AN INVESTIGATION OF A TRAJECTORY METHOD FOR FORECASTING FLOW PATTERNS AT THE 10,000-FOOT LEVEL.

Herbert G. Dorsey, Jr.
Glenn W. Brier

Introduction

Since the 10,000-foot chart has become generally accepted as a useful instrument in the preparation of both short-range and extended forecasts, research directed at improvement of the technique for constructing prognostic 10,000-foot charts represents a potential contribution toward better weather forecasts. One such work of research resulted in the method which has been tested in this investigation.

Previous work in the development of methods for forecasting 10,000-foot charts has been based largely on extrapolation technique, except for the contributions of Rossby(1). The method tested in this investigation involves a more physical basis than pure extrapolation since it applies the principle of conservation of vorticity to arbitrary initial streamline patterns. It represents Rossby's extension of the theoretical considerations involved in his simple harmonic wave formula in order to deal with the irregular flow patterns generally observed on the 10,000-foot chart.

The basic premise guiding the study of this method was a belief that the most objective evaluation of a new instrument in forecasting procedure is obtained by means of a complete statistical analysis. This analysis may make it easier to decide whether or not the method should be incorporated into any specific forecasting system.

Many forecasting procedures are difficult to evaluate statistically except through a verification of the completed forecast. In such cases it is almost impossible to differentiate between the contributions of the analytical and empirical factors, since the forecast or prognostic chart being verified represents a synthesis of many components. The contribution to be expected from the use of Rossby's trajectory method in constructing 10,000-foot prognostic charts is comparitively easy to determine objectively by verification of the winds which are forecast by computation.

This investigation was divided into two major phases, the first being concerned with obtaining a large number of trajectory-computed 10,000-foot wind forecasts, and the second being occupied with the statistical verification of these forecasts. It is the purpose of this

(1) The several contributions (including the method analyzed in this report) with examples of their application to synoptic meteorology, are conveniently summarized in Starr, V.P., "Basic Principles of Weather Forecasting," Harper and Brother, 1942

report to describe the work done in the preparation of a sufficient sample of data, to explain the breakdown and verification of these data, and to present and discuss the statistics obtained from the analysis.

1. Organizing the Investigation

This study of Rossby's proposed trajectory method was undertaken after a number of 10,000-foot prognostic charts, prepared at the University of Chicago on the basis of trajectory computations, were found to compare favorably with the observed conditions.

It was decided to test the usefulness of the method when applied to data currently received by forecasters in the United States. The choice of a summer period was imposed by the necessity of starting the program immediately. It appeared that while the normally weak flow patterns of summer would be a handicap, although most forecasting procedures are at a similar disadvantage in poorly defined situations, any favorable features still showing in Rossby's method would be quite impressive.

During the period covered by this investigation, June 30 to July 23, 1943, The University of Chicago contributed the services of a man well versed in the theory and practice of the method, who gave instruction and assisted in the trajectory computations and preparation of prognostic charts.

In the original program, which called for the construction of prognostic charts on the basis of the computed wind forecasts, it was also proposed to study the problem of obtaining the interrelationship between 10,000-foot charts and sea-level weather analyses. A similar treatment on the above plan was outlined concerning the application of this method to five-day mean charts. However, this report deals solely with the daily forecasts of 10,000-foot winds and the statistical analysis of these forecasts.

In preliminary phases of this investigation, after Rossby indicated the potentialities of the trajectory method, it appeared that a statistical analysis of the 10,000-foot wind forecasts would be desirable in order to form definite criteria governing their use in constructing prognostic charts. Actual experience in preparing prognostic charts demonstrated the need for such criteria and showed that it would be unsatisfactory to evaluate the trajectory method through verification of the prognostic ten thousand foot charts, in view of the indefinite elements of continuity, qualitative physical reasoning, and subjectivity included in their preparation.

When the statistical analysis of the computed wind forecasts for the July period was completed, considerable work had been done in all divisions of the original program, but it seemed best to report on the statistics at hand since they show the amount of purely objective information available through use of Rossby's trajectory method in constructing 10,000-foot prognostic charts. From these statistics forecasters may decide whether the method should be incorporated into their technique and may formulate certain criteria governing its use.

2. The Ten Thousand Foot Wind Forecasts June 30 - July 23, 1942

These forecasts were made daily during the above period on the basis of the 2300 E.S.T. 10,000 foot charts prepared in the Five-Day Forecast Section, United States Weather Bureau. The mechanics of the procedure from the selection of points for computation to the completed wind forecast are described in Reference 1 (pages 274-70). On the average about 10 trajectories were computed daily and extended seven days. Usually around three of the points selected fell on the axes of well-defined currents and were expected to verify fairly well, since they met the requirements of the theoretical assumptions. The balance of the points were more doubtful or experimental in nature due to their location in areas of scanty data or in poorly defined currents south of the main westerlies. Much effort was thus spent computing data of questionable value and extrapolating it to an excessive length of time, but it was done in the attempt to obtain a maximum of wind forecasts since these data were to furnish the basis for the statistical analysis.

In considering the state of the general circulation, as observed at the 10,000 foot level during the period studied, it should be noted that it is normal in summer to find the principal belt of westerlies displaced north of the aerological network over the United States. However, at the start of this test period deep westerly currents extended south of the Canadian border and there was a predominance of well-defined flow patterns for the first two weeks. A marked shift northward and trend to poor patterns was noted after July 12. The following week was essentially one of transition in which good patterns appeared on several days. The final week found the circulation definitely subnormal over the United States with meridional flow prevailing.

While the 10,000 foot wind forecasts were being computed from day to day and the individual charts were being carefully studied, it appeared that the extent to which the theoretical assumptions were met (with regard to optimum conditions affecting the trajectories) should provide a logical grouping of the forecasts. Thus in connection with the changing states noted in the general circulation during the test period, the charts were listed as having either well-defined patterns or poor patterns. Similarly, but without regard for the chart classification, when the individual points were taken on the axes of deep currents they were listed as good points--all others being considered poor points. Consequently the trajectory computations for good points on charts having well-defined (good) patterns represent data obtained insofar as possible under conditions imposed by the theoretical assumptions, and a statistical analysis of these data should enable one to evaluate the trajectory method.

The wind forecasts were also grouped according to the position of the computation point in respect to the flow pattern. Points chosen where the flow had maximum curvature were called curvature points and points chosen at minimum curvature were called inflection points. The effect of the initial speed of the trajectory on the verification was studied by dividing the points into two classes. Fast was taken as Beaufort 6 or greater and slow as Beaufort 5 or less. It was also decided to investigate the relationship between the verification and the initial angle ϕ that the trajectory made with the latitude circle. The effect of the geographic location of the computation point was studied by considering the verification as a function of the latitude and longitude of the point. Finally, for wind forecasts beyond twenty-four hours it was possible to investigate the verification as affected by the verification on the previous day, since preliminary investigations had suggested that if a particular trajectory verified well at twenty-four hours it had a better chance to verify at subsequent periods.

The various criteria mentioned above are not independent and this fact should be kept in mind when interpreting the statistics. For example, the results found when comparing good points against poor points will be associated with the comparison of fast points against slow points since it was found that a predominance of fast points were called good. Altho it was impractical to estimate the independent effect of these various quantities on the verification, in most cases where the interaction of these factors with the verification was investigated the Chi-test gave non-significant values.

3. The Statistical Analysis

The agreement of forecast with observed values was measured by two statistics. The first of these is a skill score which is measured by calculating the number of correct forecasts as a percentage on the range between the number right by chance and perfect forecasting in which all are correct. The other measure used to evaluate an individual forecast is the absolute error, computed by taking the difference between the forecast and observed values without regard to sign. Various tests of significance were used in the analysis of the data but only the final results and a statement regarding the statistical significance are discussed here. When the formal test of significance gave a value of the probability greater than .05 the term non-significant is used. A probability between .05 and .01 is referred to as significant and a probability of .01 and less is called highly significant.

A. The Verification of Wind Direction

The procedure used in the verification of wind direction is illustrated by the tabulations shown in table 1. The score is 43 and the average error is 2.82 on the 32 point wind scale. The excess of observed number right over expected number right is highly significant.

Table 1. Wind Direction Verification For One Day Forecasts All Points

Forecast Direction

Observed Direction	12	14	16	18	20	22	24	26	28	30	32	Total
2				1				2		1		4
4										1		1
6	1										1	2
8												
10												
12									1			1
14			1									1
16	2		2	1					1			6
18				2	1	2						5
20			2	2	10	12	3		1		1	31
22			1	3	6	8	5	2		1		26
24				1	5	8	4	7	3			28
26					2	3	5	19	5	1		35
28			2		1	2	4	6	5			20
30					1		1	2	4			8
32						1		2				3
Total	3		8	10	26	36	22	40	20	4	2	171

Table 2 gives a summary of the verification for all points for projections of one, two, and three days.

Table 2. Wind Direction Verification Summary for All Points.

Projection Days	Score Percent	Average Error Points	Number of Observations
1	43	2.82	171
2	26	2.95	161
3	26	3.95	129

As has been mentioned above, one grouping of the data was a division into charts having well defined patterns and those having poor patterns. Considering days one and two under this classification gives a score of 45 for good charts and 20 for poor charts, the difference being highly significant. When a classification was made (good or poor) of individual points, regardless of the type of chart on which the point appeared it was found that good points verified better than poor points, the difference being highly significant. Table 3 gives the scores for good and poor points and maps when the first two day projections are combined.

Table 3. Wind Direction Verification Scores According to Classification of Map and Point - First Two Days Combined.

		Charts: Good	Charts: Poor
Points	Good	55	48
	Poor	37	6

Since there seemed to be a definite tendency for the good points to verify better than the poor points, verification for the fifth and sixth days was carried out for the good points only. The results are summarized in Table 4.

Table 4. Wind Direction Verification for Good Points Only

Projection	Good Charts Score	Poor Charts Score	Good and Poor Charts Score	Good and Poor Charts Average Error	Number of Observations
Days	Percent	Percent	Percent	Points	
1	39	59	49	2.00	58
2	71	29	49	1.85	52
3	30	38	47	2.53	49
4	-12	36	10	3.40	40
5	2	-14	-4	3.94	31
6	21	-48	-12	4.27	30

Table 5 summarizes the data for the first three days according to the position of the original point in the flow pattern. No significant difference was found between curvature and inflection points in the verification of wind direction.

Table 5. Wind Direction Verification Scores According to Position of Computation Point in Flow Pattern

Projection	Pattern: Curvature	Pattern: Inflection
Days 1 & 2	36	34
Day 3	17	26

When projections of one and two days were combined, points having an initial speed of Beaufort 6 or greater verified with an average score of 39 and those with slower speeds had an average score of 33. This difference is not statistically significant.

The effect of the initial angle ϕ that the trajectory made with the latitude circle was studied by computing the correlation coefficient between the angle ϕ and the subsequent error in direction. The correlations within the various groups are presented in Table 6. They indicate a definite tendency for trajectories having small angles with the latitude circle to verify better than those having large angles.

Table 6. Correlation Coefficients Between Initial Angle And Subsequent Error in Direction.

Projection Days	Curvature Points	Inflection Points
1	.44*	.23*
2	.53**	.05
3	.50**	.36**

* Significant
** Highly Significant

In order to study the effect of location of the computation point on the subsequent verification, correlation coefficients were computed between the errors and the longitude and latitude of the point. The correlations for the various groups are presented in table 7.

Table 7. Correlation of Errors in Direction With Latitude And Longitude of the Computation Point.

	Projection In Days	Curvature Points		Inflection Points	
		Fast	Slow	Fast	Slow
Latitude	1	.08	.26	-.13	.12
	2	.27	.11	.16	.02
	3	.76*	-.41	-.11	.15
Longitude	1	.55**	-.25	.19	.09
	2	.74*	-.35	.35*	.14
	3	-.17	.43	.20	.32*

* Significant
** Highly Significant

When all points for the second day were combined the correlation between errors and longitude was .25, a significant value. The regression equation is.

$$E_2 = 0.43 + .025\ L$$

where E_2 is the error in forecast direction on a 32-point scale, and L is the longitude measured in degrees. This equation indicates that, in the area studied, the second day error in direction was diminished by 2.5 points for every 100° eastward that the initial point was chosen.

The relationship between verification on one day and subsequent verification was investigated by computing the correlation coefficients between errors on first and second days, and between second and third days. The following regression equations were derived:

$$E_2 = 2.21 + 0.26\, E_1, \qquad r = .28^{**}, \qquad (1)$$

$$E_3 = 2.85 + 0.40\, E_2, \qquad r = .26^{**}, \qquad (2)$$

where E_i is the error in direction on the i-th day.

It may be noted from equation (1) that a first day error of zero indicates an average second day error of 2.21, a value less than the average error for a one day forecast, viz. 2.82. This indicates that a two day forecast which has zero error on the first day is, on the average, more reliable than a one day forecast. The same reasoning can be applied to equation (2).

B. The Verification of Wind Speed

The extent of the agreement of forecast with observed speed is summarized in Table 8. The tendency for curvature points to verify better than inflection points was significant, but there was no significant difference between good and poor points.

Table 8. Verification of Wind Speed

Projection	Inflection		Curvature		Good	Poor	Total	
	Score	Average Error	Score	Average Error	Average Error	Average Error	Score	Average Error
Days	Percent	Points	Percent	Points	Points	Points	Percent	Points
1	18	1.15	27	.54	0.88	1.08	20	1.01
2	0	1.10	16	.85	1.13	0.97	4	1.04
3	4	1.11	-13	.99	0.98	1.13	-7	1.08

There was a slight tendency, not significant, for errors on the second and third days to be greater when errors on the preceding days were greater. This is shown in Table 9

Table 9. Verification of Wind Speed as Related to Errors on Preceding Days.

Verification Day 2

	Inflection Points		Curvature Points	
	Score	Av. error	Score	Av. error
Day 1 error = 0	2	1.03	29	0.62
Day 1 error > 0	0	1.14	5	1.06
		Verification Day 3		
Day 2 error = 0	21	0.85	11	0.88
Day 2 error > 0	-12	1.21	-15	1.00

When errors in speed were correlated with the initial angle Ø no significant correlations were found. For the first three days, an analysis of variance of the errors was made to determine the effect of geographic location of the point on subsequent verification. The only significant relations found were between latitudes for inflection point first day errors and between latitudes for third day errors. There was a very slight tendency for points between 10° - 19° North and 40° - 49° North to have slightly larger errors.

C. Verification of Position in Flow Pattern

As well as a forecast of direction and speed, the method under consideration also enables one to forecast whether a particle is associated with a ridge (⌒), a trough (◡), or in ascending (/) or descending (\) position in the neighbor-hood of an inflection point. Table 10 illustrates how the data were tabulated in this particular investigation. The association between the forecast and observed position was highly significant.

Table 10. Verification of Position in Flow Pattern
Day 1 - Forecast Direction (22-26)

Forecast Position (from trajectory path)

Observed Position (from Isobaric Analysis)		∪	⟋	∩	⟍	Total Number
	∪	33	1	6		40
	⟋	2		8		10
	∩	10		20	1	31
	⟍	2		4		6
		47	1	38	1	87

Scores for the first three days are given in table 11.

Table 11. Verification Scores for Position of Wind
In Flow Pattern

Projection	Forecast Direction		
Days	(22-25)	(26-32)	(12-20)
1	34%	28%	29%
2	18	7	13
3	9	7	4

4. Interpretation of Statistical Results

In concluding this report, the following discussion based on some of the statistical results should indicate how the trajectory method may be useful in forecasting flow patterns at 10,000 feet.

The verification of wind direction forecasts for the first three days appears sufficiently favorable to warrant the use of such forecasts as an aid in the preparation of prognostic charts. It would not be difficult to perform an experiment designed to estimate the relative contribution of this method when used to supplement or replace other methods now in use.

Verifying all forecasts of wind speed gave rather poor results, which were only improved slightly and not significantly after a breakdown into good and poor points. This could raise a question as to the validity of the direction forecasts derived from assumed constant speeds, but has been considered another indication that the trajectory computation should only be applied to particles well within the main westerly currents where approximately constant wind speeds may be expected.

Other results indicating the importance of selecting points on the axes of well-defined currents were those which showed that better forecasts are obtained with moderate and small initial trajectory angles and with fast wind speeds.

The fact that errors in direction are reduced systematically through progressive eastward selection of initial points suggests the influence of large-scale orographic and thermal effects arising from the geographic location of the Rocky Mountain chain near the western limit of the area studied.

Since it was shown that a trajectory verifying correctly at 24 hours has a better chance of being right at subsequent periods than a trajectory starting off poorly, the use of forecasts checked at 24 and 48 hours to supplement current computations is an important phase of the method which should not be neglected.

In brief, for prognostic charts up to 72 hours: extrapolate trajectories out to 5 days to obtain benefit of 24 and 48 hour checks, select points well within the main westerly currents, avoid points where the westerlies are within permanent or semi-permanent areas affected by topographical or thermal influences such as mountain ranges or subtropical anticyclonic cells, and consider the wind forecasts as supplemental data to be used as an additional tool along with other tested forecasting procedures.