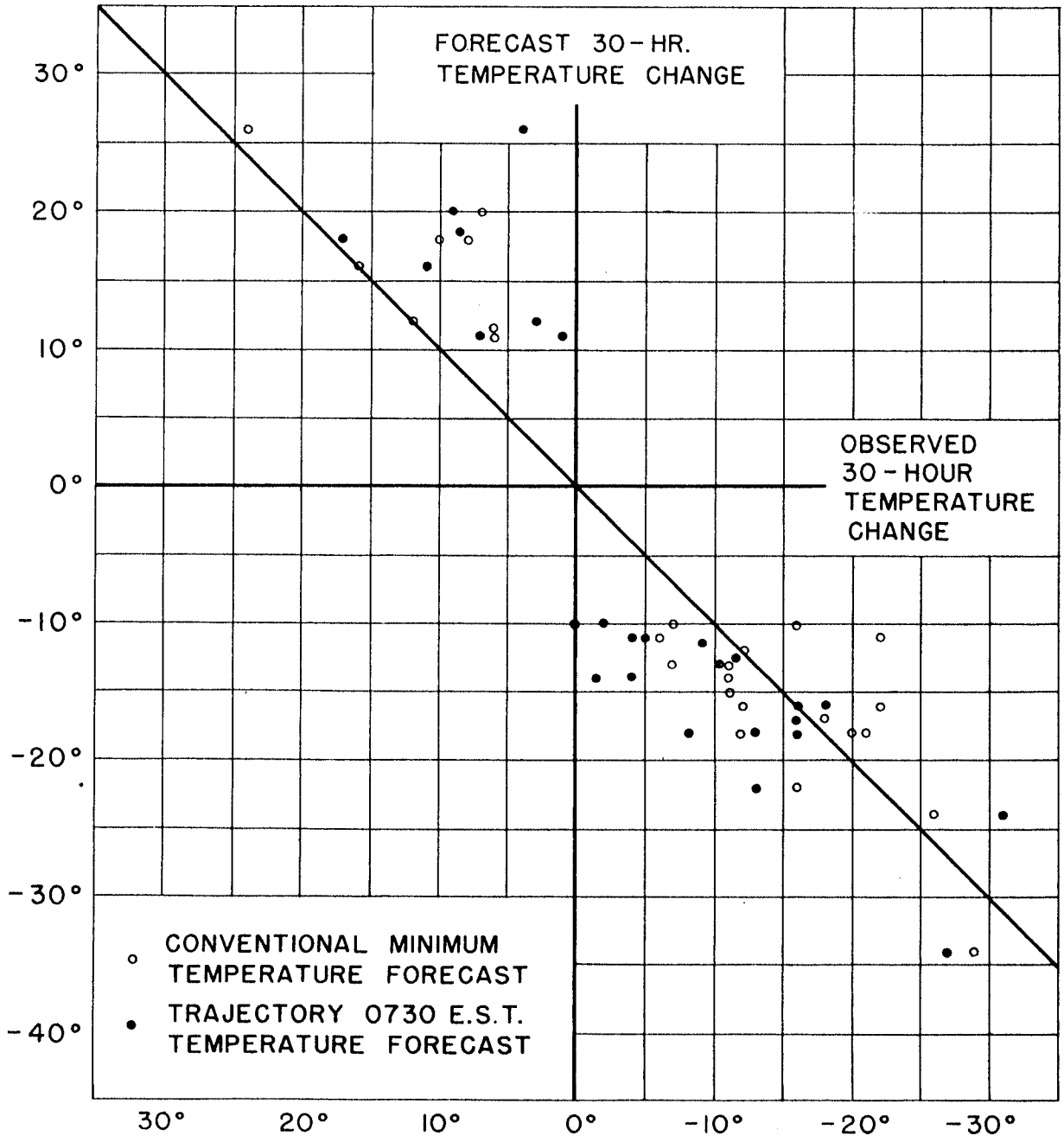


Figure 12.--Comparison of errors made using trajectory forecasting method with those made using conventional method in instances where the temperature change was observed to be ≥10°F. Cases are taken from the winter season of 1945-46.

Table 5.--Summary of errors shown in Fig. 12.

	Trajectory method	Conventional method
	°F.	°F.
Rising temperature		
Average error	7.3	5.5
Bias	-7.2	-4.6
Extreme error	-22	-13
Falling temperature		
Average error	5.1	4.0
Bias	3.8	0.6
Extreme error	+12	-8

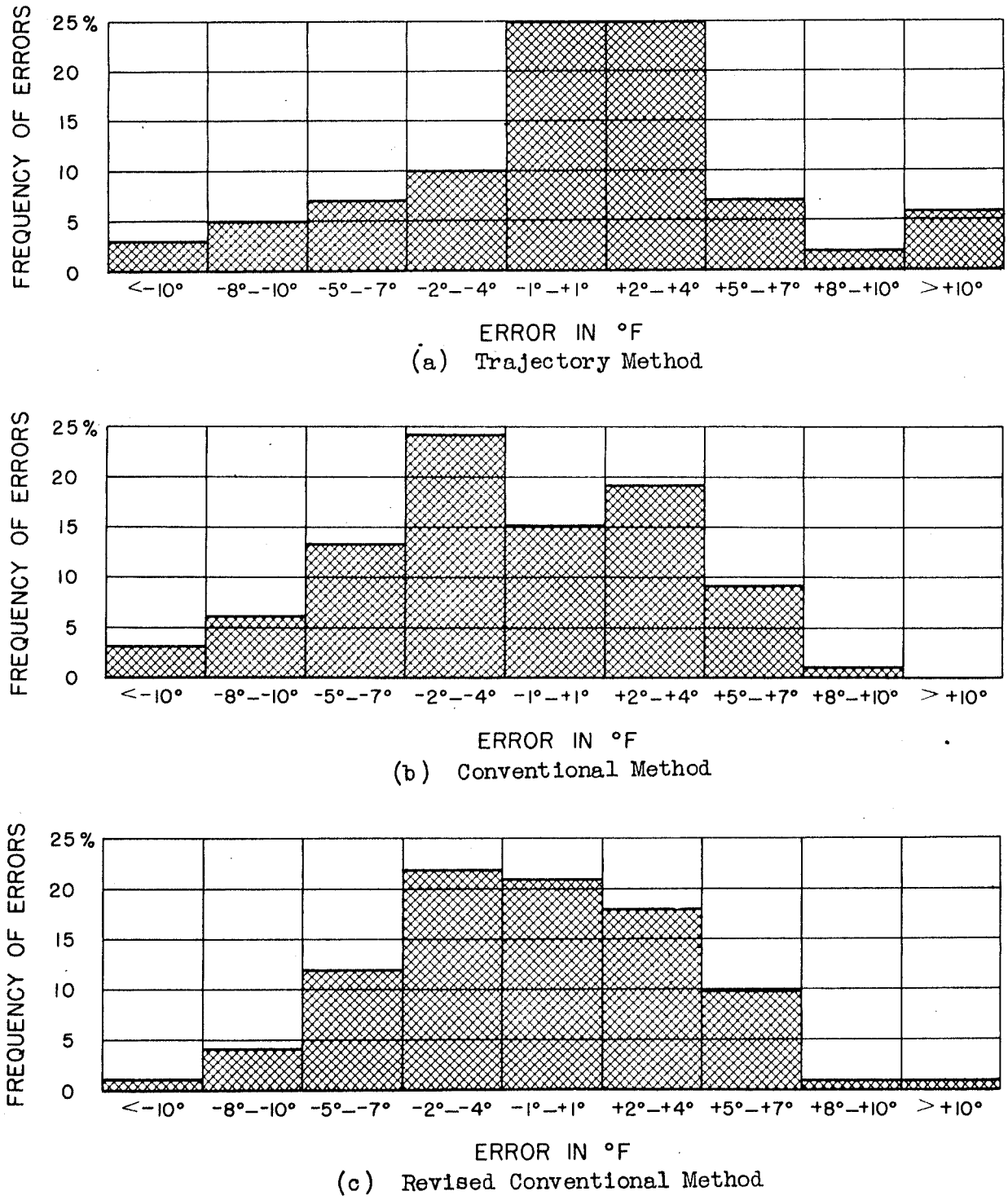


Figure 13.--Distribution of errors associated with trajectory, conventional, and revised conventional methods of forecasting, as tested during the winter, 1945-46.

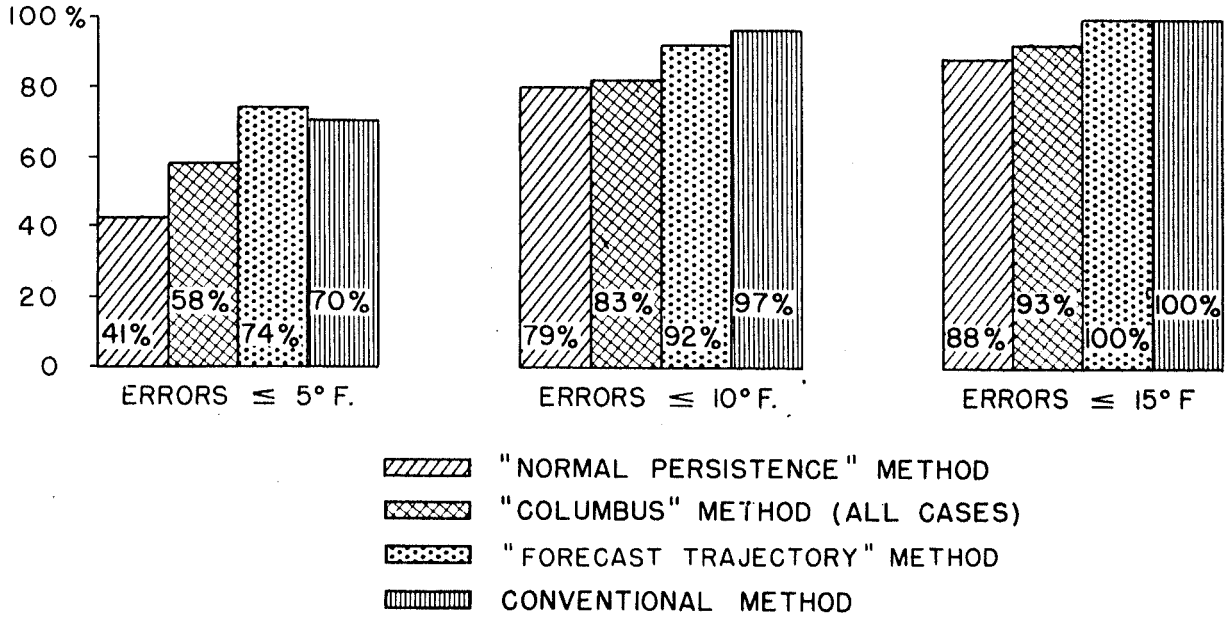


Figure 14.--Distribution of errors in four temperature forecasting methods applied to the winter 1945-46.

APPENDIX A

Detailed Instructions for Forecasting the 30-hour Trajectory of the Air Reaching Washington at 0730 E.S.T.

The charts required to obtain a trajectory of this kind are the 2300 E.S.T. (0400Z) 3-km. or 700-mb. chart, and the 0130 E.S.T. (0630Z) surface chart. In addition, certain small base weather maps are required plus a transparent overlay map of the same scale and projection as the 0130 E.S.T. surface chart. (See Charts I - IV attached.)

Chart I:

a. Mark off a point, B, 4-1/2° of longitude due west of Washington, (WA), on the 700-mb. chart.

b. At point B, using the proper wind scale, measure the contour or isobar spacing and go upwind parallel to the contours or isobars, a distance equal to 6 hours of travel at 80 percent of the speed indicated by the spacing, and mark a point, B-1.

c. At B-1, repeat the process just outlined to obtain point B-2. Repeat at B-2 to get B-3, and at B-3 to get B-4.

d. At B-4, go due east for a distance equal to 4-1/2° longitude, $x\frac{1}{4}$, and mark a point, P_{24}.

e. At B-3, go due east for a distance equal to 4-1/2° longitude, $x\frac{1}{2}$, and mark a point, P_{18}.

f. At B-2, go due east for a distance equal to 4-1/2° longitude, $x\frac{3}{4}$, and mark a point, P_{12}.

g. At B-1, go due east for a distance equal to 4-1/2° longitude, x 1, and mark a point, P_6.

Chart II:

Transfer to a blank base map of the type of Chart II the following points from Chart I: WA, P_6, P_{12}, P_{18}, P_{24}. None of the other points are to be transferred. Mark these same points (WA, P_6, P_{12}, P_{18}, P_{24}) on the transparent overlay. (Chart IV).

Chart III:

Mark P_{24} on the 0130 E.S.T. map (Chart III).

Chart IV:

a. Place the transparent overlay (Chart IV) on the 0130 E.S.T. surface map (Chart III) so that WA on the overlay is superimposed on P_{24} on the surface map, and line up the overlay and the surface map by reference to the latitude and longitude lines, so that there is no "skew".

(Note that the isobars on the surface map are readily visible through the overlay.)

b. Measure isobar spacing at WA on overlay and go upwind from WA a distance equal to 6-hour travel at 80 percent of the speed indicated by this spacing and at an angle of 30° to the isobars. Mark this point D-1 on overlay.

c. Place overlay on surface map so that P_6 on overlay is superimposed on P_{24} on surface map, and again line up overlay and map.

d. Measure isobar spacing at D-1 on overlay and go upwind from D-1 a distance determined in the manner outlined above. Mark this point D-2 on overlay.

e. Place overlay on surface map so that P_{12} on overlay is superimposed on P_{24} on surface map, and line up overlay and map.

f. Measure isobar spacing at D-2 on overlay and go upwind from D-2 a distance determined in the manner outlined. Mark this point D-3 on overlay.

g. Place overlay on surface map so that P_{18} on overlay is superimposed on P_{24} on map, and line up overlay and map.

h. Measure isobar spacing at D-3 on overlay and go upwind from D-3 a distance determined in the manner outlined. Mark this point D-4 on overlay.

i. Place overlay on surface map so that P_{24} on overlay is superimposed on P_{24} on surface map, and line up overlay and map. Overlay and map should now be in perfect superimposition.

j. Measure isobar spacing at D-4 on overlay and go upwind from D-4 a distance determined in the manner outlined. Mark this point D-5 on overlay.

Draw a line on overlay from WA to D-1 to D-2 to D-3 to D-4 to D-5. This line represents the 30-hour trajectory of the air parcel.

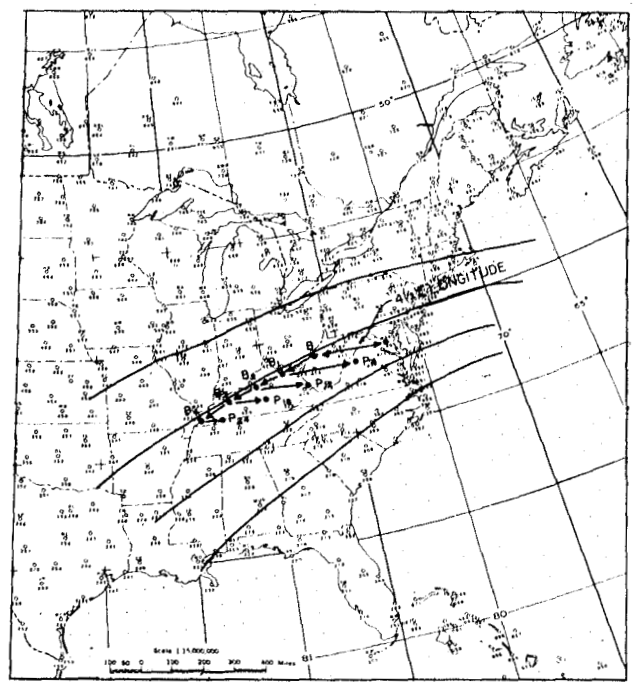


Chart I.--This chart represents the 2300 E.S.T. (0400 Z), 3-km. or 700-mb. chart.

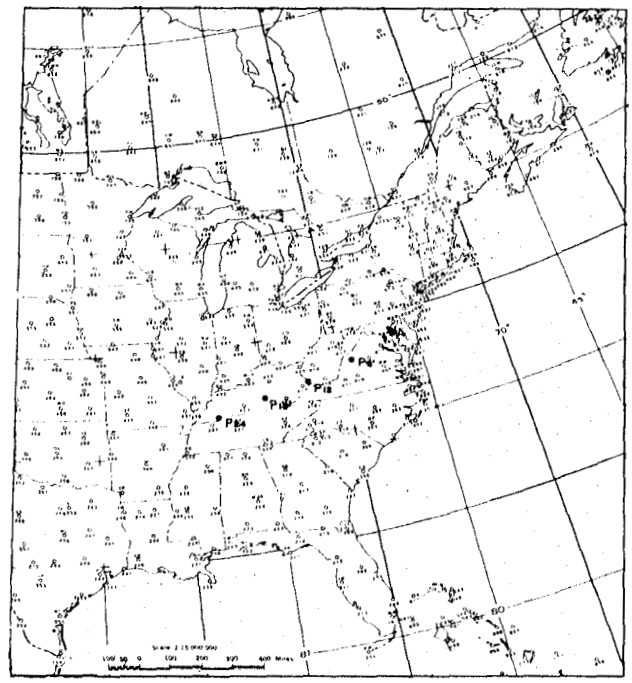


Chart II.--This chart serves merely as a record of the points found and of the trajectory, and it may be omitted without altering the process of constructing the trajectory.

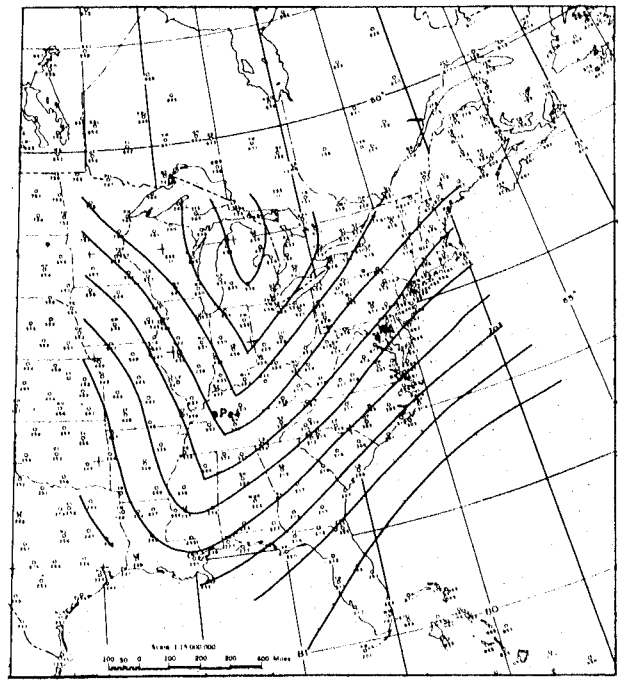


Chart III.--This chart represents the 0130 E.S.T. (0630 Z) surface map. Only P_{24} is marked on this map. (See text)

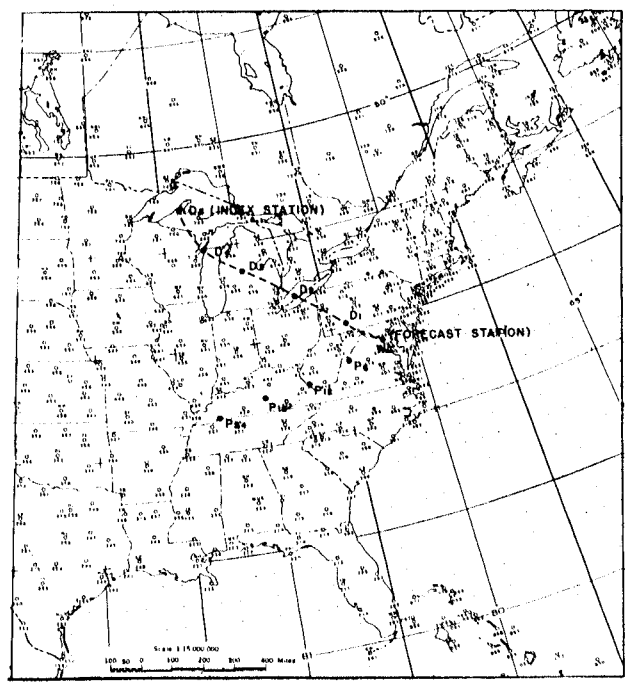


Chart IV.--This chart represents the transparent overlay referred to in the text, and shows the completed trajectory. Since the transparent overlay is used again and again, a permanent record of the trajectory is kept by transferring the "D" points to Chart II.

APPENDIX B

Reasons for Using 0730 E.S.T. Normal Temperatures

Two of the parameters involved in the N.Y.U. forecasting equations are the 0730 E.S.T. normal temperatures at both New York and the index station. In extending the 24-hour time lag to 30 hours for the Washington forecasting program, the question arose as to whether or not new 0130 E.S.T. decade normal temperatures would have to be computed. The decision to use the 0730 E.S.T. normals which had already been prepared for the New York University project was based on the following considerations:

1. 0130 E.S.T. normals were available for relatively few stations while the 0730 E.S.T. normals were available for a large number of stations.

2. If the difference between 0130 E.S.T. normals and 0730 E.S.T. normals were constant throughout the period considered for each station, no difference in accuracy would arise, as is shown below:

   (a) The standard deviation of the difference ($T_{rn}0130-T_{rn}0730$)* is .80 for the sample evaluated.

   (b) The "probable" deviation estimated from (.67) (.80) is about .54.

   (c) Weight given to this normal is .20 in the "forecast trajectory" northern land formula. Assuming that this weight would be changed but slightly if the 0130 E.S.T. normals were used in establishing the equation, the probable error generated would be roughly (.20) (.54) = .11.

   (d) For Washington, the difference is constant for December, January, and February.

   (e) This error is small compared to that expected from unevaluated influences.

* $T_{rn}0130$ = normal temperature at reference station (Washington) at 0130 E.S.T.

$T_{rn}0730$ = normal temperature at reference station (Washington) at 0730 E.S.T.